Equations

An equation is a mathematical sentence stating that two quantities are equal. For example:

$$2x = 10$$
$$x + 5 = 8$$

The idea is to find a replacement for the unknown that will make the sentence true. That's called *solving* the equation. Thus, in the first example, $x = 5$ because $2 \times 5 = 10$. In the second example, $x = 3$ because $3 + 5 = 8$.

Sometimes you can solve an equation by inspection, as with the above examples. Other equations may be more complicated and require a step-by-step solution, for example:

$$\frac{n}{4} + 1 = 3$$

The general approach is to consider an equation like a balance scale, with both sides equally balanced. Essentially, whatever you do to one side, you must also do to the other side to maintain the balance. Thus, if you were to add 2 to the left side, you would also have to add 2 to the right side.

Let's apply this *balance* concept to our complicated equation above. Remembering that if we want to solve it for *n*, we must somehow rearrange it so the *n* is isolated on one side of the equation. Its value will then be on the other side. Looking at the equation, you can see that *n* has been increased by 2, then divided by 4, and ultimately added to 1. Therefore, we will undo these operations to isolate *n*.

Begin by subtracting 1 from both sides of the equation:

$$\begin{array}{rcl} \frac{n+2}{4} + 1 & = & 3 \\ -1 & & -1 \\ \hline \frac{n+2}{4} & = & 2 \end{array}$$

Next, multiply both sides by 4:

$$\begin{array}{rcl} 4 \times \frac{n+2}{4} & = & 2 \times 4 \\ n + 2 & = & 8 \end{array}$$

Finally, subtract 2 from both sides:

$$\begin{array}{rcl} -2 & & -2 \\ \hline \end{array}$$

This isolates *n* and solves the equation:

$$n = 6$$

Notice that each operation in the original equation was undone by using the inverse operation. That is, addition was undone by subtraction, and division was undone by multiplication. In general, each operation can be undone by its *inverse:*

ALGEBRAIC INVERSES	
Operation	**Inverse**
Addition	Subtraction
Subtraction	Addition
Multiplication	Division
Division	Multiplication

After you solve an equation, check your work by plugging the answer back into the original equation to make sure it balances. Let's see what happens when we plug 6 in for *n:*

$$\begin{aligned} \tfrac{6+2}{4} + 1 &= 3 \\ \tfrac{8}{4} + 1 &= 3 \\ 2 + 1 &= 3 \\ 3 &= 3 \end{aligned}$$

Solve each equation for x:

90. $x + 5 = 12$

91. $27 = -13 + 4x$

92. $\frac{1}{4}x = 7$