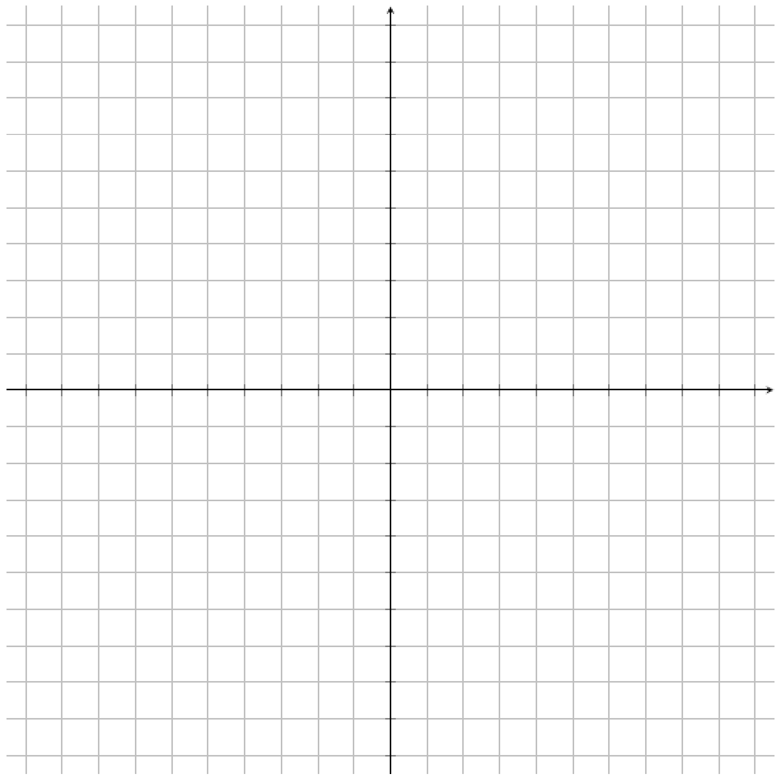
1. (*10 points*) Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is oriented.

a. (*5 pts*) $x = t^2 - 2t,\ y = t + 1,\ 0 \le t \le 4$



Answers

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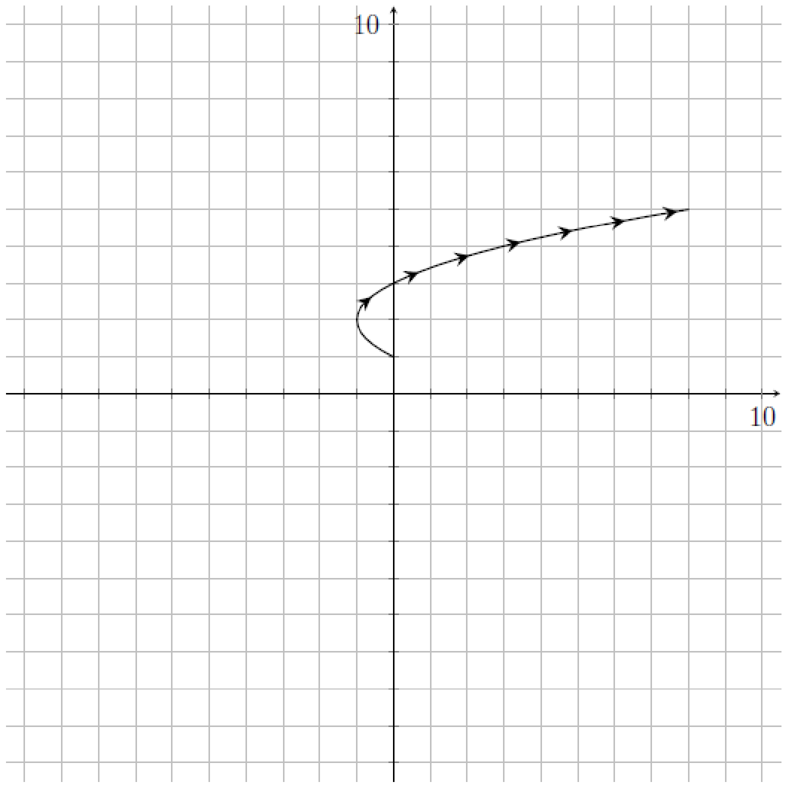
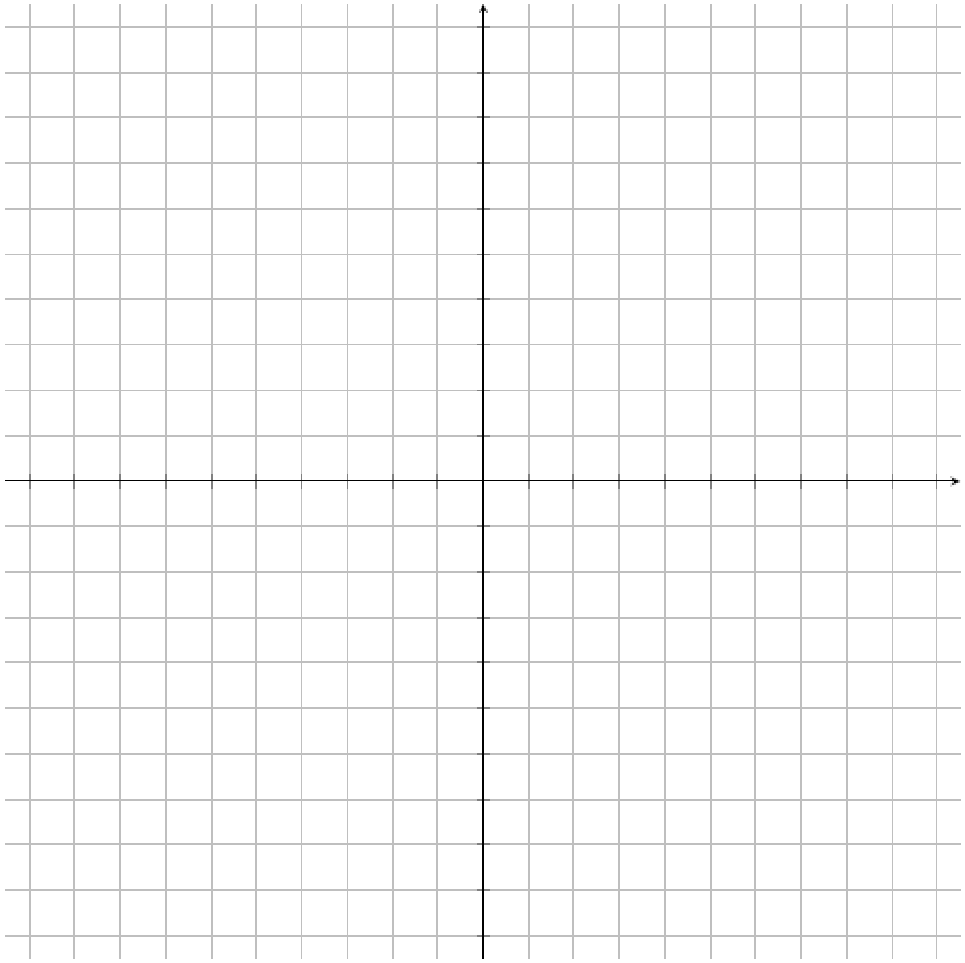
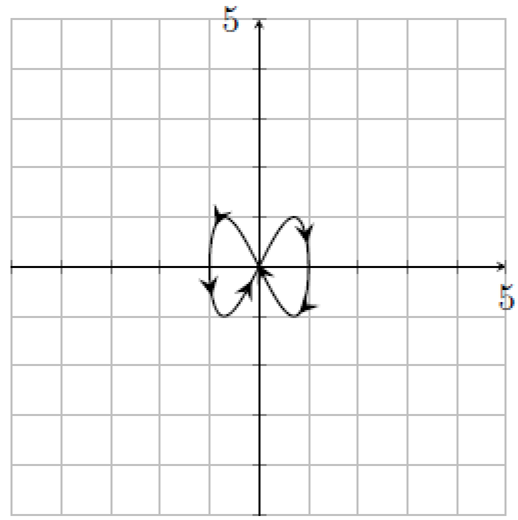


b. $(5\ pts)$ $x = \sin 2t,\ y = \sin 4t$



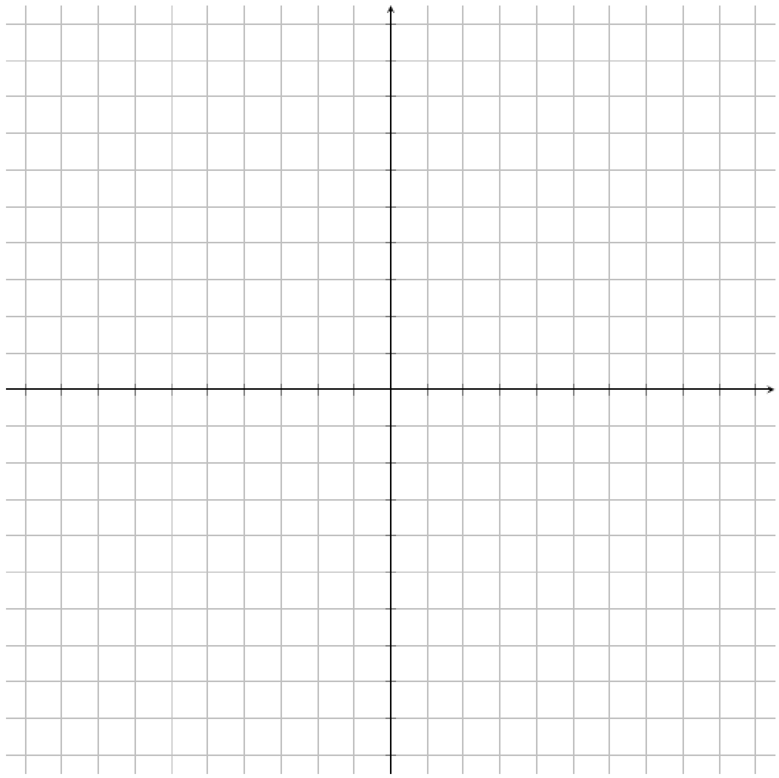
Answers

b. (*5 pts*) $x = \sin 2t,\ y = \sin 4t$



2. (*20 points*) Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is oriented. Then, eliminate the parameter to find a Cartesian equation of the curve.

a. (*10 pts*) $x = 3t + 2,\ y = 2t + 3$



Answers

2. (*20 points*) Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is oriented. Then, eliminate the parameter to find a Cartesian equation of the curve.

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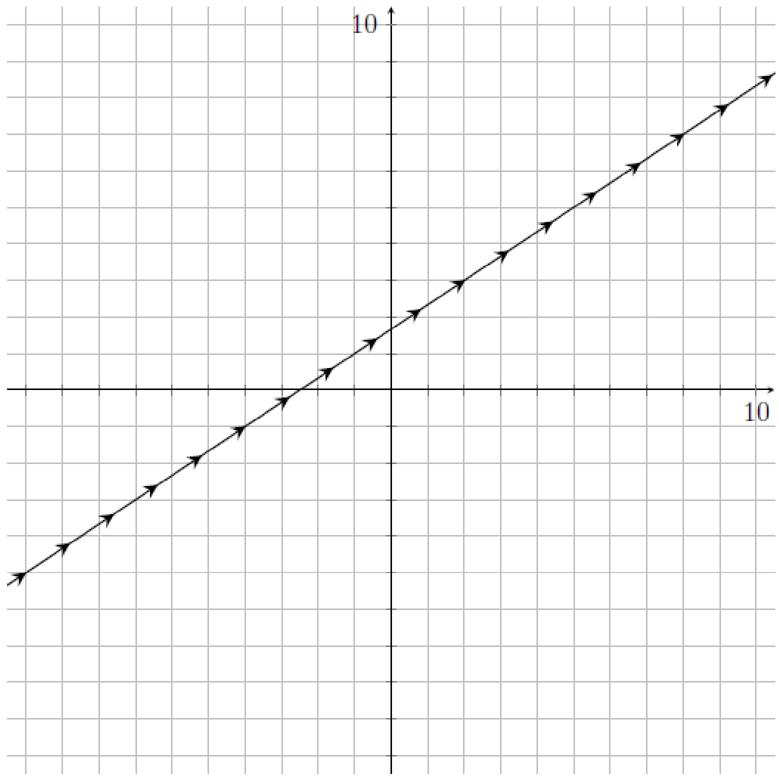
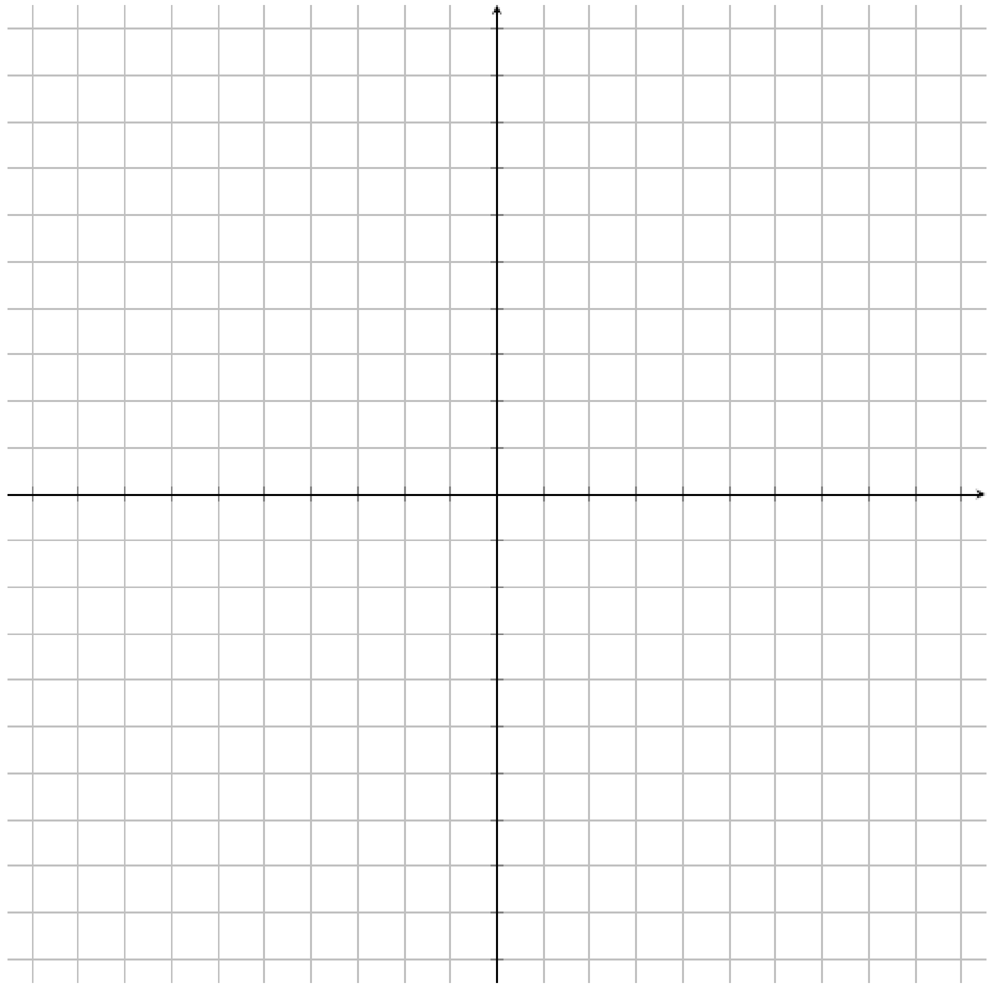


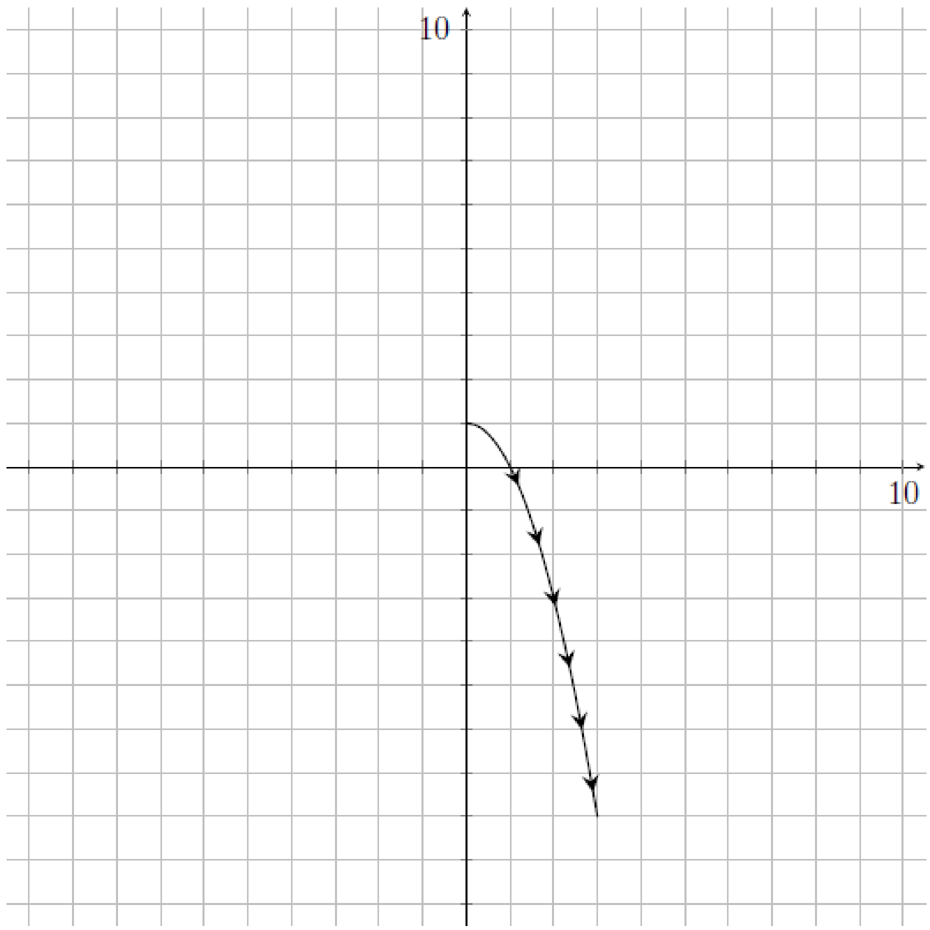
$y = \frac{2}{3}x + \frac{5}{3}$

b. (*10 pts*) $x = \sqrt{t},\ y = 1 - t,\ 0 \le t \le 9$



Answers

b. (*10 pts*) $x = \sqrt{t},\ y = 1 - t,\ 0 \le t \le 9$



$y = 1 - x^2$

3. (*12 points*) Eliminate the parameter to find a Cartesian equation of the curve.

a. (*4 pts*) $x = \frac{1}{2}\sin\theta,\ y = \frac{1}{2}\cos\theta,\ 0 \leq \theta \leq \pi$

b. (*4 pts*) $x = \sin t,\ y = \csc t,\ 0 \leq t \leq \dfrac{\pi}{2}$

c. (*4 pts*) $x = t^2,\ y = \ln t$

d. (*4 pts*) $x = \tan^2\theta,\ y = \sec\theta,\ -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

Answers

3. (*12 points*) Eliminate the parameter to find a Cartesian equation of the curve.

a. (*4 pts*) $x = \frac{1}{2}\sin\theta,\ y = \frac{1}{2}\cos\theta,\ 0 \le \theta \le \pi$

$A:\ x^2 + y^2 = \dfrac{1}{4}$

b. (*4 pts*) $x = \sin t,\ y = \csc t,\ 0 \le t \le \dfrac{\pi}{2}$

$A:\ y = \dfrac{1}{x}$

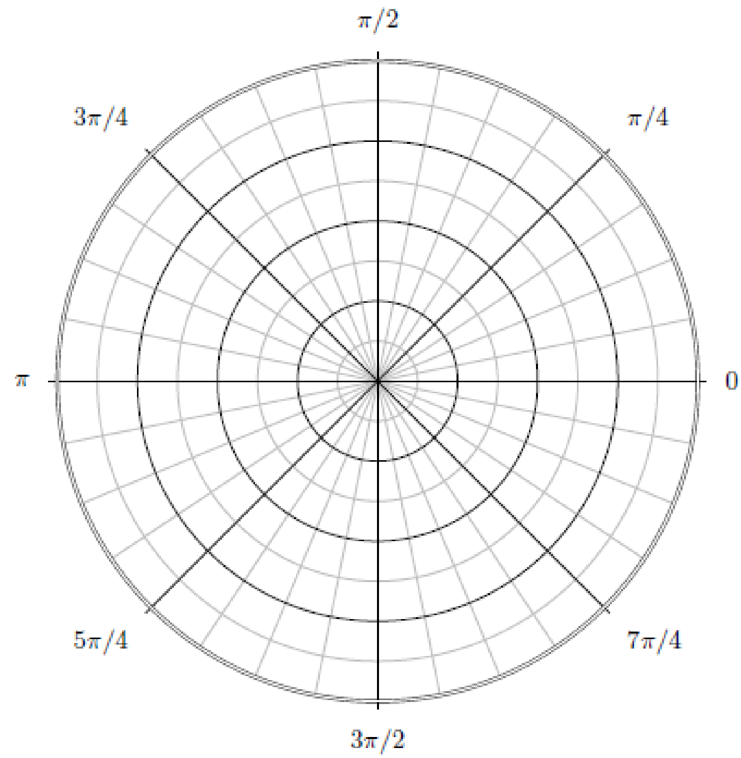
c. (*4 pts*) $x = t^2,\ y = \ln t$

$A:\ x = e^{2y}$

d. (*4 pts*) $x = \tan^2\theta,\ y = \sec\theta,\ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

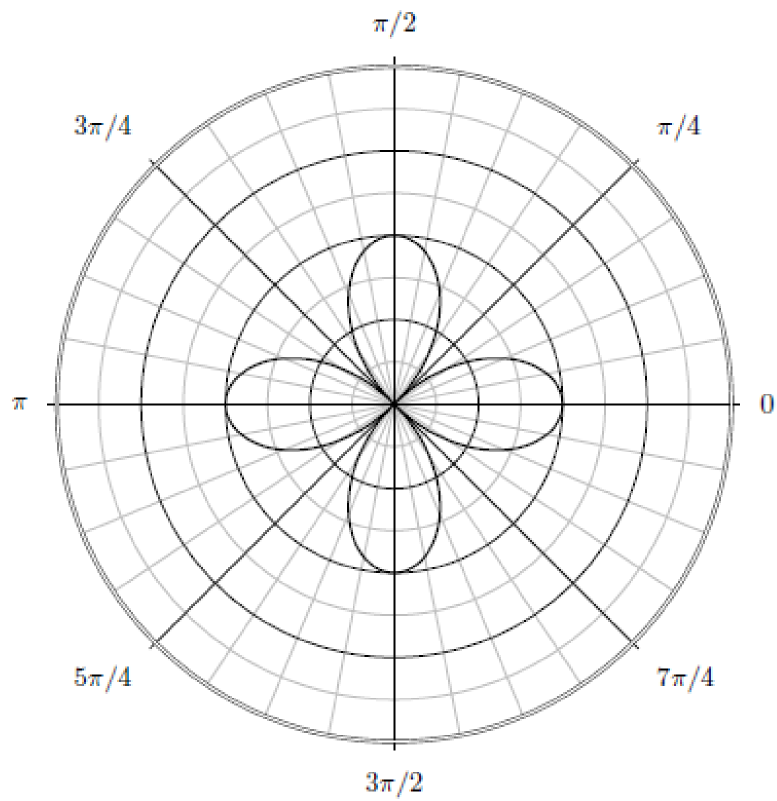
$A:\ x = y^2 - 1$

4. (*5 points*) Graph the planar curve $r = 2\cos 2\theta$.

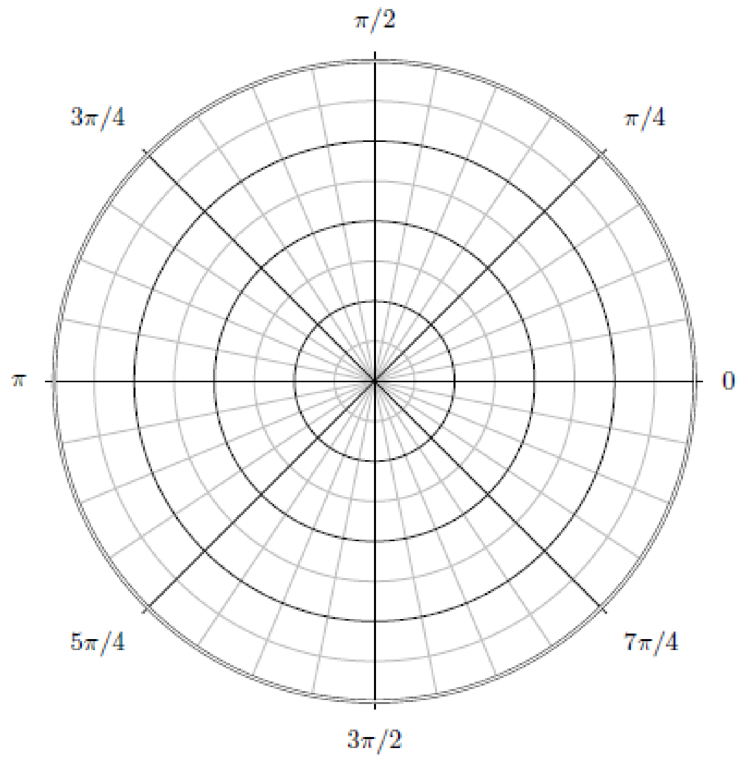


Answers

4. (*5 points*) Graph the planar curve $r = 2\cos 2\theta$.

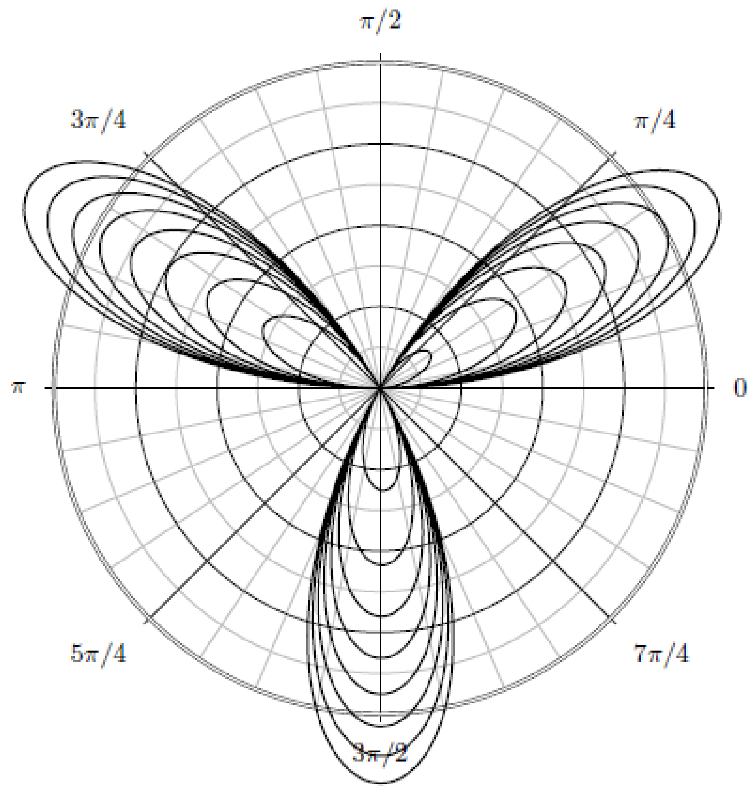


5. (*5 points*) Graph the planar curve $r = \sqrt{\theta}\sin 3\theta$.



Answers

5. (*5 points*) Graph the planar curve $r = \sqrt{\theta}\sin 3\theta$.



6. (*16 points*) Identify the curve by finding a Cartesian equation for the curve.

a. (*4 pts*) $r = 5\cos\theta$

b. (*4 pts*) $r = 2\csc\theta$

c. (*4 pts*) $\theta = \dfrac{\pi}{3}$

d. (*4 pts*) $r^2 \sin 2\theta = 1$

Answers

6. (*16 points*) Identify the curve by finding a Cartesian equation for the curve.

a. (*4 pts*) $r = 5\cos\theta$

$A:\ x^2 + y^2 = 5x$

b. (*4 pts*) $r = 2\csc\theta$

$A:\ y = 2$

c. (*4 pts*) $\theta = \frac{\pi}{3}$

$A:\ y = \sqrt{3}x$

d. (*4 pts*) $r^2 \sin 2\theta = 1$

$A:\ y = \frac{1}{2x}$

7. (*12 points*) Find a polar equation for the curve represented by the given Cartesian equation.

a. (*4 pts*) $y = 2$

b. (*4 pts*) $4y^2 = x$

c. (*4 pts*) $x^2 - y^2 = 4$

Answers

7. (*12 points*) Find a polar equation for the curve represented by the given Cartesian equation.

a. (*4 pts*) $y = 2$

$r = 2\csc\theta$

b. (*4 pts*) $4y^2 = x$

$r = \frac{1}{4}\cot\theta\csc\theta$

c. (*4 pts*) $x^2 - y^2 = 4$

$r = 2\sqrt{\sec 2\theta}$

8. (*8 points*) Find $\frac{dy}{dx}$ by 1) not eliminating the parameter

and

2) by first eliminating the parameter.

$$x = 1 + \sqrt{t},\ y = e^{t^2}$$

Answers

8. (*8 points*) Find $\frac{dy}{dx}$ by 1) not eliminating the parameter and 2) by first eliminating the parameter.

$$x = 1 + \sqrt{t},\ y = e^{t^2}$$

$A:$ 1) $\frac{dy}{dx} = 4t^{3/2}e^{t^2}$, 2) $\frac{dy}{dx} = 4e^{(x-1)^4}(x-1)^3$

9. (*8 points*) Find an equation of the tangent line to the curve at the point corresponding to the given value of the parameter.

a. (*4 pts*) $x = \sqrt{t},\ y = t^2 - 2t,\ t = 4$

b. (*4 pts*) $x = e^t \sin \pi t,\ y = e^{2t},\ t = 0$

Answers

9. (*8 points*) Find an equation of the tangent line to the curve at the point corresponding to the given value of the parameter.

a. (*4 pts*) $x = \sqrt{t},\ y = t^2 - 2t,\ t = 4$

$A:\ y = 24x - 40$

b. (*4 pts*) $x = e^t \sin \pi t,\ y = e^{2t},\ t = 0$

$A:\ y = \dfrac{2}{\pi}x + 1$

10. (*8 points*) Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$.

a. (*4 pts*) $x = t^2 + 1,\ y = e^t - 1$

b. (*4 pts*) $x = \cos t,\ y = \sin 2t$

Answers

10. (*8 points*) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

a. (*4 pts*) $x = t^2 + 1,\ y = e^t - 1$

$$A:\ \frac{dy}{dx} = \frac{e^t}{2t},\ \frac{d^2y}{dx^2} = \frac{e^t(t-1)}{4t^3}$$

b. (*4 pts*) $x = \cos t,\ y = \sin 2t$

$$A:\ \frac{dy}{dx} = \frac{2\cos 2t}{-\sin t},\ \frac{d^2y}{dx^2} = \frac{4\sin t \sin 2t + 2\cos t \cos 2t}{-\sin^3 t}$$

11. (*12 points*) Find the slope of the tangent line to the given polar curve at the point specified by the value of θ.

a. (*4 pts*) $r = 2 + \sin 3\theta,\ \theta = \pi/4$

b. (*4 pts*) $r = \cos(\theta/3), \theta = \pi$

c. (*4 pts*) $r = 1 + 2\cos\theta,\ \theta = \pi/3$

Answers

11. (*12 points*) Find the slope of the tangent line to the given polar curve at the point specified by the value of θ.

a. (*4 pts*) $r = 2 + \sin 3\theta,\ \theta = \pi/4$

$A:\ \dfrac{-1+\sqrt{2}}{-2-\sqrt{2}}$

b. (*4 pts*) $r = \cos(\theta/3), \theta = \pi$

$A:\ -\sqrt{3}$

c. (*4 pts*) $r = 1 + 2\cos\theta,\ \theta = \pi/3$

$A:\ \dfrac{\sqrt{3}}{9}$

12. (*5 points*) Find the arc length of the curve on the given interval.

$$x = 6t^2,\ y = 2t^3,\ 1 \leq t \leq 4$$

Answers

12. (*5 points*) Find the arc length of the curve on the given interval.

$$x = 6t^2,\ y = 2t^3,\ 1 \leq t \leq 4$$

$A:\ 70\sqrt{5}$

13. (*5 points*) Find the area of the surface generated by revolving the curve about the given axis.

$$x = a\cos\theta,\ y = a\sin\theta,\ 0 \le \theta \le \frac{\pi}{2} \text{ about the } x\text{-axis}$$

Answers

13. (*5 points*) Find the area of the surface generated by revolving the curve about the given axis.

$$x = a\cos\theta,\ y = a\sin\theta,\ 0 \le \theta \le \frac{\pi}{2} \text{ about the } x\text{-axis}$$

$A:\ S = 2\pi a^2$

14. (*5 points*) Find the area between the loops of $r = 3 - 6\sin\theta$.

Answers

14. (*5 points*) Find the area between the loops of $r = 3 - 6\sin\theta$.

$A:\ 9\pi + 27\sqrt{3}$

15. (*5 points*) Find the slope of the tangent line to the polar curve

$$r = 2\cos\theta + 3\sin\theta \text{ where } \theta = \pi/4.$$

Answers

15. (*5 points*) Find the slope of the tangent line to the polar curve

$$r = 2\cos\theta + 3\sin\theta \text{ where } \theta = \pi/4.$$

$A:\ -\dfrac{3}{2}$

16. (*5 points*) Find the arc length of the curve over the given interval.

$$r = 2a\cos\theta,\ -\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$$

Answers

16. (*5 points*) Find the arc length of the curve over the given interval.

$$r = 2a\cos\theta,\ -\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$$

$A:\ a\pi$

17. (*5 points*) Find the area of the surface formed by revolving the polar equation about the given line.

$$r = 12\sin\theta,\ 0 \leq \theta \leq \frac{\pi}{2},\ \theta = \frac{\pi}{2}$$

Answers

17. (*5 points*) Find the area of the surface formed by revolving the polar equation about the given line.

$$r = 12\sin\theta,\ 0 \le \theta \le \frac{\pi}{2},\ \theta = \frac{\pi}{2}$$

$A:\ 144\pi$