1. (*18 points*) Give the first six terms for the following sequences.

a. (*6 pts*) $a_n = \dfrac{(-1)^{n-1}n^2}{n!}$

b. (*6 pts*) $a_n = \dfrac{n}{2^n}$

c. (*6 pts*) $a_n = n \sin \dfrac{n\pi}{6}$

Answers

1. (*18 points*) Give the first six terms for the following sequences.

a. (*6 pts*) $\left\{1, -2, \frac{3}{2}, -\frac{2}{3}, \frac{5}{24}, -\frac{1}{20}\right\}$

b. (*6 pts*) $\left\{\frac{1}{2}, \frac{1}{2}, \frac{3}{8}, \frac{1}{4}, \frac{5}{32}, \frac{3}{32}\right\}$

c. (*6 pts*) $\left\{\frac{1}{2}, \sqrt{3}, 3, 2\sqrt{3}, \frac{5}{2}, 0\right\}$

2. (*9 points*) Give the general term a_n for the following sequences, assuming the pattern holds.

a. (*3 pts*) $\left\{-\frac{1}{3}, \frac{1}{2}, -\frac{3}{5}, \frac{2}{3}, -\frac{5}{7}, \cdots\right\}$

b. (*3 pts*) $\left\{\frac{1}{3}, \frac{4}{5}, \frac{9}{7}, \frac{16}{9}, \frac{25}{11}, \cdots\right\}$

c. (*3 pts*) $\{0, 1, 0, -1, 0, 1, 0, -1, 0, \cdots\}$

Answers

2. (*9 points*)

Give the general term a_n for the following sequences, assuming the pattern holds.

a. (*3 pts*) $a_n = \dfrac{(-1)^n n}{n+2}$

b. (*3 pts*) $a_n = \dfrac{n^2}{2n+1}$

c. (*3 pts*) $a_n = \sin\left(\dfrac{(n-1)\pi}{2}\right)$

3. *(20 points)*

State whether the sequence is convergent or divergent.

If it is convergent, find the limit.

a. *(5 pts)* $a_n = \dfrac{n^3 - 2}{2n^3 + n + 9}$

b. *(5 pts)* $a_n = \dfrac{2^n}{n^2 + n}$

c. *(5 pts)* $\{4n^3 e^{-n}\}$

d. *(5 pts)* $\{\ln(3n^4 + 1) - \ln(7n^4 - n^2 - 11)\};\ n \geq 2$

Answers

3. (*20 points*)

State whether the sequence is convergent or divergent.
If it is convergent, find the limit.

a. (*5 pts*) Convergent, $\frac{1}{2}$

b. (*5 pts*) Divergent

c. (*5 pts*) Convergent, 0

d. (*5 pts*) Convergent, $\ln \frac{3}{7}$

4. (*4 points*) Calculate the sum of the series $\sum_{n=1}^{\infty} a_n$

whose partial sums are $s_n = \dfrac{2n^3+7}{5n^3-1}$.

Answers

4. *(4 points)* $\frac{2}{5}$

5. (*8 points*) Is the geometric series convergent?

If it is, give the sum.

a. (*4 pts*) $2 - 5 + \frac{25}{2} - \frac{125}{4} + \cdots$

b. (*4 pts*) $\frac{1}{3} + \frac{1}{4} + \frac{3}{16} + \frac{9}{64} + \cdots$

Answers

5. (*8 points*) Is the geometric series convergent?
If it is, give the sum.

a. (*4 pts*) Divergent

b. (*4 pts*) Convergent, $\frac{4}{3}$

6. (*20 points*)

Confirm that the integral test can be applied to each series. Then use the integral test to determine if the series is convergent or divergent.

a. (*5 pts*) $\sum_{n=1}^{\infty} n^{-0.7}$

b. (*5 pts*) $\sum_{n=1}^{\infty} \frac{2}{5n-1}$

c. (*5 pts*) $1 + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} + \frac{1}{5\sqrt{5}} + \cdots$

d. (*5 pts*) $\sum_{n=2}^{\infty} \frac{\ln n}{n^2}$

Answers

6. *(20 points)*

Confirm that the integral test can be applied to each series. Then use the integral test to determine if the series is convergent or divergent.

a. *(5 pts)* Divergent

b. *(5 pts)* Divergent

c. *(5 pts)* Convergent

d. *(5 pts)* Convergent

7. (*20 points*) Determine if the p-series is convergent or divergent.

a. (*4 pts*) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}}$

b. (*4 pts*) $\displaystyle\sum_{n=1}^{\infty} \frac{4}{n^{8/7}}$

c. (*4 pts*) $\displaystyle\sum_{n=1}^{\infty} \frac{e}{n^{\pi}}$

d. (*4 pts*) $1 + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} + \frac{1}{5\sqrt{5}} + \cdots$

e. (*4 pts*) $1 + \frac{1}{\sqrt[4]{8}} + \frac{1}{\sqrt[4]{27}} + \frac{1}{\sqrt[4]{64}} + \frac{1}{\sqrt[4]{125}} + \cdots$

Answers

7. *(20 points)*

Determine if the p-series is convergent or divergent.

a. *(4 pts)* Divergent

b. *(4 pts)* Convergent

c. *(4 pts)* Convergent

d. *(4 pts)* Convergent

e. *(4 pts)* Divergent

8. (*20 points*) Test the series for convergence or divergence.

a. (*4 pts*) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n+1}$

b. (*4 pts*) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n}}{e^n}$

c. (*4 pts*) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{\ln(n+1)}$

d. (*4 pts*) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n} \cos n\pi$

e. (*4 pts*) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{1 \cdot 4 \cdot 7 \cdots (3n-2)}$

Answers

8. (*20 points*) Test the series for convergence or divergence.

a. (*4 pts*) Convergent

b. (*4 pts*) Convergent

c. (*4 pts*) Divergent

d. (*4 pts*) Convergent

e. (*4 pts*) Convergent

9. (*24 points*) Use the ratio test to determine convergence or divergence.

a. (*4 pts*) $\displaystyle\sum_{n=1}^{\infty} \frac{n^4}{4^n}$

b. (*4 pts*) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{e^n}$

c. (*4 pts*) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{(n+2)!}$

d. (*4 pts*) $\displaystyle\sum_{n=0}^{\infty} \frac{e^{n+1}}{n!}$

e. (*4 pts*) $\displaystyle\sum_{n=0}^{\infty} \frac{(n!)^2}{(4n)!}$

f. (*4 pts*) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 3^{n-1}}{n!}$

Answers

9. (*24 points*)

Use the ratio test to determine convergence or divergence.

a. (*4 pts*) Convergent

b. (*4 pts*) Convergent

c. (*4 pts*) Convergent

d. (*4 pts*) Convergent

e. (*4 pts*) Convergent

f. (*4 pts*) Convergent

10. (*20 points*) Use the root test to determine convergence or divergence.

a. (*4 pts*) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^n}$

b. (*4 pts*) $\displaystyle\sum_{n=1}^{\infty} (2\sqrt[n]{n} + 1)^n$

c. (*4 pts*) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{500}\right)^n$

d. (*4 pts*) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n^2}\right)^n$

e. (*4 pts*) $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^n}{(n^n)^2}$

Answers

10. *(20 points)*

Use the root test to determine convergence or divergence.

a. *(4 pts)* Convergent

b. *(4 pts)* Divergent

c. *(4 pts)* Divergent

d. *(4 pts)* Convergent

e. *(4 pts)* Divergent

11. *(8 points)*

Find the nth Taylor polynomial for the function, centered at c.

a. *(4 pts)* $f(x) = \frac{2}{x},\ n = 3,\ c = 1$

b. *(4 pts)* $f(x) = \sqrt{x},\ n = 2,\ c = 9$

Answers

11. (*8 points*)

Find the nth Taylor polynomial for the function, centered at c.

a. (*4 pts*) $2-2(x-1)+2(x-1)^2-2(x-1)^3$

b. (*4 pts*) $3+\frac{1}{6}(x-9)-\frac{1}{216}(x-9)^2$

12. *(8 points)*

Find the nth Maclaurin polynomial for the function.

a. *(4 pts)* $f(x) = xe^x,\ n = 4$

b. *(4 pts)* $f(x) = \ln(1 - x),\ n = 2$

Answers

12. (*8 points*) Find the nth Maclaurin polynomial for the function.

a. (*4 pts*) $x + x^2 + \frac{x^3}{2} + \frac{x^4}{6}$

b. (*4 pts*) $-x - \frac{1}{2}x^2$

13. *(8 points)*

Approximate the given quantity using a Taylor polynomial with 3 nonzero terms.

a. *(4 pts)* $e^{0.12}$

b. *(4 pts)* $\sqrt{101}$

Answers

13. *(8 points)*

Approximate the given quantity using a Taylor polynomial with 3 nonzero terms.

a. *(4 pts)* 1.1272

b. *(4 pts)* 10.04988

14. *(20 points)*

Find the radius of convergence and interval of convergence for the series.

a. *(5 pts)* $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{\sqrt[3]{n}}$

b. *(5 pts)* $\displaystyle\sum_{n=1}^{\infty} \frac{x^{2n}}{n!}$

c. *(5 pts)* $\displaystyle\sum_{n=1}^{\infty} (n+1)!(5x+3)^n$

Answers

14. *(20 points)*

Find the radius of convergence and interval of convergence for the series.

a. (*5 pts*) $1, \ (-1, 1]$

b. (*5 pts*) $\infty, \ (-\infty, \infty)$

c. (*5 pts*) $0, \ \left\{ -\frac{3}{5} \right\}$

15. (*15 points*)

Find a power series for the function, centered at 0, and determine the interval of convergence.

a. (*5 pts*) $\dfrac{1}{1-3x}$

b. (*5 pts*) $\dfrac{2x^3}{1-x}$

c. (*5 pts*) $\dfrac{4x^{12}}{1-2x}$

Answers

15. *(15 points)*

Find a power series for the function, centered at 0, and determine the interval of convergence.

a. *(5 pts)* $\sum_{n=0}^{\infty}(3x)^n, \ \left(-\frac{1}{3}, \frac{1}{3}\right)$

b. *(5 pts)* $\sum_{n=0}^{\infty} 2x^{n+3}, \ (-1, 1)$

c. *(5 pts)* $\sum_{n=0}^{\infty} 2^{n+2}x^{n+12}, \ \left(-\frac{1}{2}, \frac{1}{2}\right)$

16. *(12 points)*

Find the Taylor series, centered at c, for the given function.

a. *(4 pts)* $f(x) = \sin x,\ c = \dfrac{\pi}{2}$

b. *(4 pts)* $f(x) = \dfrac{1}{x},\ c = 1$

c. *(4 pts)* $f(x) = \ln x,\ c = 3$

Answers

16. (*12 points*) Find the Taylor series, centered at c, for the given function.

a. (*4 pts*) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}\left(x-\frac{\pi}{2}\right)^{2n}$

b. (*4 pts*) $\displaystyle\sum_{n=0}^{\infty}(-1)^n(x-1)^n$

c. (*4 pts*) $\displaystyle\ln 3+\sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{n3^n}(x-3)^{n+1}$

17. (*18 points*)

Find the Maclaurin series for the given function.

Then find the interval of convergence.

a. (*6 pts*) $f(x) = e^{-x}$

b. (*6 pts*) $f(x) = (1 + x^2)^{-1}$

c. (*6 pts*) $f(x) = \tan^{-1} x$

Answers

17. (*18 points*) Find the Maclaurin series for the given function.

Then find the interval of convergence.

a. (*6 pts*) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^n, \ (-\infty, \infty)$

b. (*6 pts*) $\displaystyle\sum_{n=0}^{\infty} (-1)^n x^{2n}, \ (-1, 1)$

c. (*6 pts*) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}, \ [-1, 1]$

18. (*12 points*)

Use the first 6 terms of the Maclaurin series to estimate the given integral.

a. (*6 pts*) $\int_{-1}^{1} e^{-x^3}\, dx$

b. (*6 pts*) $\int_{0}^{\pi/2} \sin(x^2)\, dx$

Answers

18. *(12 points)*

Use the first 6 terms of the Maclaurin series to estimate the given integral.

a. *(6 pts)* 2.149267

b. *(6 pts)* 0.8281