CHAPTER 2

Reasoning and Proof

Then

You used segment and angle relationships.

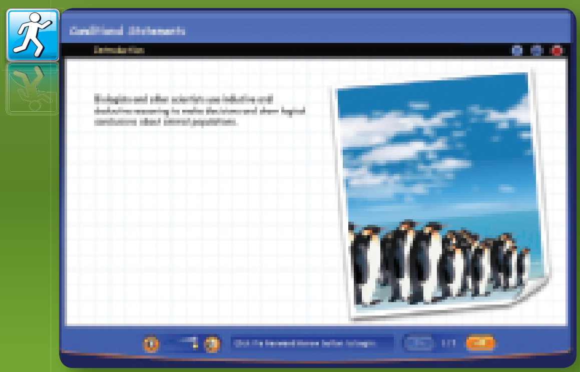
Now

In this chapter, you will:

- Make conjectures and find counterexamples for statements.
- Use deductive reasoning to reach valid conclusions.
- Write proofs involving segment and angle theorems.

Why?

SCIENCE AND NATURE Biologists and other scientists use inductive and deductive reasoning to make decisions and draw logical conclusions about animal populations.



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Get Ready for the Chapter

Diagnose Readiness | You have two options for checking prerequisite skills.

1 Textbook Option

Take the Quick Check below. Refer to the Quick Review for help.

QuickCheck

Evaluate each expression for the given value of x.

1. $4x + 7;\ x = 6$

2. $(x - 2)180;\ x = 8$

3. $5x^2 - 3x;\ x = 2$

4. $\frac{x(x - 3)}{2};\ x = 5$

5. $x + (x + 1) + (x + 2);\ x = 3$

Write each verbal expression as an algebraic expression.

6. eight less than five times a number

7. three more than the square of a number

Solve each equation.

8. $8x - 10 = 6x$

9. $18 + 7x = 10x + 39$

10. $3(11x - 7) = 13x + 25$

11. $3x + 8 = \frac{1}{2}x + 35$

12. $\frac{2}{3}x + 1 = 5 - 2x$

13. **CLOTHING** Nancy bought 4 shirts at the mall for \$52. Write and solve an equation to find the average cost of one shirt.

Refer to the figure in Example 3.

14. Identify a pair of vertical angles that appear to be obtuse.

15. Identify a pair of adjacent angles that appear to be complementary.

16. Identify a linear pair.

17. If $m\angle DXB = 116$ and $m\angle EXA = 3x + 2$, find x.

18. If $m\angle BXC = 90$, $m\angle CXD = 6x - 13$, and $m\angle DXE = 10x + 7$, find x.

QuickReview

Example 1

Evaluate $x^2 - 2x + 11$ for $x = 6$.

$x^2 - 2x + 11$	Original expression
$= (6)^2 - 2(6) + 11$	Substitute 6 for x.
$= 36 - 2(6) + 11$	Evaluate the exponent.
$= 36 - 12 + 11$	Multiply.
$= 35$	Simplify.

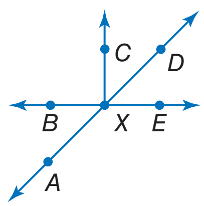
Example 2

Solve $36x - 14 = 16x + 58$.

$36x - 14 = 16x + 58$	Original equation
$36x - 14 - 16x = 16x + 58 - 16x$	Subtract $16x$ from each side.
$20x - 14 = 58$	Simplify.
$20x - 14 + 14 = 58 + 14$	Add 14 to each side.
$20x = 72$	Simplify.
$\frac{20x}{20} = \frac{72}{20}$	Divide each side by 20.
$x = 3.6$	Simplify.

Example 3

If $m\angle BXA = 3x + 5$ and $m\angle DXE = 56$, find x.



$m\angle BXA = m\angle DXE$	Vertical ∡ are ≅.
$3x + 5 = 56$	Substitution
$3x = 51$	Subtract 5 from each side.
$x = 17$	Divide each side by 3.

2 Online Option

Take an online self-check Chapter Readiness Quiz at connectED.mcgraw-hill.com.

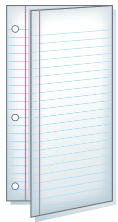
Get Started on the Chapter

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 2. To get ready, identify important terms and organize your resources. You may refer to Chapter 0 to review prerequisite skills.

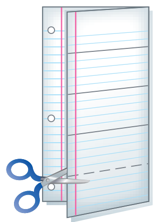
FOLDABLES® StudyOrganizer

Reasoning and Proof Make this Foldable to help you organize your Chapter 2 notes about logic, reasoning, and proof. Begin with one sheet of notebook paper.

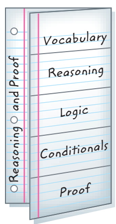
1 **Fold** lengthwise to the holes.



2 **Cut** five tabs in the top sheet.



3 **Label** the tabs as shown.



NewVocabulary

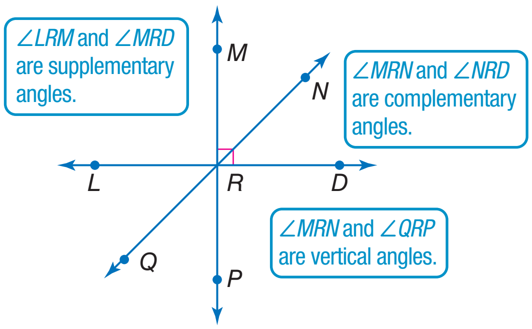
English		Español
inductive reasoning	p. 91	razonamiento inductivo
conjecture	p. 91	conjetura
counterexample	p. 94	contraejemplo
negation	p. 99	negación
if-then statement	p. 107	enunciado si-entonces
hypothesis	p. 107	hipótesis
conclusion	p. 107	conclusión
converse	p. 109	recíproco
inverse	p. 109	inverso
postulate	p. 127	postulado
proof	p. 128	demostración
theorem	p. 129	teorema

ReviewVocabulary

complementary angles ángulos complementarios two angles with measures that have a sum of 90

supplementary angles ángulos suplementarios two angles with measures that have a sum of 180

vertical angles ángulos opuestos por el vértice two nonadjacent angles formed by intersecting lines



LESSON 2-1

Inductive Reasoning and Conjecture

Then

- You used data to find patterns and make predictions.

Now

1. Make conjectures based on inductive reasoning.
2. Find counterexamples.

Why?

- Market research is conducted by an analyst to answer specific questions about products. For example, a company that creates video games might hire focus group testers to play an unreleased video game. The process of using patterns to analyze the effectiveness of a product involves inductive reasoning.

NewVocabulary

inductive reasoning
conjecture
counterexample

1 Make Conjectures

Inductive reasoning is reasoning that uses a number of specific examples to arrive at a conclusion. When you assume that an observed pattern will continue, you are applying inductive reasoning. A concluding statement reached using inductive reasoning is called a **conjecture**.

Example 1 Patterns and Conjecture

Write a conjecture that describes the pattern in each sequence. Then use your conjecture to find the next item in the sequence.

a. Movie show times: 8:30 A.M., 9:45 A.M., 11:00 A.M., 12:15 P.M., . . .

Step 1 Look for a pattern.

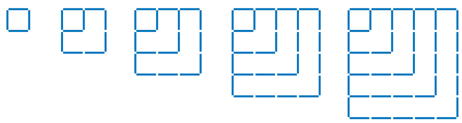
8:30 A.M., 9:45 A.M., 11:00 A.M., 12:15 P.M., . . .

+1 hr 15 min +1 hr 15 min +1 hr 15 min

Step 2 Make a conjecture.

The show time is 1 hour and fifteen minutes greater than the previous show time. The next show time will be 12:15 P.M. + 1:15 or 1:30 P.M.

b.



4 10 18 28 40 . . .

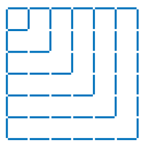
Step 1 4, 10, 18, 28, 40

+6 +8 +10 +12

The numbers increase by 6, 8, 10, and 12.

Step 2 The next figure will increase by 12 + 2 or 14 segments. So, the next figure will have 40 + 14 or 54 segments.

CHECK Draw the next figure to check your conjecture ✓



54

Junko Kimura/Getty Images

StudyTip

Figural Patterns Patterns that involve a sequence of figures, like those in Example 1b and in Guided Practice 1C, are called *figural patterns.*

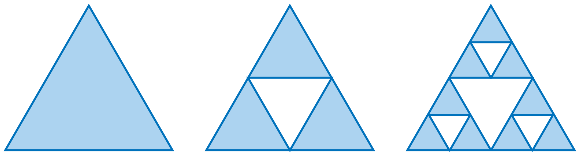
GuidedPractice

Write a conjecture that describes the pattern in each sequence. Then use your conjecture to find the next item in the sequence.

1A. Follow-up visits: Dec., May, Oct., Mar., . . .

1B. 10, 4, −2, −8, . . .

1C.



To make some algebraic and geometric conjectures, you will need to provide examples.

Example 2 Algebraic and Geometric Conjectures

Make a conjecture about each value or geometric relationship. List or draw some examples that support your conjecture.

a. the sum of two odd numbers

Step 1 List examples.

$1 + 3 = 4 \qquad 1 + 5 = 6 \qquad 3 + 5 = 8 \qquad 7 + 9 = 16$

Step 2 Look for a pattern.

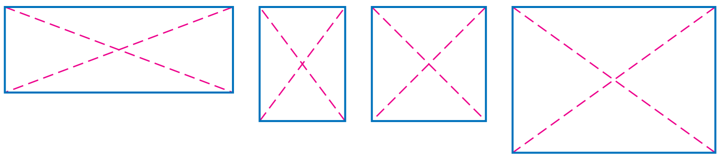
Notice that the sums 4, 6, 8, and 16 are all even numbers.

Step 3 Make a conjecture.

The sum of two odd numbers is an even number.

b. segments joining opposite vertices of a rectangle

Step 1



Step 2 Notice that the segments joining opposite vertices of each rectangle appear to have the same measure. Use a ruler or compass to confirm this.

Step 3 Conjecture: the segments joining opposite vertices of a rectangle are congruent.

StudyTip

CCSS Arguments Examples that support a conjecture are not enough to show that a conjecture is true. To show that an algebraic or geometric conjecture is true, you must offer a logical argument called a proof. You will learn more about proofs in Lesson 2-5.

GuidedPractice

2A. the sum of two even numbers

2B. the relationship between AB and EF, if $AB = CD$ and $CD = EF$

2C. the sum of the squares of two consecutive natural numbers

Real-world conjectures are often made based on data gathered about a specific topic of interest.

Real-World Example 3 Make Conjectures from Data

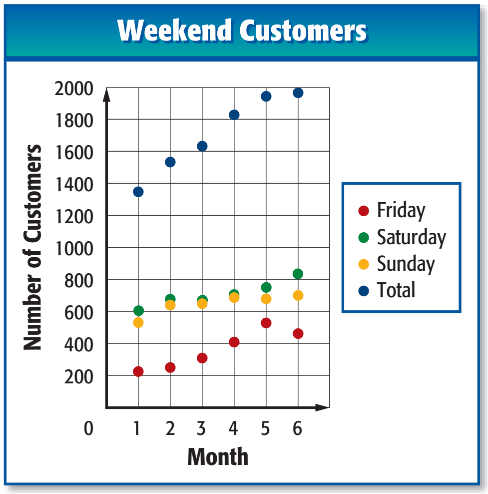
BUSINESS **The owner of a hair salon collected data on the number of customers her salon had each Friday, Saturday, and Sunday for 6 months to decide whether she should increase the number of stylists working each weekend. The data she collected are shown below.**

Number of Customers on the Weekend

Day	Month 1	Month 2	Month 3	Month 4	Month 5	Month 6
Friday	225	255	321	406	540	450
Saturday	603	658	652	712	746	832
Sunday	552	635	642	692	685	705
Total	1380	1548	1615	1810	1971	1987

a. Make a statistical graph that best displays the data.

Since you want to look for a pattern over time, use a scatter plot to display the data. Label the horizontal axis with the months and the vertical axis with the number of customers. Plot each set of data using a different color, and include a legend.



b. Make a conjecture based on the data, and explain how this conjecture is supported by your graph.

Look for patterns in the data. The number of customers on each day usually increases each month, and the total number of customers increases every single month.

Survey data supports a conjecture that the amount of business on the weekends has increased, so the owner should schedule more stylists to work on those days.

Real-World Career

Hair Stylist Hair stylists work in salons where various services, including skin and nail treatments, may be provided in addition to hair care. About 48% of hair stylists are self-employed and own their own businesses. Hair stylists must attend cosmetology school and obtain a license.

Guided Practice

3. **POSTAGE** The table at the right shows the price of postage for the years 1982 through 2009.

A. Make a statistical graph that best displays the data.

B. Predict the postage rate in 2015 based on the graph.

C. Does it make sense that the pattern of the data will continue over time? If not, how will it change? Explain your reasoning.

Year	Rate (cents)
1982	20
1987	22
1992	29
1997	32
2002	37
2007	41
2009	44

2 Find Counterexamples

To show that a conjecture is true for all cases, you must prove it. It takes only one false example, however, to show that a conjecture is not true. This false example is called a **counterexample**, and it can be a number, a drawing, or a statement.

Vocabulary Link

Counterexample

Everyday Use The prefix *counter-* means *the opposite of.*

Math Use A counterexample is the opposite of an example.

Example 4 Find Counterexamples

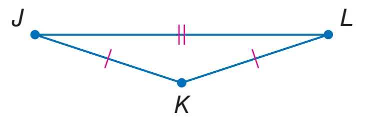
Find a counterexample to show that each conjecture is false.

a. If n is a real number, then $n^2 > n$.

When n is 1, the conjecture is false, since $1^2 \not> 1$.

b. If $JK = KL$, then K is the midpoint of $\overline{JL}$.

When J, K, and L are noncollinear, the conjecture is false. In the figure, $JK = KL$, but K is not the midpoint of $\overline{JL}$.



Guided Practice

4A. If n is a real number, then $-n$ is a negative number.

4B. If $\angle ABC \cong \angle DBE$, then $\angle ABC$ and $\angle DBE$ are vertical angles.

Check Your Understanding

● = Step-by-Step Solutions begin on page R14.

Example 1 **Write a conjecture that describes the pattern in each sequence. Then use your conjecture to find the next item in the sequence.**

1. Costs: \$4.50, \$6.75, \$9.00, . . .

2. Appointment times: 10:15 A.M., 11:00 A.M., 11:45 A.M., . . .

3.

4.

5 3, 3, 6, 9, 15, . . .

6. 2, 6, 14, 30, 62, . . .

Example 2 **Make a conjecture about each value or geometric relationship.**

7. the product of two even numbers

8. the relationship between a and b if $a + b = 0$

9. the relationship between the set of points in a plane equidistant from point A

10. the relationship between $\overline{AP}$ and $\overline{PB}$ if M is the midpoint of $\overline{AB}$ and P is the midpoint of $\overline{AM}$

Example 3 **11. CELL PHONES** Refer to the table of the number of wireless subscriptions in the United States by year.

a. Make a graph that shows U.S. wireless use from 2002 to 2007.

b. Make a conjecture about U.S. wireless use in 2012.

U.S. Wireless Subscribership

Year	Subscribers (Millions)
2002	140.8
2003	158.7
2004	182.1
2005	207.9
2006	233.0
2007	255.4

Source: Cellular Telecommunications and Internet Association

Example 4 CCSS **CRITIQUE** **Find a counterexample to show that each conjecture is false.**

12. If ∠*A* and ∠*B* are complementary angles, then they share a common side.

13 If a ray intersects a segment at its midpoint, then the ray is perpendicular to the segment.

Practice and Problem Solving

Extra Practice is on page R2.

Example 1 **Write a conjecture that describes the pattern in each sequence. Then use your conjecture to find the next item in the sequence.**

14. 0, 2, 4, 6, 8

15. 3, 6, 9, 12, 15

16. 4, 8, 12, 16, 20

17. 2, 22, 222, 2222

18. 1, 4, 9, 16

19. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$

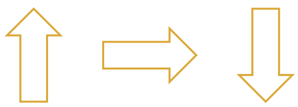
20. Arrival times: 3:00 P.M., 12:30 P.M., 10:00 A.M., . . .

21. Percent humidity: 100%, 93%, 86%, . . .

22. Work-out days: Sunday, Tuesday, Thursday, . . .

23. Club meetings: January, March, May, . . .

24.



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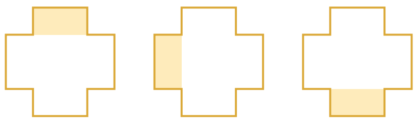

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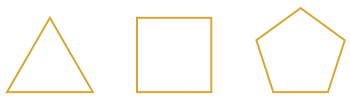

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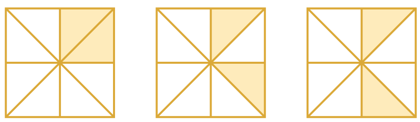


28. FITNESS Gabriel started training with the track team five weeks ago. During the first week, he ran 0.5 mile at each practice. The next three weeks he ran 0.75 mile, 1 mile, and 1.25 miles at each practice. If he continues this pattern, how many miles will he be running at each practice during the 7th week?

29. CONSERVATION When there is a shortage of water, some municipalities limit the amount of water each household is allowed to consume. Most cities that experience water restrictions are in the western and southern parts of the United States. Make a conjecture about why water restrictions occur in these areas.

30. VOLUNTEERING Carrie collected canned food for a homeless shelter in her area each day for one week. On day one, she collected 7 cans of food. On day two, she collected 8 cans. On day three, she collected 10 cans. On day four, she collected 13 cans. If Carrie wanted to give at least 100 cans of food to the shelter and this pattern of can collecting continued, did she meet her goal?

Example 2 **Make a conjecture about each value or geometric relationship.**

31. the product of two odd numbers

32. the product of two and a number, plus one

33. the relationship between a and c if $ab = bc$, $b \neq 0$

34. the relationship between a and b if $ab = 1$

35. the relationship between $\overline{AB}$ and the set of points equidistant from A and B

36. the relationship between the angles of a triangle with all sides congruent

37. the relationship between the areas of a square with side x and a rectangle with sides x and $2x$

38. the relationship between the volume of a prism and a pyramid with the same base

Example 3

39. **SPORTS** Refer to the table of Americans over the age of 7 that played hockey.
    a. Make a statistical graph that best displays the data.
    b. Make a conjecture based on the data, and explain how this conjecture is supported by your graph.

Year	Number of Participants (millions)
2000	1.9
2002	2.1
2004	2.4
2006	2.6

Example 4 CCSS **CRITIQUE** **Determine whether each conjecture is *true* or *false*. Give a counterexample for any false conjecture.**

40. If n is a prime number, then $n + 1$ is not prime.

41. If x is an integer, then $-x$ is positive.

42. If $\angle 2$ and $\angle 3$ are supplementary angles, then $\angle 2$ and $\angle 3$ form a linear pair.

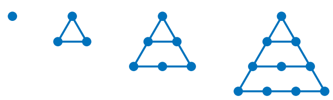
43. If you have three points A, B, and C, then A, B, C are noncollinear.

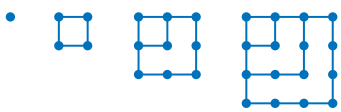
44. If in $\triangle ABC$, $(AB)^2 + (BC)^2 = (AC)^2$, then $\triangle ABC$ is a right triangle.

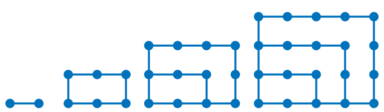
45. If the area of a rectangle is 20 square meters, then the length is 10 meters and the width is 2 meters.

FIGURAL NUMBERS **Numbers that can be represented by evenly spaced points arranged to form a geometric shape are called figural numbers. For each figural pattern below,**

a. write the first four numbers that are represented,
b. write a conjecture that describes the pattern in the sequence,
c. explain how this numerical pattern is shown in the sequence of figures,
d. find the next two numbers, and draw the next two figures.

46. 

47. 

48. 

49. 

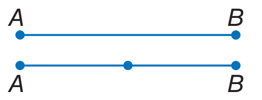
50. The sequence of odd numbers, 1, 3, 5, 7, . . . can also be a sequence of figural numbers. Use a figural pattern to represent this sequence.

51. GOLDBACH'S CONJECTURE Goldbach's conjecture states that every even number greater than 2 can be written as the sum of two primes. For example, $4 = 2 + 2$, $6 = 3 + 3$, and $8 = 3 + 5$.

a. Show that the conjecture is true for the even numbers from 10 to 20.

b. Given the conjecture *All odd numbers greater than 2 can be written as the sum of two primes*, is the conjecture *true* or *false*? Give a counterexample if the conjecture is false.

52. SEGMENTS Two collinear points form one segment, as shown for $\overline{AB}$. If a collinear point is added to $\overline{AB}$, the three collinear points form three segments.



a. How many distinct segments are formed by four collinear points? by five collinear points?

b. Make a conjecture about the number of distinct segments formed by *n* collinear points.

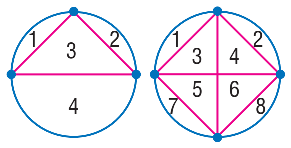
c. Test your conjecture by finding the number of distinct segments formed by six points.

53. CCSS TOOLS Using dynamic geometry software, Nora calculates the perimeter *P* and area *A* of a regular hexagon with a side length of 2 units. The change to the perimeter and area after three doublings of this side length are listed in the table. Analyze the patterns in the table. Then make a conjecture as to the effects on the perimeter and area of a regular hexagon when the side length is doubled. Explain.

Side (units)	*P* (units)	*A* (units²)
2	12	$6\sqrt{3}$
4	24	$24\sqrt{3}$
8	48	$96\sqrt{3}$
16	96	$384\sqrt{3}$

H.O.T. Problems Use Higher-Order Thinking Skills

54. CHALLENGE If you draw points on a circle and connect every pair of points, the circle is divided into regions. For example, two points form two regions, three points form four regions, and four points form eight regions.



a. Make a conjecture about the relationship between the number of points on a circle and the number of regions formed in the circle.

b. Does your conjecture hold true when there are six points? Support your answer with a diagram.

55. ERROR ANALYSIS Juan and Jack are discussing prime numbers. Juan states a conjecture that all prime numbers are odd. Jack disagrees with the conjecture and states that not all prime numbers are odd. Is either of them correct? Explain.

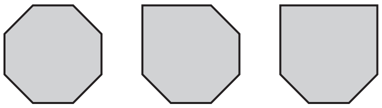
56. OPEN ENDED Write a number sequence that can be generated by two different patterns. Explain your patterns.

57. REASONING Consider the conjecture *If two points are equidistant from a third point, then the three points are collinear*. Is the conjecture *true* or *false*? If false, give a counterexample.

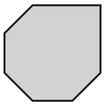
58. WRITING IN MATH Suppose you are conducting a survey. Choose a topic and write three questions you would include in your survey. How would you use inductive reasoning with your responses?

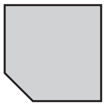
Standardized Test Practice

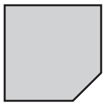
59. Look at the pattern below.

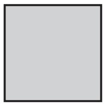


If the pattern continues, what will be the next shape?

A


B


C


D


60. GRIDDED RESPONSE What is the value of the expression below if $a = 10$ and $b = 1$?

$$2b + ab \div (a + b)$$

61. ALGEBRA A chemistry student mixed a 30% copper sulfate solution with a 40% copper sulfate solution to obtain 100 mL of a 32% copper sulfate solution. How much of the 30% copper sulfate solution did the student use in the mixture?

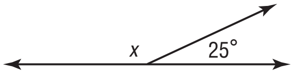
F 90 mL

G 80 mL

H 60 mL

J 20 mL

62. SAT/ACT Which of the following is equal to $2x$?



A 50°

B 78°

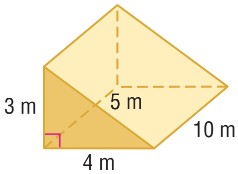
C 155°

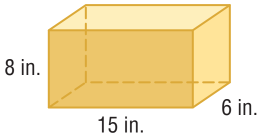
D 310°

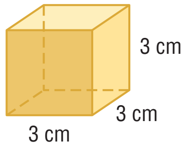
E 360°

Spiral Review

Find the surface area and volume of each solid. (Lesson 1-7)

63.


64.


65.


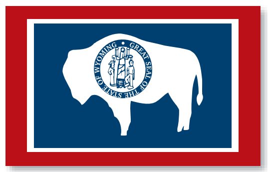
Find the perimeter of $\triangle ABC$ to the nearest hundredth, given the coordinates of its vertices. (Lesson 1-6)

66. $A(1, 6)$, $B(1, 2)$, $C(3, 2)$

67. $A(-3, 2)$, $B(2, -9)$, $C(0, -10)$

68. ALGEBRA The measures of two complementary angles are $16z - 9$ and $4z + 3$. Find the measures of the angles. (Lesson 1-5)

69. FLAGS The Wyoming state flag is shown at the right. Name the geometric term modeled by this flag: point, line, or plane. (Lesson 1-1)



70. ALGEBRA Evaluate $5|x + y| - 3|2 - z|$ if $x = 3$, $y = -4$, and $z = -5$. (Lesson 0-4)

Skills Review

ALGEBRA Determine which values in the replacement set make each inequality true.

71. $x - 3 > 12$
{6, 10, 14, 18}

72. $6 + x > 9$
{8, 6, 4, 2}

73. $2x - 4 > 10$
{5, 6, 7, 8}

LESSON 2-2

Logic

Then

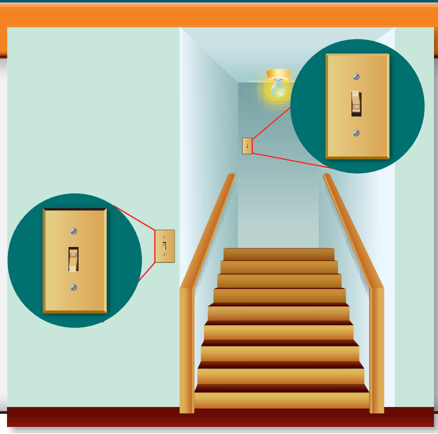
- You found counterexamples for false conjectures.

Now

1. Determine truth values of negations, conjunctions, and disjunctions, and represent them using Venn diagrams.
2. Find counterexamples.

Why?

- Many electrical circuits operate by evaluating a series of tests that are either true or false. For example, a single light can be controlled by two different switches connected on a circuit. The positions of both switches, either up or down, determine whether the light is on or off.



NewVocabulary

statement
truth value
negation
compound statement
conjunction
disjunction
truth table

1 Determine Truth Values

A **statement** is a sentence that is either true or false. The **truth value** of a statement is either true (T) or false (F). Statements are often represented using a letter such as p or q.

p: A rectangle is a quadrilateral. Truth value: T

The **negation** of a statement has the opposite meaning, as well as an opposite truth value. For example, the negation of the statement above is *not* p or $\sim p$.

$\sim p$: A rectangle is not **a quadrilateral.** Truth value: F

Two or more statements joined by the word *and* or *or* form a **compound statement**. A compound statement using the word *and* is called a **conjunction**. A conjunction is true only when both statements that form it are true.

p: A rectangle is a quadrilateral. Truth value: T

q: A rectangle is convex. Truth value: T

p and q: **A rectangle is a quadrilateral,** and **a rectangle is convex.**

Since both p and q are true, the conjunction p and q, also written $p \wedge q$, is true.

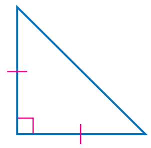
Example 1 Truth Values of Conjunctions

Use the following statements to write a compound statement for each conjunction. Then find its truth value. Explain your reasoning.

p: The figure is a triangle.

q: The figure has two congruent sides.

r: The figure has three acute angles.



a. p **and** r

p and r: The figure is a triangle, and the figure has three acute angles.

Although p is true, r is false. So, p and r is false.

b. $q \wedge \sim r$

$q \wedge \sim r$: The figure has two congruent sides, and the figure does not have three acute angles.

Both q and $\sim r$ are true, so $q \wedge \sim r$ is true.

GuidedPractice

1A. $p \wedge q$

1B. not p and not r

A compound statement that uses the word *or* is called a **disjunction**.

p: **Malik studies geometry**

q: **Malik studies chemistry.**

p or *q*: **Malik studies geometry,** or **Malik studies chemistry**.

A disjunction is true if at least one of the statements is true. If Malik studies either geometry or chemistry or both subjects, the disjunction p or q, also written as $p \vee q$, is true. If Malik studies neither geometry nor chemistry, p or q is false.

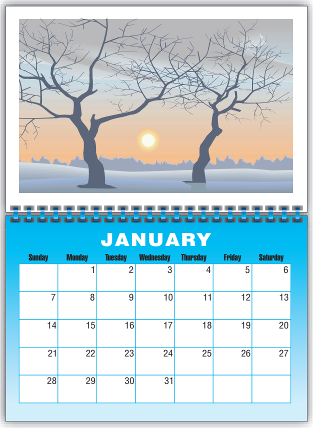
Example 2 Truth Values of Disjunctions

Use the following statements to write a compound statement for each disconjunction. Then find its truth value. Explain your reasoning.

***p*: January is a fall month.**

***q*: January has only 30 days.**

***r*: January 1 is the first day of a new year.**



a. p **or** r

q or r: January has only 30 days, or January 1 is the first day of a new year.

q or r is true because r is true. It does not matter that q is false.

b. $p \vee q$

$p \vee q$: January is a fall month, or January has only 30 days.

Since both p and q are false, $p \vee q$ is false.

c. $\sim p \vee r$

$\sim p \vee r$: January is *not* a fall month, or January 1 is the first day of a new year.

Not p or r is true, because not p is true and r is true.

WatchOut!

Negation Just as the opposite of an integer is not always negative, the negation of a statement is not always false. The negation of a statement has the opposite truth value of the original statement.

GuidedPractice

2A. r or p **2B.** $q \vee \sim r$ **2C.** $p \vee \sim q$

ConceptSummary Negation, Conjunction, Disconjunction

Statement	Words	Symbols
negation	a statement that has the opposite meaning and truth value of an original statement	$\sim p$, read not p
conjunction	a compound statement formed by joining two or more statements using the word *and*	$p \wedge q$, read p and q
disconjunction	a compound statement formed by joining two or more statements using the word *or*	$p \vee q$, read p or q

A convenient method for organizing the truth values of statements is to use a **truth table**. Truth tables can be used to determine truth values of negations and compound statements.

Negation	
p	$\sim p$
T	F
F	T

Conjunction		
p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction		
p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

StudyTip

Truth Tables The truth tables for a conjunction and a disjunction are easier to recall if you remember the following.

- A conjunction is true only when both statements are true.
- A disjunction is false only when both statements are false.

You can use the truth values for negation, conjunction, and disjunction to construct truth tables for more complex compound statements.

Example 3 Construct Truth Tables

Construct a truth table for $\sim p \vee q$.

1. Make columns with headings that include each original statement, any negations of these statements, and the compound statement itself.

p	q	$\sim p$	$\sim p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

2. List the possible combinations of truth values.

3. Use the truth values of p to determine the truth values of its negation.

4. Use the truth values for each part of the compound statement to determine the truth value of the statement.

GuidedPractice

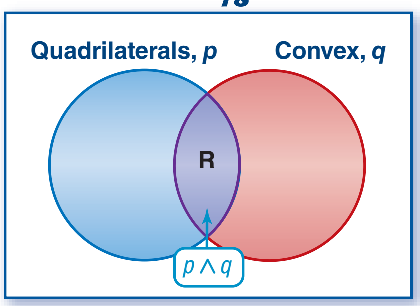
3. Construct a truth table for $\sim p \wedge \sim q$.

2 Venn Diagrams

Conjunctions can be illustrated with Venn diagrams. Consider the conjunction given at the beginning of the lesson.

p and q: **A rectangle is a quadrilateral,** and **a rectangle is convex.**

All Polygons



The Venn diagram shows that a rectangle (R) is located in the *intersection* of the set of quadrilaterals and the set of convex polygons. In other words, rectangles must be in the set containing quadrilaterals *and* in the set of convex polygons.

VocabularyLink

Intersection

Everyday Use the point at which two or more objects overlap

Math Use The intersection of two sets is the set of elements that are common to both.

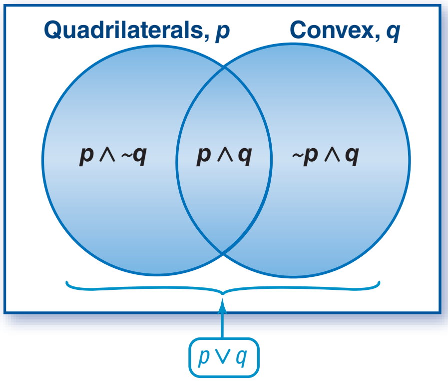
A disjunction can also be illustrated with a Venn diagram. Consider the following statements.

p: A figure is a quadrilateral.
q: A figure is convex.
p or *q*: A figure is a quadrilateral or convex.

VocabularyLink
Union
Everyday Use the joining of two or more objects
Math Use The union of two sets is the set of elements that appear in either of the sets.

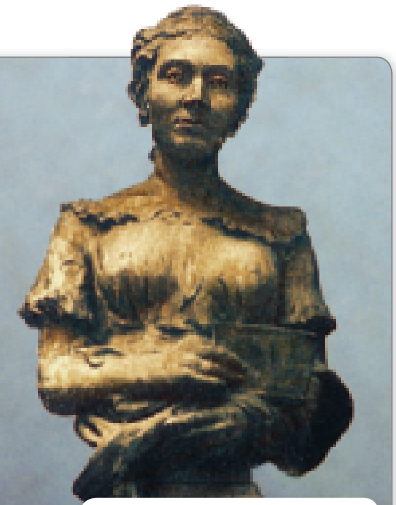
In the Venn diagram, the disjunction is represented by the union of the two sets. The union includes all polygons that are quadrilaterals, convex, *or* both.

All Polygons



The disjunction includes these three regions:

$p \wedge \sim q$ quadrilaterals that are *not* convex
$\sim p \wedge q$ convex polygons that are *not* quadrilaterals
$p \wedge q$ polygons that are both quadrilaterals and convex

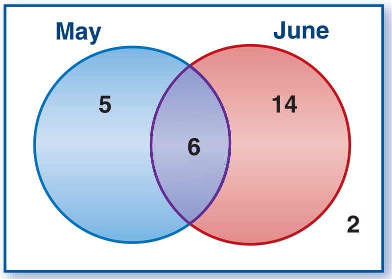


Math HistoryLink
Sophie Germain
(1776–1831) Sophie Germain was born in Paris, France. Like Goldbach, she studied relationships involving prime numbers. In order to pursue her passion for mathematics, she assumed a man's identity.

Real-World Example 4 Use Venn Diagrams

SCHEDULING The Venn diagram shows the number of people who can or cannot attend the May or the June Spanish Club meetings.

Spanish Club Meeting



a. How many people can attend the May or the June meeting?
The people who can attend either the May meeting or the June meeting are represented by the union of the sets. There are 5 + 6 + 14 or 25 people who can attend either night.

b. How many people can attend both the May and the June meetings?
The people who can attend both the May and the June meetings are represented by the intersection of the two sets. There are 6 people who can attend both meetings.

c. Describe the meetings that the 14 people located in the nonintersecting portion of the June region can attend.
These 14 people can attend the June meeting but not the May meeting.

GuidedPractice

4. PROM The Venn diagram shows the number of graduates last year who did or did not attend their junior or senior prom.

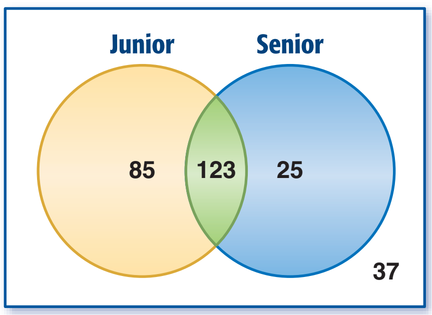
A. How many graduates attended their senior but not their junior prom?

B. How many graduates attended their junior and senior proms?

C. How many graduates did not attend either of their proms?

D. How many students graduated last year? Explain your reasoning.

Prom Attendance



Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Examples 1–2 **Use the following statements to write a compound statement for each conjunction or disjunction. Then find its truth value. Explain your reasoning.**

p: A week has seven days.

q: There are 20 hours in a day.

r: There are 60 minutes in an hour.

1. p and r **2.** $p \wedge q$ **3.** $q \vee r$

4. $\sim p$ or q **5.** $p \vee r$ **6.** $\sim p \wedge \sim r$

Example 3 **7.** Copy and complete the truth table at the right.

p	q	$\sim q$	$p \vee \sim q$
T	T	F	
T	F		
F	T		
F	F		

Construct a truth table for each compound statement.

8. $p \wedge q$ **9.** $\sim p \vee \sim q$

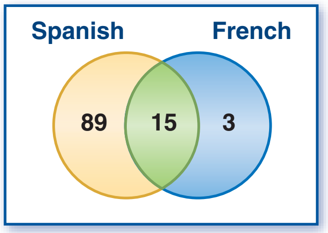
Example 4 **10. CLASSES** Refer to the Venn diagram that represents the foreign language classes students selected in high school.

a. How many students chose only Spanish?

b. How many students chose Spanish and French?

c. Describe the class(es) the three people in the nonintersecting portion of the French region chose.

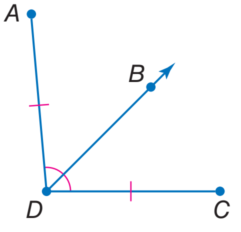
Foreign Language Classes Selected



Practice and Problem Solving

Extra Practice is on page R2.

Examples 1–2 **Use the following statements and figure to write a compound statement for each conjunction or disjunction. Then find its truth value. Explain your reasoning.**



p: $\overrightarrow{DB}$ is the angle bisector of $\angle ADC$.

q: Points C, D, and B are collinear.

r: $\overline{AD} \cong \overline{DC}$

11. p and r **12.** q or p **13.** r or $\sim p$

14. r and q **15.** $\sim p$ or $\sim r$ **16.** $\sim p$ and $\sim r$

CCSS **REASONING** **Use the following statements to write a compound statement for each conjunction or disjunction. Then find its truth value. Explain your reasoning.**

p: Springfield is the capital of Illinois.

q: Illinois borders the Atlantic Ocean.

r: Illinois shares a border with Kentucky.

s: Illinois is to the west of Missouri.

17. $p \wedge r$ **18.** $p \wedge q$ **19.** $\sim r \vee s$

20. $r \vee q$ **21.** $\sim p \wedge \sim r$ **22.** $\sim s \vee \sim p$

Example 3 **Copy and complete each truth table.**

23.

p	q	$\sim p$	$\sim p \wedge q$
T		F	
T		F	
F		T	
F		T	

24.

p	q	$-p$	$\sim q$	$\sim p \vee q$
T			F	
T			T	
F			F	
F			T	

Construct a truth table for each compound statement.

25. $p \wedge r$

26. $r \wedge q$

27. $p \vee r$

28. $q \vee r$

29. $\sim p \wedge r$

30. $\sim q \vee \sim r$

Example 4

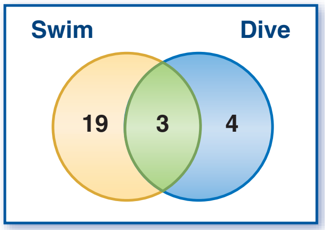
31 **WATER SPORTS** Refer to the Venn diagram that represents the number of students who swim and dive at a high school.

a. How many students dive?

b. How many students participate in swimming or diving or both?

c. How many students swim and dive?

Swimming and Diving



32. CCSS **REASONING** Venus has switches at the top and bottom of her stairs to control the light for the stairwell. She notices that when the upstairs switch is up and the downstairs switch is down, the light is turned on.

a. Copy and complete the truth table.

b. If both the upstairs and downstairs switches are in the up position, will the light be on? Explain your reasoning.

c. If the upstairs switch is in the down position and the downstairs switch is in the up position, will the light be on?

d. In general, how should the two switches be positioned so that the light is on?

Position of Switch		Light On
Upstairs	Downstairs	
up		
up	down	T

33. **ELECTRONICS** A group of 330 teens were surveyed about what type of electronics they used. They chose from a cell phone, a portable media player, and a DVR. The results are shown in the Venn diagram.

a. How many teens used only a portable media player and DVR?

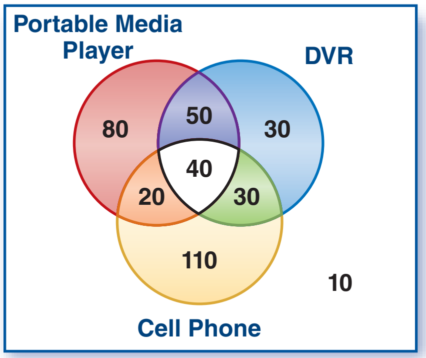
b. How many said they used all three types of electronics?

c. How many said they used only a cell phone?

d. How many teens said they used only a portable media player and a cell phone?

e. Describe the electronics that the 10 teens outside of the regions use.

Type of Electronics Used



Construct a truth table for each compound statement. Determine the truth value of each compound statement if the given statements are true.

34. $p \wedge (q \wedge r)$; p, q **35** $p \wedge (\sim q \vee r)$; p, r **36.** $(\sim p \vee q) \wedge r$; q, r

37. $p \wedge (\sim q \wedge \sim r)$; p, q, r **38.** $\sim p \wedge (\sim q \wedge \sim r)$; p, q, r **39.** $(\sim p \vee q) \vee \sim r$; p, q

40. CCSS **REASONING** A travel agency surveyed 70 of their clients who had visited Europe about international travel. Of the 70 clients who had visited Europe, 60 had traveled to England, France, or both. Of those 60 clients, 45 had visited England, and 50 had visited France.

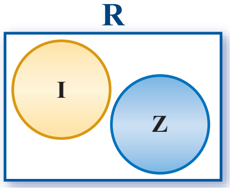
a. Make a Venn diagram to show the results of the survey.

b. If p represents a client who has visited England and q represents a client who has visited France, write a compound statement to represent each area of the Venn diagram. Include the compound statements on your Venn diagram.

c. What is the probability that a randomly chosen participant in the survey will have visited both England and France? Explain your reasoning.

H.O.T. Problems Use Higher-Order Thinking Skills

41. **REASONING** Irrational numbers and integers both belong to the set of real numbers (R). Based upon the Venn diagram, is it *sometimes*, *always*, or *never* true that integers (Z) are irrational numbers (I)? Explain your reasoning.



CHALLENGE To negate a statement containing the words *all* or *for every*, you can use the phrase *at least one* or *there exists*. To negate a statement containing the phrase *there exists*, use the phrase *for all* or *for every*.

p: All polygons are convex.

$\sim p$: *At least one* polygon is *not* convex.

q: *There exists* a problem that has no solution.

$\sim q$: *For every* problem, there is a solution.

Sometimes these phrases may be implied. For example, *The square of a real number is nonnegative* implies the following conditional and its negation.

p: *For every* real number x, $x^2 \geq 0$.

$\sim p$: *There exists* a real number x such that $x^2 < 0$.

Use the information above to write the negation of each statement.

42. Every student at Hammond High School has a locker.

43. All squares are rectangles.

44. There exists a real number x such that $x^2 = x$.

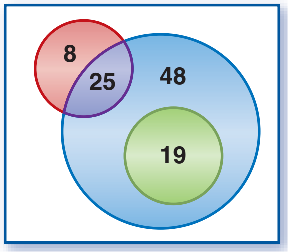
45. There exists a student who has at least one class in C-Wing.

46. Every real number has a real square root.

47. There exists a segment that has no midpoint.

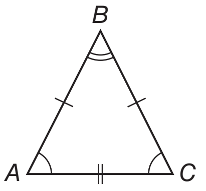
48. **WRITING IN MATH** Describe a situation that might be depicted using the Venn diagram shown.

49. **OPEN ENDED** Write a compound statement that results in a true conjunction.



Standardized Test Practice

50. Which statement about $\triangle ABC$ has the same truth value as $AB = BC$?



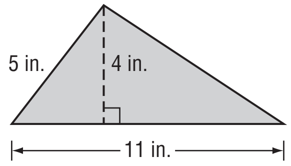
A $m\angle A = m\angle C$

B $m\angle A = m\angle B$

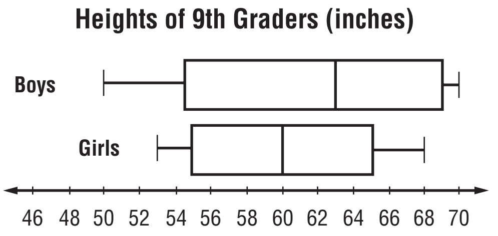
C $AC = BC$

D $AB = AC$

51. EXTENDED RESPONSE What is the area of the triangle shown below? Explain how you found your answer.



52. STATISTICS The box-and-whisker plot below represents the heights of 9th graders at a certain high school. How much greater was the median height of the boys than the median height of the girls?



F 3 inches **H** 5 inches

G 4 inches **J** 6 inches

53. SAT/ACT Heather, Teresa, and Nina went shopping for new clothes. Heather spent twice as much as Teresa, and Nina spent three times what Heather spent. If they spent a total of \$300, how much did Teresa spend?

A \$33.33 **D** \$100.00

B \$50.00 **E** \$104.33

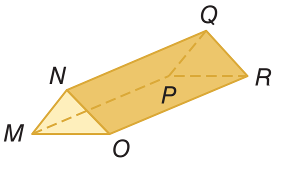
C \$66.33

Spiral Review

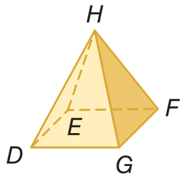
54. LUNCH For the past four Tuesdays, Jason's school has served chicken sandwiches for lunch. Jason assumes that chicken sandwiches will be served for lunch on the next Tuesday. What type of reasoning did he use? Explain. (Lesson 2-1)

Identify each solid. Name the bases, faces, edges, and vertices. (Lesson 1-7)

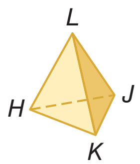
55.



56.



57.



ALGEBRA Solve each equation. (Lesson 0-5)

58. $\frac{y}{2} - 7 = 5$

59. $3x + 9 = 6$

60. $4(m - 5) = 12$

61. $6(w + 7) = 0$

62. $2x - 7 = 11$

63. $\frac{y}{5} + 4 = 9$

Skills Review

ALGEBRA Evaluate each expression for the given values.

64. $2y + 3x$ if $y = 3$ and $x = -1$

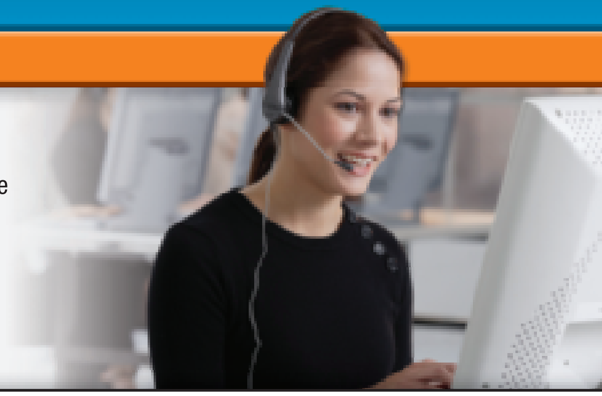
65. $4d - c$ if $d = 4$ and $c = 2$

66. $m^2 + 7n$ if $m = 4$ and $n = -2$

67. $ab - 2a$ if $a = -2$ and $b = -3$

LESSON 2-3

Conditional Statements



Then

- You used logic and Venn diagrams to determine truth values of negations, conjunctions, and disjunctions.

Now

1. Analyze statements in if-then form.
2. Write the converse, inverse, and contrapositive of if-then statements.

Why?

- Call centers route calls to the appropriate departments using menus that allow callers to choose from a number of options. The recorded directions are frequently in the form of conditional statements.

NewVocabulary

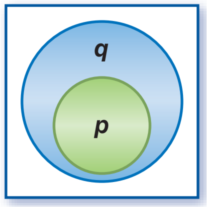
conditional statement
if-then statement
hypothesis
conclusion
related conditionals
converse
inverse
contrapositive
logically equivalent

1 If-Then Statements

A **conditional statement** is a statement that can be written in *if-then form*. The direction given below is an example of a conditional statement.

If you would like to speak to a representative, **then** you will press 0 now.

KeyConcept Conditional Statement

Words	Symbols	Model
An **if-then statement** is of the form *if p, then q.*	$p \to q$ read *if p then q*, or *p implies q*	 $p \to q$
The **hypothesis** of a conditional statement is the phrase immediately following the word *if.*	p	
The **conclusion** of a conditional statement is the phrase immediately following the word *then.*	q	

When a conditional statement is written as an if-then statement, you can quickly identify its hypothesis and conclusion.

Example 1 Identify the Hypothesis and Conclusion

Identify the hypothesis and conclusion of each conditional statement.

a. If the forecast is rain, then I will take an umbrella.

Hypothesis: The forecast is rain.

Conclusion: I will take an umbrella.

b. A number is divisible by 10 if its last digit is a 0.

Hypothesis: The last digit of a number is zero.

Conclusion: The number is divisible by 10.

GuidedPractice

1A. If a polygon has six sides, then it is a hexagon.

1B. Another performance will be scheduled if the first one is sold out.

Many conditional statements are written without using the words *if* and *then*. To write these statements in if-then form, identify the hypothesis and conclusion.

ReadingMath

If and Then The word *if* is not part of the hypothesis. The word *then* is not part of the conclusion.

Points will be deducted from any **paper turned in after Wednesday's deadline.**

Conclusion: Points will be deducted — Hypothesis: paper turned in after Wednesday's deadline

If **a paper is turned in after Wednesday's deadline**, then **points will be deducted.**

Remember, the conclusion depends upon the hypothesis.

Example 2 Write a Conditional in If-Then Form

Identify the hypothesis and conclusion for each conditional statement. Then write the statement in if-then form.

a. A mammal is a warm-blooded animal.

Hypothesis: An animal is a mammal.

Conclusion: It is warm-blooded.

If an animal is a mammal, then it is warm-blooded.

b. A prism with bases that are regular polygons is a regular prism.

Hypothesis: A prism has bases that are regular polygons.

Conclusion: It is a regular prism.

If a prism has bases that are regular polygons, then it is a regular prism.

GuidedPractice

2A. Four quarters can be exchanged for a $1 bill.

2B. The sum of the measures of two supplementary angles is 180.

The hypothesis and the conclusion of a conditional statement can have a truth value of true or false, as can the conditional statement itself. Consider the following conditional.

If **Tom finishes his homework**, then **he will clean his room**.

Hypothesis	Conclusion	Conditional	
Tom finishes his homework.	Tom cleans his room.	If Tom finishes his homework, then he will clean his room.	
T	T	T	If Tom *does* finish his homework and he *does* clean his room, then the conditional is true.
T	F	F	If Tom does *not* clean his room after he *does* finish his homework, then he has not fulfilled his promise and the conditional is false.
F	T	?	The conditional only indicates what will happen if Tom *does* finish his homework. He could clean his room or not clean his room if he does *not* finish his homework.
F	F	?	

ReadingMath

Not False If a statement is *not false*, logic dictates that it must be *true*.

When the hypothesis of a conditional is not met, the truth of a conditional cannot be determined. When the truth of a conditional statement cannot be determined, it is considered true by default.

The results from the previous page can be used to create a truth table for conditional statements.

Conditional Statements		
p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Notice that a conditional is false *only* when its hypothesis is true and its conclusion is false.

Notice too that when a hypothesis is false, the conditional will *always* be considered true, regardless of whether the conclusion is true or false.

WatchOut!

Analyzing Conditionals When analyzing a conditional, do not try to determine whether the argument makes sense. Instead, analyze the form of the argument to determine whether the conclusion follows logically from the hypothesis.

To show that a conditional is true, you must show that for each case when the hypothesis is true, the conclusion is also true. To show that a conditional is false, you only need to find one counterexample.

Example 3 Truth Values of Conditionals

Determine the truth value of each conditional statement. If *true*, explain your reasoning. If *false*, give a counterexample.

a. If you divide an integer by another integer, the result is also an integer.

Counterexample: When you divide 1 by 2, the result is 0.5.

Since 0.5 is not an integer, the conclusion is false.
Since you can find a counterexample, the conditional statement is false.

b. If next month is August, then this month is July.

When the hypothesis is true, the conclusion is also true, since August is the month that follows July. So, the conditional statement is true.

c. If a triangle has four sides, then it is concave.

The hypothesis is false, since a triangle can never have four sides. A conditional with a false hypothesis is always true.

GuidedPractice

3A. If $\angle A$ is an acute angle, then $m\angle A$ is 35.

3B. If $\sqrt{x} = -1$, then $(-1)^2 = -1$.

2 Related Conditionals

There are other statements that are based on a given conditional statement. These are known as **related conditionals**.

KeyConcept Related Conditionals

Words	Symbols	Examples
A conditional statement is a statement that can be written in the form *if p, then q*.	$p \to q$	If $m\angle A$ is 35, then $\angle A$ is an acute angle.
The **converse** is formed by exchanging the hypothesis and conclusion of the conditional.	$q \to p$	If $\angle A$ is an acute angle, then $m\angle A$ is 35.
The **inverse** is formed by negating both the hypothesis and conclusion of the conditional.	$\sim p \to \sim q$	If $m\angle A$ is *not* 35, then $\angle A$ is *not* an acute angle.
The **contrapositive** is formed by negating both the hypothesis and the conclusion of the converse of the conditional.	$\sim q \to \sim p$	If $\angle A$ is *not* an acute angle, then $m\angle A$ is *not* 35.

A conditional and its contrapositive are either both true or both false. Similarly, the converse and inverse of a conditional are either both true or both false. Statements with the same truth values are said to be **logically equivalent**.

KeyConcept Logically Equivalent Statements

- A conditional and its contrapositve are logically equivalent.
- The converse and inverse of a conditional are logically equivalent.

If a conditional is true, the converse may or may not be true.

You can use logical equivalence to check the true value of statements. Notice that in Example 4, both the conditional and contrapositive are true. Also, both the converse and inverse are false.

PT

Real-World Example 4 Related Conditionals

NATURE Write the converse, inverse, and contrapositive of the following true statement. Then use the information at the left to determine whether each related conditional is *true* or *false*. If a statement is false, find a counterexample.

Lions are cats that can roar.

Conditional: First, rewrite the conditional in if-then form.

If an animal is a lion, then it is a cat that can roar.

Based on the information at the left, this statement is true.

Converse: If an animal is a cat that can roar, then it is a lion.

Counterexample: A tiger is a cat that can roar, but it is not a lion.

Therefore, the converse is false.

Inverse: If an animal is not a lion, then it is not a cat that can roar.

Counterexample: A tiger is not a lion, but it is a cat that can roar.

Therefore, the inverse is false.

Contrapositive: If an animal is not a cat that can roar, then it is not a lion.

Based on the information at the left, this statement is true.

CHECK Check to see that logically equivalent statements have the same truth value.

Both the conditional and contrapositive are true. ✔

Both the converse and inverse are false. ✔

Real-WorldLink

Cats in the genus *Panthera* include the leopard, jaguar, lion, and tiger. These are the only cats that can roar. They cannot, however, purr.

Source: *Encyclopaedia Britannica*

GuidedPractice

Write the converse, inverse, and contrapositive of each true conditional statement. Determine whether each related conditional is *true* or *false*. If a statement is false, find a counterexample.

4A. Two angles that have the same measure are congruent.

4B. A hamster is a rodent.

Masterfile

Check Your Understanding

● = Step-by-Step Solutions begin on page R14.

Example 1 **Identify the hypothesis and conclusion of each conditional statement.**

1. If today is Friday, then tomorrow is Saturday.
2. If $2x + 5 > 7$, then $x > 1$.
3. If two angles are supplementary, then the sum of the measures of the angles is 180.
4. If two lines form right angles, then the lines are perpendicular.

Example 2 **Write each statement in if-then form.**

5. Sixteen-year-olds are eligible to drive.
6. Cheese contains calcium.
7. The measure of an acute angle is between 0 and 90.
8. Equilateral triangles are equiangular.
9. **WEATHER** Various kinds of precipitation form under different conditions. Write the three conditionals below in if-then form.
   a. Moisture in the air condenses and falls to form rain.
   b. Supercooled moisture in cumulonimbus clouds forms hail.
   c. When the temperature is freezing in all or most of the atmosphere, precipitation falls as snow.

Example 3 **Determine the truth value of each conditional statement. If *true*, explain your reasoning. If *false*, give a counterexample.**

10. If $x^2 = 16$, then $x = 4$.
11. If you live in Charlotte, then you live in North Carolina.
12. If tomorrow is Friday, then today is Thursday.
13. If an animal is spotted, then it is a Dalmatian.
14. If the measure of a right angle is 95, then bees are lizards.
15. If pigs can fly, then $2 + 5 = 7$.

Example 4 CCSS **ARGUMENTS** **Write the converse, inverse, and contrapositive of each true conditional statement. Determine whether each related conditional is *true* or *false*. If a statement is false, find a counterexample.**

16. If a number is divisible by 2, then it is divisible by 4.
17. All whole numbers are integers

Practice and Problem Solving

Extra Practice is on page R2.

Example 1 **Identify the hypothesis and conclusion of each conditional statement.**

18. If two angles are adjacent, then they have a common side.
19. If you lead, then I will follow.
20. If $3x - 4 = 11$, then $x = 5$.
21. If two angles are vertical, then they are congruent.

Identify the hypothesis and conclusion of each conditional statement.

22. If the degree measure of an angle is between 90 and 180, then the angle is obtuse.

23. "If there is no struggle, there is no progress." (Frederick Douglass)

24. If a quadrilateral has four congruent sides, then it is a square.

25. If a convex polygon has five sides, then it is a pentagon.

Example 2 **Write each statement in if-then form.**

26. Get a free water bottle with a one-year membership.

27. Everybody at the party received a gift.

28. The intersection of two planes is a line.

29. The area of a circle is πr^2.

30. Collinear points lie on the same line.

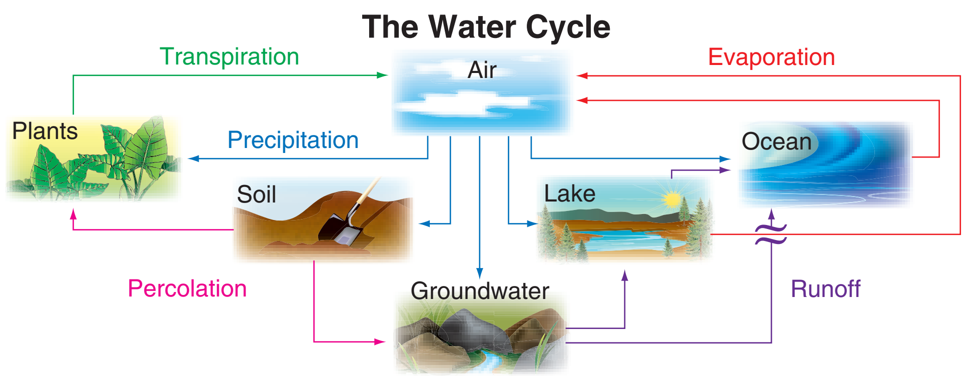
31. A right angle measures 90 degrees.

32. **MUSIC** Different instruments are emphasized in different types of music. Write each statement in if-then form.

- Jazz music often incorporates trumpet or saxophone.
- Rock music emphasizes guitar and drums.
- In hip-hop music, the bass is featured.

33. **ART** Write the following statement in if-then form: At the Andy Warhol Museum in Pittsburgh, Pennsylvania, most of the collection is Andy Warhol's artwork.

34. **SCIENCE** The water on Earth is constantly changing through a process called the *water cycle*. Write the three conditionals below in if-then form.



a. As runoff, water flows into bodies of water.

b. Plants return water to the air through transpiration.

c. Water bodies return water to the air through evaporation.

Example 3 **CCSS ARGUMENTS** **Determine the truth value of each conditional statement. If *true*, explain your reasoning. If *false*, give a counterexample.**

35 If a number is odd, then it is divisible by 5.

36. If a dog is an amphibian, then the season is summer.

37. If an angle is acute, then it has a measure of 45.

38. If a polygon has six sides, then it is a regular polygon.

Determine the truth value of each conditional statement. If *true*, explain your reasoning. If *false*, give a counterexample.

39. If an angle's measure is 25, then the measure of the angle's complement is 65.

40. If North Carolina is south of Florida, then the capital of Ohio is Columbus.

41. If red paint and blue paint mixed together make white paint, then $3 - 2 = 0$.

42. If two angles are congruent, then they are vertical angles.

43. If an animal is a bird, then it is an eagle.

44. If two angles are acute, then they are supplementary.

45. If two lines intersect, then they form right angles.

46. If a banana is blue, then an apple is a vegetable.

Example 4 **Write the converse, inverse, and contrapositive of each true conditional statement. Determine whether each related conditional is *true* or *false*. If a statement is false, find a counterexample.**

47. If you live in Chicago, you live in Illinois.

48. If a bird is an ostrich, then it cannot fly.

49. If two angles have the same measure, then the angles are congruent.

50. All squares are rectangles.

51. All congruent segments have the same length.

52. A right triangle has an angle measure of 90.

CCSS **ARGUMENTS** Write the statement indicated, and determine the truth value of each statement. If a statement is false, give a counterexample.

Animals with stripes are zebras.

53. conditional **54.** converse **55** inverse **56.** contrapositive

57. **SCIENCE** Chemical compounds are grouped and described by the elements that they contain. Acids contain hydrogen (H). Bases contain hydroxide (OH). Hydrocarbons contain only hydrogen (H) and carbon (C).

Compound	Example	Chemical Formula
Acid	Hydrochloric Acid	HCl
Base	Sodium Hydroxide	NaOH
Hydrocarbon	Methane	CH_4

a. Write three conditional statements in if-then form for classifying chemical compounds.

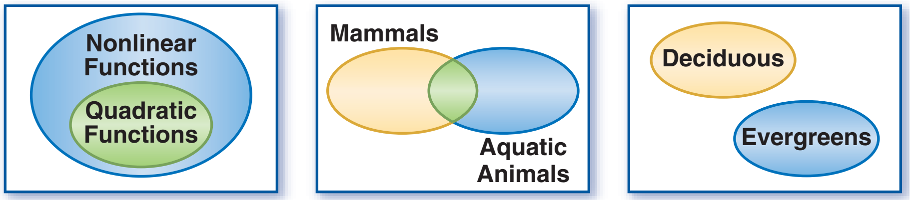
b. Write the converse of the three true conditional statements. State whether each is *true* or *false*. If a statement is false, find a counterexample.

58. **SPORTS** In football, touchdowns are worth 6 points, extra point conversions are worth 2 points, and safeties are worth 2 points.

a. Write three conditional statements in if-then form for scoring in football.

b. Write the converse of the three true conditional statements. State whether each is *true* or *false*. If a statement is false, find a counterexample.

Use the Venn diagrams below to determine the truth value of each conditional. Explain your reasoning.



59 If a function is nonlinear, then it is quadratic.

60. If an animal is a mammal, then it cannot be aquatic.

61. If a tree is deciduous, then it is not an evergreen.

62. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate a law of logic by using conditionals.

a. Logical Write three true conditional statements, using each consecutive conclusion as the hypothesis for the next statement.

b. Graphical Create a Venn diagram to model your series of statements.

c. Logical Write a conditional using the hypothesis of your first conditional and the conclusion of your third conditional. Is the conditional true if the hypothesis is true?

d. Verbal Given two conditionals *If a, then b* and *If b, then c*, make a conjecture about the truth value of *c* when *a* is true. Explain your reasoning.

H.O.T. Problems Use Higher-Order Thinking Skills

63. **CCSS CRITIQUE** Nicole and Kiri are evaluating the conditional *If 15 is a prime number, then 20 is divisible by 4*. Both think that the conditional is true, but their reasoning differs. Is either of them correct? Explain.

Nicole	Kiri
The conclusion is true, because 20 is divisible by 4, so the conditional is true.	*The hypothesis is false, because 15 is not a prime number, so the conditional is true.*

64. **CHALLENGE** You have learned that statements with the same truth value are logically equivalent. Use logical equivalence to create a truth table that summarizes the conditional, converse, inverse, and contrapositive for the statements *p* and *q*.

65. **REASONING** You are evaluating a conditional statement in which the hypothesis is true, but the conclusion is false. Is the inverse of the statement true or false? Explain your reasoning.

66. **OPEN ENDED** Write a conditional statement in which the converse, inverse, and contrapositive are all true. Explain your reasoning.

67. **CHALLENGE** The inverse of conditional *A* is given below. Write conditional *A*, its converse, and its contrapositive. Explain your reasoning.

If I received a detention, then I did not arrive at school on time.

68. **WRITING IN MATH** Describe the relationship between a conditional, its converse, its inverse, and its contrapositive.

Standardized Test Practice

69. *If the sum of the measures of two angles is 90, then the angles are complementary angles.*

Which of the following is the converse of the conditional above?

A If the angles are complementary angles, then the sum of the measures of two angles is 90.

B If the angles are not complementary angles, then the sum of the measures of the angles is 90.

C If the angles are complementary angles, then the sum of the measures of the angles is not 90.

D If the angles are not complementary angles, then the sum of the measures of two angles is not 90.

70. ALGEBRA What is $\frac{10a^2 - 15ab}{4a^2 - 9b^2}$ reduced to lowest terms?

F $\frac{5a}{2a - 2b}$ **H** $\frac{a}{2a + 3b}$

G $\frac{5a}{2a + 3b}$ **J** $\frac{a}{2a - 3b}$

71. SHORT RESPONSE What is the standard notation for the following expression?

$$4.62 \times 10^{-3}$$

72. SAT/ACT What is the greatest common prime factor of 18 and 33?

A 1 **D** 5

B 2 **E** 11

C 3

Spiral Review

Construct a truth table for each compound statement. (Lesson 2-2)

73. p and q **74.** p or $\sim q$ **75.** $\sim p \wedge q$ **76.** $\sim p \wedge \sim q$

Make a conjecture based on the given information. Draw a figure to illustrate your conjecture. (Lesson 2-1)

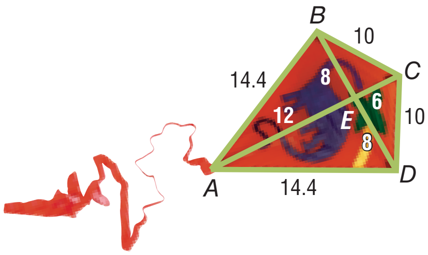
77. Points H, J, and K are each located on different sides of a triangle.

78. Collinear points X, Y, and Z; Z is between X and Y.

79. $R(3, -4)$, $S(-2, -4)$, and $T(0, -4)$

80. $A(-1, -7)$, $B(4, -7)$, $C(4, -3)$, and $D(-1, -3)$

81. KITES Kite making has become an art form. The kite shown is known as a diamond kite. The measures are in inches. Name all of the congruent segments in the figure. (Lesson 1-2)



Refer to the conversion charts inside the back cover of your textbook and in Lesson 0-2. (Lesson 0-2)

82. RUNNING Ling is participating in a 5-kilometer charity run next weekend. About how many miles is the race?

83. NATURE An African elephant weighs about 9 tons. About how many kilograms is this?

84. SPORTS A football field is 120 yards long from one end zone to the other. How many feet long is a football field?

Skills Review

ALGEBRA Identify the operation used to change Equation (1) to Equation (2).

85. (1) $8(y - 11) = 32$
(2) $y - 11 = 4$

86. (1) $x + 9 = 4 - 3x$
(2) $4x + 9 = 4$

87. (1) $\frac{1}{3}m = 2$
(2) $m = 6$

EXTEND 2-3

Geometry Lab
Biconditional Statements

Amy is the starting pitcher for her high school softball team. If she is elected by the district coaches, she will make the All-Star Team. If she makes the All-Star Team, she has been elected by the district coaches.

p: Amy is elected by the district coaches
q: Amy makes the All-Star Team

$p \rightarrow q$: If Amy is elected by the district coaches, then she makes the All-Star Team.

$q \rightarrow p$: If Amy made the All-Star Team, then she was elected by the district coaches.

In this case, both the conditional and its converse are true. The conjunction of the two statements is called a **biconditional**.

KeyConcept Biconditional Statement

Words	A biconditional statement is the conjunction of a conditional and its converse.
Symbols	$(p \rightarrow q) \wedge (q \rightarrow p) \rightarrow (p \leftrightarrow q)$, read *p if and only if q*

If and only if can be abbreviated *iff*.

So, the biconditional statement is as follows.

$p \leftrightarrow q$: Amy makes the All-Star Team if and only if she is elected by the district coaches.

Examples

Write each biconditional as a conditional and its converse. Then determine whether the biconditional is *true* or *false*. If false, give a counterexample.

a. An angle is a right angle if and only if its measure is 90.

Conditional: If an angle measures 90, then the angle is right.
Converse: If an angle is right, then the angle measures 90.
Both the conditional and the converse are true, so the biconditional is true.

b. $x > -2$ iff x is positive.

Conditional: If x is positive, then $x > -2$.
Converse: If $x > -2$, then x is positive.
Let $x = -1$. Then $-1 > -2$, but -1 is not positive. So, the biconditional is false.

Exercises

Write each biconditional as a conditional and its converse. Then determine whether the biconditional is *true* or *false*. If false, give a counterexample.

1. Two angles are complements if and only if their measures have a sum of 90.
2. There is no school if and only if it is Saturday.
3. Two lines intersect if and only if they are not horizontal.
4. $|2x| = 4$ iff $x = 2$.
5. Use logical equivalence to create a truth table that summarizes the conditional, converse, and biconditional for statements p and q.

LESSON 2-4

Deductive Reasoning

Then

- You used inductive reasoning to analyze patterns and make conjectures.

Now

1. Use the Law of Detachment.
2. Use the Law of Syllogism.

Why?

- When detectives are trying to solve a case, they use techniques like fingerprinting to analyze evidence. Then they use this evidence to eliminate suspects and eventually identify the person responsible for the crime.

NewVocabulary
deductive reasoning
valid
Law of Detachment
Law of Syllogism

1 Law of Detachment

The process that detectives use to identify who is most likely responsible for a crime is called deductive reasoning. Unlike inductive reasoning, which uses a pattern of examples or observations to make a conjecture, **deductive reasoning** uses facts, rules, definitions, or properties to reach logical conclusions from given statements.

Real-World Example 1 Inductive and Deductive Reasoning

Determine whether each conclusion is based on *inductive* or *deductive* reasoning.

a. Every time Katie has worn her favorite socks to a softball game, she has gotten at least one hit. Katie is wearing her favorite socks to a game tonight, so she concludes that she will get at least one hit.

Katie is basing her conclusion on a pattern of observations, so she is using inductive reasoning.

b. If John is late making his car insurance payment, he will be assessed a late fee of $50. John's payment is late this month, so he concludes that he will be assessed a late fee of $50.

John is basing his conclusion on facts provided to him by his insurance company, so he is using deductive reasoning.

GuidedPractice

1A. All of the signature items on the restaurant's menu shown are noted with a special symbol. Kevin orders a menu item that has this symbol next to it, so he concludes that the menu item that he has ordered is a signature item.

1B. None of the students who ride Raul's bus own a car. Ebony rides a bus to school, so Raul concludes that Ebony does not own a car.

While one counterexample is enough to disprove a conjecture reached using inductive reasoning, it is not a logically correct, or **valid**, method of proving a conjecture. To prove a conjecture requires deductive reasoning. One valid form of deductive reasoning is the **Law of Detachment**.

KeyConcept Law of Detachment

Words If $p \longrightarrow q$ is a true statement and p is true, then q is true.

Example *Given*: If a car is out of gas, then it will not start.
Sarah's car is out of gas.

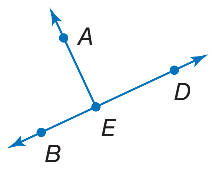
Valid Conclusion: Sarah's car will not start.

As long as the facts given are true, the conclusion reached using deductive reasoning will also be true.

Example 2 Law of Detachment

Determine whether each conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning.

**a. Given: If two angles form a linear pair, then their noncommon sides are opposite rays.
∠*AED* and ∠*AEB* form a linear pair.**



Conclusion: $\overrightarrow{ED}$ and $\overrightarrow{EB}$ are opposite rays.

Step 1 Identify the hypothesis p and the conclusion q of the true conditional.

***p*: Two angles form a linear pair.**

***q*: Their noncommon sides are opposite rays.**

Step 2 Analyze the conclusion.

The given statement *∠AED and ∠AEB form a linear pair* satisfies the hypothesis, so p is true. By the Law of Detachment, $\overrightarrow{ED}$ and $\overrightarrow{EB}$ *are opposite rays*, which matches q, is a true or valid conclusion.

**b. Given: If Mika goes to the beach, she will wear sunscreen.
Mika is wearing sunscreen.**

Conclusion: Mika is at the beach.

Step 1 ***p*: Mika goes to the beach.**

***q*: Mika wears sunscreen.**

Step 2 The given statement *Mika is wearing sunscreen* satisfies the conclusion q of the true conditional. However, knowing that a conditional statement and its conclusion are true does not make the hypothesis true. Mika could be wearing sunscreen because she is at the pool. The conclusion is invalid.

GuidedPractice

2A. Given: If three points are noncollinear, they determine a plane.
Points A, B, and C lie in plane G.

Conclusion: Points A, B, and C are noncollinear.

2B. Given: If a student turns in a permission slip, then the student can go on the field trip.
Felipe turned in his permission slip.

Conclusion: Felipe can go on the field trip.

StudyTip

Given Information From this point forward in this text, all given information can be assumed true.

You can also use a Venn diagram to test the validity of a conclusion.

Tom Brakefield/Getty Images

Real-WorldLink

The easiest way to distinguish monkeys from other primates is to look for a tail. Most monkey species have tails, but apes do not.

Source: *Encyclopaedia Britannica*

Example 3 Judge Conclusions Using Venn Diagrams

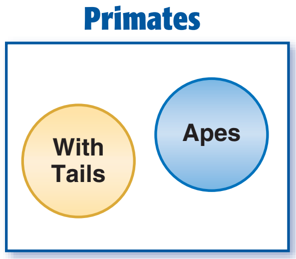
NATURE Determine whether each conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning using a Venn diagram.

Given: **If a primate is an ape, then it does not have a tail.**

Koko is a primate who does not have a tail.

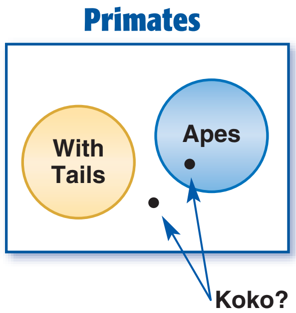
Conclusion: **Koko is an ape.**

Understand Draw a Venn diagram. According to the conditional, an ape does not have a tail, so draw a circle for apes that does not intersect the circle for primates with tails.



Plan Since we are only given that Koko does not have a tail, we can only conclude that Koko belongs outside the circle for primates with tails.

Solve This could put her in the area inside or outside of the Apes circle, so the conclusion is invalid.



Check From the given information, we know that apes are primates that do not have tails. We also know that Koko is a primate that does not have a tail. It is possible for Koko to be a primate without a tail and still not be an ape. Therefore, the conclusion *is* invalid. ✓

StudyTip

CCSS Arguments An *argument* consists of reasons, proof, or evidence to support a position. A *logical argument* such as the one shown is supported by the rules of logic. This is different from a *statistical argument*, which is supported by examples or data.

GuidedPractice

3. Given: If a figure is a square, then it is a polygon.
Figure A is a square.

Conclusion: Figure A is a polygon.

2 Law of Syllogism

The **Law of Syllogism** is another valid form of deductive reasoning. This law allows you to draw conclusions from two true conditional statements when the conclusion of one statement is the hypothesis of the other.

KeyConcept Law of Syllogism

Words	If $p \rightarrow q$ and $q \rightarrow r$ are true statements, then $p \rightarrow r$ is a true statement.
Example	*Given:* If you get a job, then you will earn money. If you earn money, then you will buy a car. *Valid Conclusion:* If you get a job, then you will buy a car.

It is important to remember that if the conclusion of the first statement is *not* the hypothesis of the second statement, no valid conclusion can be drawn.

Standardized Test Example 4 Law of Syllogism

Determine which statement follows logically from the given statements.

(1) If you like musicals, then you enjoy theater productions.

(2) If you are an actor, then you enjoy theater productions.

A If you are an actor, then you like musicals.

B If you like musicals, then you are an actor.

C If you do not enjoy musicals, then you are not an actor.

D There is no valid conclusion.

Read the Test Item

Let p, q, and r represent the parts of the given conditional statements.

p: **You like musicals.**
q: **You enjoy theater productions.**
r: **You are an actor.**

Solve the Test Item

Analyze the logic of the given conditional statement using symbols.

Statement (1): $p \rightarrow q$ Statement (2): $r \rightarrow q$

Both statements are considered true. However, the Law of Syllogism does not apply since q, the conclusion of the Statement (1), is not the hypothesis of the second statement. While choices A, B, and C may be true, the logic used to draw these conclusions is not valid. Therefore, choice D is correct.

Test-Taking Tip

True vs. Valid Conclusions
A true conclusion is not the same as a valid conclusion. True conclusions that are reached using invalid deductive reasoning are still invalid.

Guided Practice

4. Determine which statement follows logically from the given statements.

(1) If you do not get enough sleep, then you will be tired.

(2) If you are tired, then you will not do well on the test.

F If you are tired, then you will not get enough sleep.

G If you do not get enough sleep, then you will not do well on the test.

H If you do not do well on the test, then you did not get enough sleep.

J There is no valid conclusion.

Example 5 Apply Laws of Deductive Reasoning

Draw a valid conclusion from the given statements, if possible. Then state whether your conclusion was drawn using the Law of Detachment or the Law of Syllogism. If no valid conclusion can be drawn, write *no valid conclusion* and explain your reasoning.

Given: If you are 16 years old, then you can apply for a driver's license. Nate is 16 years old.

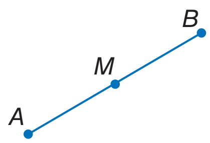
p: **You are 16 years old.**

q: **You can apply for a driver's license.**

Since *Nate is 16 years old* satisfies the hypothesis p is true. By the Law of Detachment, a valid conclusion is *Nate can apply for a driver's license.*

Guided Practice

5. Given: The midpoint divides a segment into two congruent segments. If two segments are congruent, then their measures are equal. M is the midpoint of $\overline{AB}$.



Check Your Understanding

● = Step-by-Step Solutions begin on page R14.

Example 1 **Determine whether each conclusion is based on *inductive* or *deductive* reasoning.**

1. Students at Olivia's high school must have a B average in order to participate in sports. Olivia has a B average, so she concludes that she can participate in sports at school.

2. Holly notices that every Saturday, her neighbor mows his lawn. Today is Saturday. Holly concludes her neighbor will mow his lawn.

Example 2 **Determine whether the stated conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning.**

3. **Given:** If a number is divisible by 4, then the number is divisible by 2. 12 is divisible by 4.

   Conclusion: 12 is divisible by 2.

4. **Given:** If Elan stays up late, he will be tired the next day. Elan is tired.

   Conclusion: Elan stayed up late.

Example 3 **Determine whether the stated conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning using a Venn diagram.**

5. **Given:** If a beach is public, then it does not have a lifeguard. Bayview does not have a lifeguard.

   Conclusion: Bayview is a public beach.

6. **Given:** If students pass an entrance exam, they will be accepted into college. Latisha passed the entrance exam.

   Conclusion: Latisha will be accepted into college.

Example 4

7. **MULTIPLE CHOICE** Determine which statement follows logically from the given statements.

   (1) If a triangle is a right triangle, then it has an angle that measures 90.

   (2) If a triangle has an angle that measures 90, then its acute angles are complementary.

   A If a triangle is not a right triangle, then it has an angle that measures 90.

   B If an angle of a triangle measures 90, then its acute angles are not complementary.

   C If a triangle is a right triangle, then its acute angles are complementary.

   D If a triangle has an angle that measures 90, then it is not a right triangle.

Example 5 **CCSS ARGUMENTS** **Draw a valid conclusion from the given statements, if possible. Then state whether your conclusion was drawn using the Law of Detachment or the Law of Syllogism. If no valid conclusion can be drawn, write *no valid conclusion* and explain your reasoning.**

8. **Given:** If Dalila finishes her chores, she will receive her allowance.

   If Dalila receives her allowance, she will buy a CD.

9. **Given:** Vertical angles are congruent.

   $\angle 1 \cong \angle 2$

Practice and Problem Solving

Extra Practice is on page R2.

Example 1 **Determine whether each conclusion is based on *inductive* or *deductive* reasoning.**

10. At Fumio's school if you are late five times, you will receive a detention. Fumio has been late to school five times; therefore he will receive a detention.

11. A dental assistant notices a patient has never been on time for an appointment. She concludes the patient will be late for her next appointment.

12. A person must have a membership to work out at a gym. Jesse is working out at a gym. Jesse has a membership to the gym.

13. If Eduardo decides to go to a concert tonight, he will miss football practice. Tonight, Eduardo went to a concert. Eduardo missed football practice.

14. Every Wednesday Lucy's mother calls. Today is Wednesday, so Lucy concludes her mother will call.

15. Whenever Juanita has attended a tutoring session she notices that her grades have improved. Juanita attends a tutoring session and she concludes her grades will improve.

Example 2 CCSS **CRITIQUE** **Determine whether the stated conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning.**

16. **Given:** Right angles are congruent. $\angle 1$ and $\angle 2$ are right angles.

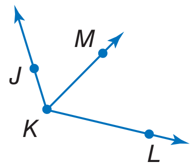
Conclusion: $\angle 1 \cong \angle 2$

17. **Given:** If a figure is a square, it has four right angles. Figure *ABCD* has four right angles.

Conclusion: Figure *ABCD* is a square.

18. **Given:** An angle bisector divides an angle into two congruent angles. $\overrightarrow{KM}$ is an angle bisector of $\angle JKL$.

Conclusion: $\angle JKM \cong \angle MKL$



19. **Given:** If you leave your lights on while your car is off, your battery will die. Your battery is dead.

Conclusion: You left your lights on while the car was off.

20. **Given:** If Dante obtains a part-time job, he can afford a car payment. Dante can afford a car payment.

Conclusion: Dante obtained a part-time job.

21. **Given:** If 75% of the prom tickets are sold, the prom will be held at the country club. 75% of the prom tickets were sold.

Conclusion: The prom will be held at the country club.

22. **COMPUTER GAMES** Refer to the game ratings at the right. Determine whether the stated conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning.

Given: If a title is rated E, then it has content that may be suitable for ages 6 and older. Cesar buys a computer game that he believes is suitable for his little sister, who is 7.

Conclusion: The game Cesar purchased has a rating of E.

Game Ratings

Rating	Age
EC	3 and older
E	6 and older
E10+	10 and older
T	13 and older
M	17 and older

Example 3 **Determine whether the stated conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning using a Venn diagram.**

23. **Given:** If the temperature drops below 32°F, it may snow. The temperature did not drop below 32°F on Monday.

Conclusion: It did not snow on Monday.

24. **Given:** If a person is a Missouri resident, he or she does not live by a beach. Michelle does not live by the beach.

Conclusion: Michelle is a Missouri resident.

25. **Given:** Some nurses wear blue uniforms. Sabrina is a nurse.

Conclusion: Sabrina wears a blue uniform.

26. **Given:** All vegetarians do not eat meat. Theo is a vegetarian.

Conclusion: Theo does not eat meat.

27 **TRANSPORTATION** There are many types of vehicles and they are classified using different sets of criteria. Determine whether the stated conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning using a Venn diagram.

Given: If a vehicle is a sport-utility vehicle, then it is a four-wheel-drive car built on a truck chassis. Ms. Rodriguez has just purchased a vehicle that has four-wheel drive.

Conclusion: Ms. Rodriguez has just purchased a sport-utility vehicle.

Examples 4–5 **28.** **GOLF** Zach Johnson won the Masters Tournament in 2007. Use the Law of Syllogism to draw a valid conclusion from each set of statements, if possible. If no valid conclusion can be drawn, write *no valid conclusion* and explain your reasoning.

(1) If Zach Johnson's score is lower than the other golfers at the end of the tournament, then he wins the tournament.

(2) If a golfer wins the Masters Tournament, then he gets a green jacket.

CCSS ARGUMENTS **Use the Law of Syllogism to draw a valid conclusion from each set of statements, if possible. If no valid conclusion can be drawn, write *no valid conclusion* and explain your reasoning.**

29. If you interview for a job, then you wear a suit.

If you interview for a job, then you will update your resume.

30. If Tina has a grade point average of 3.0 or greater, she will be on the honor roll.

If Tina is on the honor roll, then she will have her name in the school paper.

31. If two lines are perpendicular, then they intersect to form right angles.

Lines r and s form right angles.

32. If the measure of an angle is between 90 and 180, then it is obtuse.

If an angle is obtuse, then it is not acute.

33. If two lines in a plane are not parallel, then they intersect.

If two lines intersect, then they intersect in a point.

34. If a number ends in 0, then it is divisible by 2.

If a number ends in 4, then it is divisible by 2.

Draw a valid conclusion from the given statements, if possible. Then state whether your conclusion was drawn using the Law of Detachment or the Law of Syllogism. If no valid conclusion can be drawn, write *no valid conclusion* and explain your reasoning.

35. Given: If a figure is a square, then all the sides are congruent.

Figure *ABCD* is a square.

36. Given: If two angles are complementary, the sum of the measures of the angles is 90.

$\angle 1$ and $\angle 2$ are complements of each other.

37. Given: Ballet dancers like classical music.

If you like classical music, then you enjoy the opera.

38. Given: If you are athletic, then you enjoy sports.

If you are competitive, then you enjoy sports.

39. Given: If a polygon is regular, then all of its sides are congruent.

All sides of polygon *WXYZ* are congruent.

40. Given: If Bob completes a course with a grade of C, then he will not receive credit.

If Bob does not receive credit, he will have to take the course again.

41. DATA ANALYSIS The table shows the number of at bats and hits for some of the members of the Florida Marlins in a recent season.

a. Construct a scatter plot to represent the data.

b. Predict the number of hits a player with 300 at bats would get. Identify and explain your reasoning.

c. Did the player with 157 at bats or the player with 240 at bats get more hits? What type of reasoning did you use? Explain.

At Bats	Hits
13	6
576	195
240	79
502	139
157	36
64	11

Source: ESPN

H.O.T. Problems Use Higher-Order Thinking Skills

42. WRITING IN MATH Explain why the Law of Syllogism cannot be used to draw a conclusion from these conditionals.

If you wear winter gloves, then you will have warm hands.

If you do not have warm hands, then your gloves are too thin.

43. CHALLENGE Use the symbols from Lesson 2-2 for *conjunction* and *disjunction*, and the symbol for implies from Lesson 2-3 to represent the Law of Detachment and the Law of Syllogism symbolically. Let *p* represent the hypothesis, and let *q* represent the conclusion.

44. OPEN ENDED Write a pair of statements in which the Law of Syllogism can be used to reach a valid conclusion. Specify the conclusion that can be reached.

45. CCSS REASONING Students in Mr. Kendrick's class are divided into two groups for an activity. Students in group A must always tell the truth. Students in group B must always lie. Jonah and Janeka are in Mr. Kendrick's class. When asked if he and Janeka are in group A or B, Jonah says, "We are both in Group B." To which group does each student belong? Explain your reasoning.

46. WRITING IN MATH Compare and contrast inductive and deductive reasoning when making conclusions and proving conjectures.

Standardized Test Practice

47. Determine which statement follows logically from the given statements.

If you order two burritos, then you also get nachos.
Michael ordered two burritos.

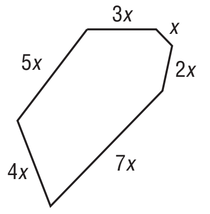
A Michael ordered one burrito.
B Michael will order two burritos.
C Michael ordered nachos.
D Michael got nachos.

48. ALGEBRA Solve for x:

$$4(x + 2) = x - 1$$

F -3
G -5
H -6
J -8

49. SHORT RESPONSE If the perimeter of the figure shown is 52 units, what is the value of x?



50. SAT/ACT If 30% of x is 50, then 60% of x is

A 300
B 250
C 175
D 150
E 100

Spiral Review

51. TIME All states in the United States observe daylight savings time except for Arizona and Hawaii. (Lesson 2-3)

a. Write a true conditional statement in if-then form for daylight savings time.

b. Write the converse of the true conditional statement. State whether the statement is *true* or *false*. If false, find a counterexample.

Construct a truth table for each compound statement. (Lesson 2-2)

52. a and b

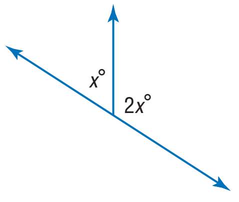
53. $\sim p$ or $\sim q$

54. k and $\sim m$

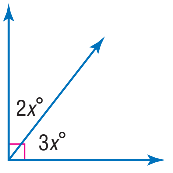
55. $\sim y$ or z

ALGEBRA Find x. (Lesson 1-5)

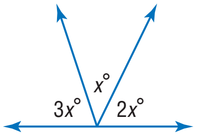
56.



57.

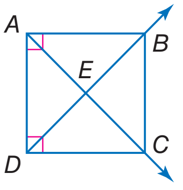


58.



Skills Review

Determine whether each statement can be assumed from the figure. Explain.



59. $\angle DAB$ is a right angle.

60. $\angle AEB \cong \angle DEC$

61. $\angle ADB$ and $\angle BDC$ are complementary.

62. $\angle DAE \cong \angle ADE$

63. $\overline{AB} \perp \overline{BC}$

64. $\angle AEB$ and $\angle BEC$ are supplementary.

EXPLORE 2-5

Geometry Lab
Necessary and Sufficient Conditions

We all know that water is a *necessary* condition for plants to survive. However, it is not a *sufficient* condition. For example, plants also need sunlight to survive.

Marc Romanelli/Photodisc/Getty Images

Necessary and sufficient conditions are important in mathematics. Consider the property of having four sides. While *having four sides* is a necessary condition for something being a square, that single condition is not, by itself, a sufficient condition to guarantee that it is a square. Trapezoids are four-sided figures that are not squares.

Condition	Definition	Examples
necessary	A condition *A* is said to be *necessary* for a condition *B*, if and only if the falsity or nonexistence of *A* guarantees the falsity or nonexistence of *B*.	Having opposite sides parallel is a necessary condition for something being a square.
sufficient	A condition *A* is said to be *sufficient* for a condition *B*, if and only if the truth or existence of *A* guarantees the truth or existence of *B*.	Being a square is a sufficient condition for something being a rectangle.

Exercises

Determine whether each statement is *true* or *false*. If false, give a counterexample.

1. Being a square is a necessary condition for being a rectangle.

2. Being a rectangle is a necessary condition for being a square.

3. Being greater than 5 is a necessary condition for being less than 10.

4. Being less than 18 is a sufficient condition for being less than 25.

5. Walking on four legs is a sufficient condition for being a dog.

6. Breathing air is a necessary condition for being a human being.

7. Being an equilateral rectangle is both a necessary and sufficient condition for being a square.

Determine whether I is a *necessary* condition for II, a *sufficient* condition for II, or *both*. Explain.

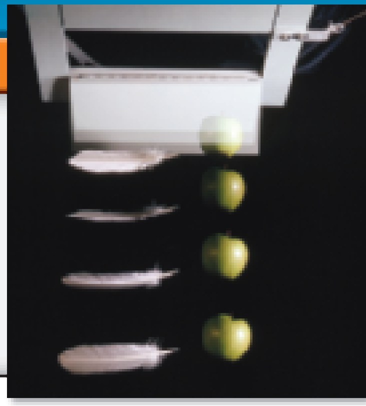
8. I. Two points are given.
II. An equation of a line can be written.

9. I. Two planes are parallel.
II. Two planes do not intersect.

10. I. Two angles are acute.
II. Two angles are complementary.

LESSON 2-5

Postulates and Paragraph Proofs



Then

- You used deductive reasoning by applying the Law of Detachment and the Law of Syllogism.

Now

1. Identify and use basic postulates about points, lines, and planes.
2. Write paragraph proofs.

Why?

- If a feather and an apple are dropped from the same height in a vacuum chamber, the two objects will fall at the same rate. This demonstrates one of Sir Isaac Newton's laws of gravity and inertia. These laws are accepted as fundamental truths of physics. Some laws in geometry also must be assumed or accepted as true.

NewVocabulary

postulate
axiom
proof
theorem
deductive argument
paragraph proof
informal proof

Common Core State Standards

Content Standards

G.MG.3 Apply geometric methods to solve problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). ★

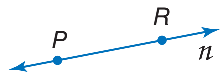
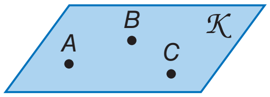
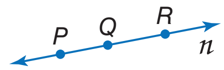
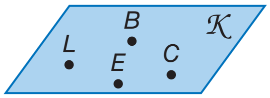
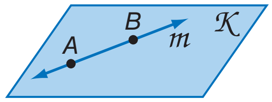
Mathematical Practices

2 Reason abstractly and quantitatively.
3 Construct viable arguments and critique the reasoning of others.

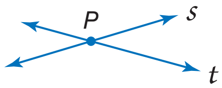
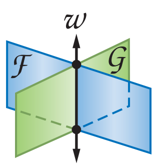
1 Points, Lines, and Planes

A **postulate** or **axiom** is a statement that is accepted as true without proof. Basic ideas about points, lines, and planes can be stated as postulates.

Postulates Points, Lines, and Planes

Words	Example	
2.1 Through any two points, there is exactly one line.		Line n is the only line through points P and R.
2.2 Through any three noncollinear points, there is exactly one plane.		Plane $\mathcal{K}$ is the only plane through noncollinear points A, B, and C.
2.3 A line contains at least two points.		Line n contains points P, Q, and R.
2.4 A plane contains at least three noncollinear points.		Plane $\mathcal{K}$ contains noncollinear points L, B, C, and E.
2.5 If two points lie in a plane, then the entire line containing those points lies in that plane.		Points A and B lie in plane $\mathcal{K}$ and line m contains points A and B, so line m is in plane $\mathcal{K}$.

KeyConcept Intersections of Lines and Planes

Words	Example	
2.6 If two lines intersect, then their intersection is exactly one point.		Lines s and t intersect at point P.
2.7 If two planes intersect, then their intersection is a line.		Planes $\mathcal{F}$ and $\mathcal{G}$ intersect in line w.

Jim Sugar/CORBIS

StudyTip

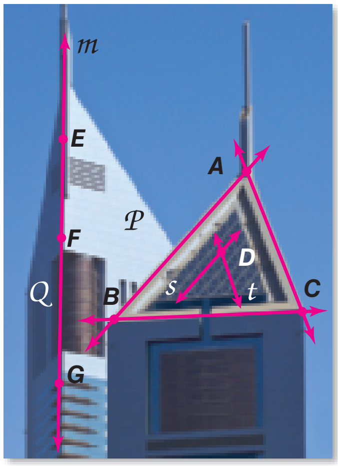
Undefined Terms Recall from Lesson 1-1 that points, lines, and planes are *undefined terms*. The postulates that you have learned in this lesson describe special relationships between them.

These additional postulates form a foundation for proofs and reasoning about points, lines, and planes.

PT

Real-World Example 1 Identifying Postulates

ARCHITECTURE Explain how the picture illustrates that each statement is true. Then state the postulate that can be used to show each statement is true.



a. Line *m* contains points *F* and *G*. Point *E* can also be on line *m*.

The edge of the building is a straight line *m*. Points *E*, *F*, and *G* lie along this edge, so they lie along a line *m*. Postulate 2.3, which states that a line contains at least two points, shows that this is true.

b. Lines *s* and *t* intersect at point *D*.

The lattice on the window of the building forms intersecting lines. Lines *s* and *t* of this lattice intersect at only one location, point *D*. Postulate 2.6, which states that if two lines intersect, then their intersection is exactly one point, shows that this is true.

GuidedPractice

1A. Points *A*, *B*, and *C* determine a plane. **1B.** Planes $\mathcal{P}$ and *Q* intersect in line *m*.

You can use postulates to explain your reasoning when analyzing statements.

PT

Example 2 Analyze Statements Using Postulates

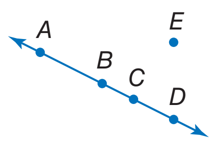
Determine whether each statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

a. If two coplanar lines intersect, then the point of intersection lies in the same plane as the two lines.

Always; Postulate 2.5 states that if two points lie in a plane, then the entire line containing those points lies in that plane. So, since both points lie in the plane, any point on those lines, including their point of intersection, also lies in the plane.

b. Four points are noncollinear.

Sometimes; Postulate 2.3 states that a line contains at least two points. This means that a line can contain two *or more* points. So four points can be noncollinear, like *A*, *E*, *C*, and *D*, or collinear, like points *A*, *B*, *C*, and *D*.



GuidedPractice

2A. Two intersecting lines determine a plane. **2B.** Three lines intersect in two points.

StudyTip

Axiomatic System An axiomatic system is a set of axioms, from which some or all axioms can be used to logically derive theorems.

2 Paragraph Proofs

To prove a conjecture, you use deductive reasoning to move from a hypothesis to the conclusion of the conjecture you are trying to prove. This is done by writing a **proof**, which is a logical argument in which each statement you make is supported by a statement that is accepted as true.

Jochem Wijnands/age fotostock

Once a statement or conjecture has been proven, it is called a **theorem**, and it can be used as a reason to justify statements in other proofs.

KeyConcept The Proof Process

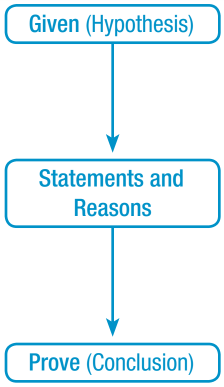
Step 1 List the given information and, if possible, draw a diagram to illustrate this information.

Step 2 State the theorem or conjecture to be proven.

Step 3 Create a **deductive argument** by forming a logical chain of statements linking the given to what you are trying to prove.

Step 4 Justify each statement with a reason. Reasons include definitions, algebraic properties, postulates, and theorems.

Step 5 State what it is that you have proven.



StudyTip

Proposition A *proposition* is a statement that makes an assertion that is either false or true. In mathematics, a proposition is usually used to mean a true assertion and can be synonymous with theorem.

One method of proving statements and conjectures, a **paragraph proof**, involves writing a paragraph to explain why a conjecture for a given situation is true. Paragraph proofs are also called **informal proofs**, although the term *informal* is not meant to imply that this form of proof is any less valid than any other type of proof.

Example 3 Write a Paragraph Proof

Given that M is the midpoint of $\overline{XY}$ write a paragraph proof to show that $\overline{XM} \cong \overline{MY}$.

Steps 1 and 2 →

Given: M is the midpoint of $\overline{XY}$.

Prove: $\overline{XM} \cong \overline{MY}$

Steps 3 and 4 →

If M is the midpoint of $\overline{XY}$, then from the definition of midpoint of a segment, we know that $XM = MY$. This means that $\overline{XM}$ and $\overline{MY}$ have the same measure. By the definition of congruence, if two segments have the same measure, then they are congruent.

Step 5 →

Thus, $\overline{XM} \cong \overline{MY}$.

Problem-SolvingTip

Work Backward One strategy for writing a proof is to *work backward*. Start with what you are trying to prove, and work backward step by step until you reach the given information.

GuidedPractice

3. Given that C is between A and B and $\overline{AC} \cong \overline{CB}$, write a paragraph proof to show that C is the midpoint of $\overline{AB}$.

Once a conjecture has been proven true, it can be stated as a theorem and used in other proofs. The conjecture in Example 3 is known as the Midpoint Theorem.

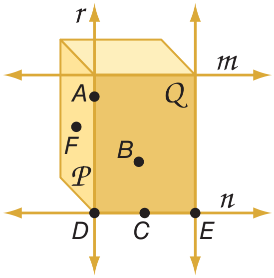
Theorem 2.1 Midpoint Theorem

If M is the midpoint of $\overline{AB}$, then $\overline{AM} \cong \overline{MB}$.

Check Your Understanding

(green circle) = Step-by-Step Solutions begin on page R14.

Example 1 **Explain how the figure illustrates that each statement is true. Then state the postulate that can be used to show each statement is true.**

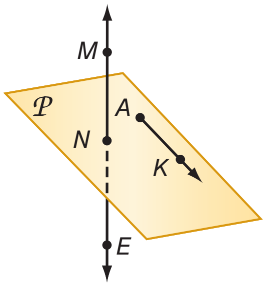


1. Planes $\mathcal{P}$ and $\mathcal{Q}$ intersect in line r.
2. Lines r and n intersect at point D.
3. Line n contains points C, D, and E.
4. Plane $\mathcal{P}$ contains the points A, F, and D.
5. Line n lies in plane $\mathcal{Q}$.
6. Line r is the only line through points A and D.

Example 2 **Determine whether each statement is *always*, *sometimes*, or *never* true. Explain your reasoning.**

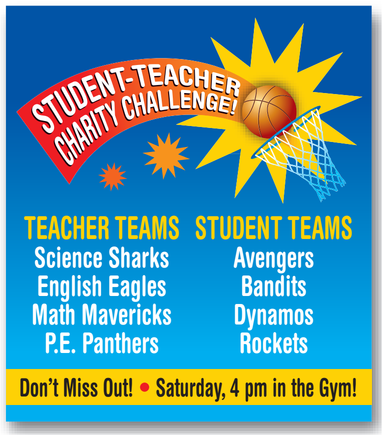
7. The intersection of three planes is a line.
8. Line r contains only point $\mathcal{P}$.
9. Through two points, there is exactly one line.

In the figure, $\overrightarrow{AK}$ is in plane $\mathcal{P}$ and M is on $\overleftrightarrow{NE}$. State the postulate that can be used to show each statement is true.



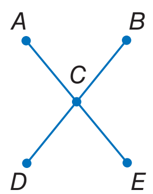
10. M, K, and N are coplanar.
11. $\overleftrightarrow{NE}$ contains points N and M.
12. N and K are collinear.
13. Points N, K, and A are coplanar.

14. **SPORTS** Each year, Jennifer's school hosts a student vs. teacher basketball tournament to raise money for charity. This year, there are eight teams participating in the tournament. During the first round, each team plays all of the other teams.
    a. How many games will be played in the first round?
    b. Draw a diagram to model the number of first round games. Which postulate can be used to justify your diagram?
    c. Find a numerical method that you could use regardless of the number of the teams in the tournament to calculate the number of games in the first round.



Example 3

15. **CCSS ARGUMENTS** In the figure at the right, $\overline{AE} \cong \overline{DB}$ and C is the midpoint of $\overline{AE}$ and $\overline{DB}$. Write a paragraph proof to show that $AC = CB$.

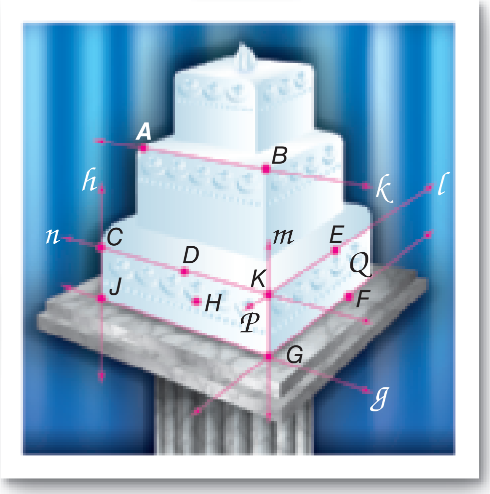


Practice and Problem Solving

Extra Practice is on page R2.

Example 1

CAKES **Explain how the picture illustrates that each statement is true. Then state the postulate that can be used to show each statement is true.**



16. Lines n and ℓ intersect at point K.

17. Planes $\mathcal{P}$ and $\mathcal{Q}$ intersect in line m.

18. Points D, K, and H determine a plane.

19. Point D is also on the line n through points C and K.

20. Points D and H are collinear.

21. Points E, F, and G are coplanar.

22. $\overleftrightarrow{EF}$ lies in plane $\mathcal{Q}$.

23. Lines h and g intersect at point J.

Example 2

Determine whether each statement is *always*, *sometimes*, or *never* true. Explain.

24. There is exactly one plane that contains noncollinear points A, B, and C.

25. There are at least three lines through points J and K.

26. If points M, N, and P lie in plane $\mathcal{X}$, then they are collinear.

27. Points X and Y are in plane $\mathcal{Z}$. Any point collinear with X and Y is in plane $\mathcal{Z}$.

28. The intersection of two planes can be a point.

29. Points A, B, and C determine a plane.

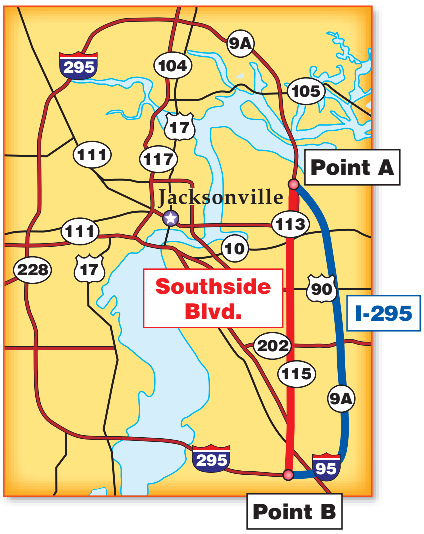
Example 3

30. **PROOF** Point Y is the midpoint of $\overline{XZ}$. Z is the midpoint of $\overline{YW}$. Prove that $\overline{XY} \cong \overline{ZW}$.

31. **PROOF** Point L is the midpoint of $\overline{JK}$. $\overline{JK}$ intersects $\overline{MK}$ at K. If $\overline{MK} \cong \overline{JL}$, prove that $\overline{LK} \cong \overline{MK}$.

32. CCSS **ARGUMENTS** Last weekend, Emilio and his friends spent Saturday afternoon at the park. There were several people there with bikes and skateboards. There were a total of 11 bikes and skateboards that had a total of 36 wheels. Use a paragraph proof to show how many bikes and how many skateboards there were.

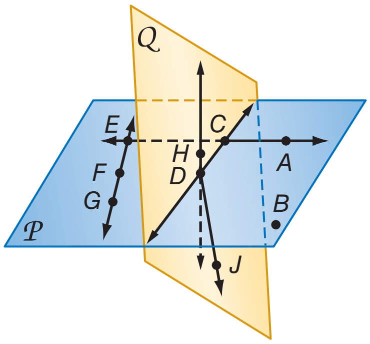
33. **DRIVING** Keisha is traveling from point A to point B. Two possible routes are shown on the map. Assume that the speed limit on Southside Boulevard is 55 miles per hour and the speed limit on I–295 is 70 miles per hour.



a. Which of the two routes covers the shortest distance? Explain your reasoning.

b. If the distance from point A to point B along Southside Boulevard is 10.5 miles and the distance along I-295 is 11.6 miles, which route is faster, assuming that Keisha drives the speed limit?

In the figure at the right, $\overleftrightarrow{CD}$ and $\overleftrightarrow{CE}$ lie in plane $\mathcal{P}$ and $\overleftrightarrow{DH}$ and $\overleftrightarrow{DJ}$ lie in plane Q. State the postulate that can be used to show each statement is true.



34. Points C and B are collinear.

35 $\overleftrightarrow{EG}$ contains points E, F, and G.

36. $\overleftrightarrow{DA}$ lies in plane $\mathcal{P}$.

37. Points D and F are collinear.

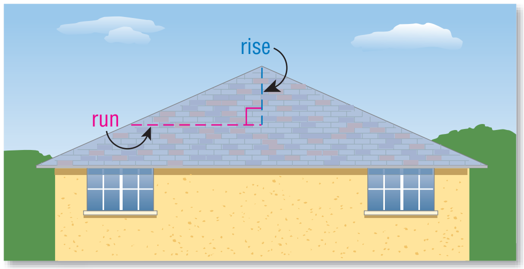
38. Points C, D, and B are coplanar.

39. Plane Q contains the points C, H, D, and J.

40. $\overleftrightarrow{AC}$ and $\overleftrightarrow{FG}$ intersect at point E.

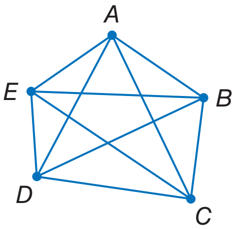
41. Plane $\mathcal{P}$ and plane Q intersect at $\overleftrightarrow{CD}$.

42. CCSS **ARGUMENTS** Roofs are designed based on the materials used to ensure that water does not leak into the buildings they cover. Some roofs are constructed from waterproof material, and others are constructed for watershed, or gravity removal of water. The pitch of a roof is the rise over the run, which is generally measured in rise per foot of run. Use the statements below to write a paragraph proof justifying the following statement: The pitch of the roof in Den's design is not steep enough.



- Waterproof roofs should have a minimum slope of $\frac{1}{4}$ inch per foot.
- Watershed roofs should have a minimum slope of 4 inches per foot.
- Den is designing a house with a watershed roof.
- The pitch in Den's design is 2 inches per foot.

43. **NETWORKS** Diego is setting up a network of multiple computers so that each computer is connected to every other. The diagram at the right illustrates this network if Diego has 5 computers.



a. Draw diagrams of the networks if Diego has 2, 3, 4, or 6 computers.

b. Create a table with the number of computers and the number of connections for the diagrams you drew.

c. If there are n computers in the network, write an expression for the number of computers to which each of the computers is connected.

d. If there are n computers in the network, write an expression for the number of connections there are.

44. CCSS **SENSE-MAKING** The photo is of the rotunda in the capitol building in St. Paul, Minnesota. A rotunda is a round building, usually covered by a dome. Use Postulate 2.1 to help you answer parts **a–c.**

a. If you were standing in the middle of the rotunda, which arched exit is the closest to you?

b. What information did you use to formulate your answer?

c. What term describes the shortest distance from the center of a circle to a point on the circle?

H.O.T. Problems Use Higher-Order Thinking Skills

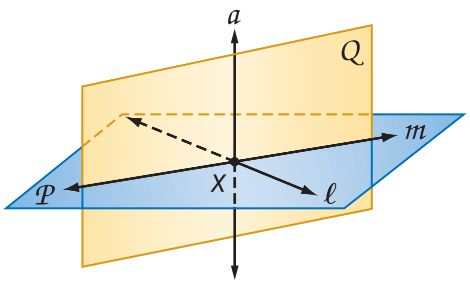
45. **ERROR ANALYSIS** Omari and Lisa were working on a paragraph proof to prove that if $\overline{AB}$ is congruent to $\overline{BD}$ and A, B, and D are collinear, then B is the midpoint of $\overline{AD}$. Each student started his or her proof in a different way. Is either of them correct? Explain your reasoning.

Omari	Lisa
If B is the midpoint of $\overline{AB}$, then B divides $\overline{AD}$ into two congruent segments.	$\overline{AB}$ is congruent to $\overline{BD}$ and A, B, and D are collinear.

46. **OPEN ENDED** Draw a figure that satisfies five of the seven postulates you have learned. Explain which postulates you chose and how your figure satisfies each postulate.

47. **CHALLENGE** Use the following true statement and the definitions and postulates you have learned to answer each question.

Two planes are perpendicular if and only if one plane contains a line perpendicular to the second plane.



a. Through a given point, there passes one and only one plane perpendicular to a given line. If plane Q is perpendicular to line ℓ at point X and line ℓ lies in plane $\mathcal{P}$, what must also be true?

b. Through a given point, there passes one and only one line perpendicular to a given plane. If plane Q is perpendicular to plane $\mathcal{P}$ at point X and line a lies in plane Q, what must also be true?

REASONING **Determine if each statement is *sometimes*, *always*, or *never* true. Explain your reasoning or provide a counterexample.**

48. Through any three points, there is exactly one plane.

49. Three coplanar lines have two points of intersection.

50. **WRITING IN MATH** How does writing a proof require logical thinking?

Standardized Test Practice

51. ALGEBRA Which is one of the solutions of the equation $3x^2 - 5x + 1 = 0$?

A $\frac{5 + \sqrt{13}}{6}$

B $\frac{-5 - \sqrt{13}}{6}$

C $\frac{5}{6} - \sqrt{13}$

D $-\frac{5}{6} + \sqrt{13}$

52. GRIDDED RESPONSE Steve has 20 marbles in a bag, all the same size and shape. There are 8 red, 2 blue, and 10 yellow marbles in the bag. He will select a marble from the bag at random. What is the probability that the marble Steve selects will be yellow?

53. Which statement *cannot* be true?

F Three noncollinear points determine a plane.

G Two lines intersect in exactly one point.

H At least two lines can contain the same two points.

J A midpoint divides a segment into two congruent segments.

54. SAT/ACT What is the greatest number of regions that can be formed if 3 distinct lines intersect a circle?

A 3

B 4

C 5

D 6

E 7

Spiral Review

Determine whether a valid conclusion can be reached from the two true statements using the Law of Detachment or the Law of Syllogism. If a valid conclusion is possible, state it and the law that is used. If a valid conclusion does not follow, write *no conclusion*. (Lesson 2-4)

55. (1) If two angles are vertical, then they do not form a linear pair.
(2) If two angles form a linear pair, then they are not congruent.

56. (1) If an angle is acute, then its measure is less than 90.
(2) $\angle EFG$ is acute.

Write each statement in if-then form. (Lesson 2-3)

57. Happy people rarely correct their faults.

58. A champion is afraid of losing.

Use the following statements to write a compound statement for each conjunction. Then find its truth value. Explain your reasoning. (Lesson 2-2)

p: ***M* is on $\overline{AB}$.**

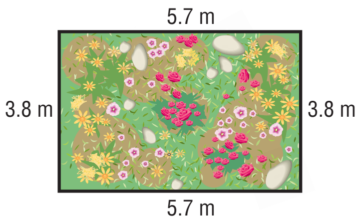
q: $AM + MB = AB$

r: ***M* is the midpoint of $\overline{AB}$.**

59. $p \wedge q$

60. $\sim p \vee \sim r$

61. GARDENING A landscape designer is putting black plastic edging around a rectangular flower garden that has length 5.7 meters and width 3.8 meters. The edging is sold in 5-meter lengths. Find the perimeter of the garden and determine how much edging the designer should buy. (Lesson 1-6)



62. HEIGHT Taylor is 5 feet 8 inches tall. How many inches tall is Taylor? (Lesson 0-1)

Skills Review

ALGEBRA Solve each equation.

63. $4x - 3 = 19$

64. $\frac{1}{3}x + 6 = 14$

65. $5(x^2 + 2) = 30$

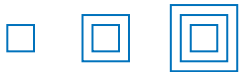
CHAPTER 2

Mid-Chapter Quiz

Lessons 2-1 through 2-5

Write a conjecture that describes the pattern in each sequence. Then use your conjecture to find the next item in the sequence. (Lesson 2-1)

1. 5, 5, 10, 15, 25, . . .

2. 

Find a counterexample to show that each conjecture is false. (Lesson 2-1)

3. If $AB = BC$, then B is the midpoint of $\overline{AC}$.

4. If n is a real number, then $n^3 > n$.

Use the following statements to write a compound statement for each conjunction or disjunction. Then find its truth value. Explain your reasoning. (Lesson 2-2)

p: A dollar is equal to 100 cents.

q: There are 4 quarters in a dollar.

r: February is the month before January.

5. $p \wedge r$

6. p and q

7. $p \wedge \sim r$

8. Copy and complete the truth table. (Lesson 2-2)

p	q	$\sim q$	$p \vee \sim q$
T	F		
F	T		
F	F		
T	T		

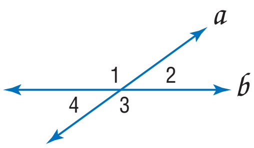
Identify the hypothesis and conclusion of each conditional statement. (Lesson 2-3)

9. If a polygon has five sides, then it is a pentagon.

10. If $4x - 6 = 10$, then $x = 4$.

11. An angle with a measure less than 90 is an acute angle.

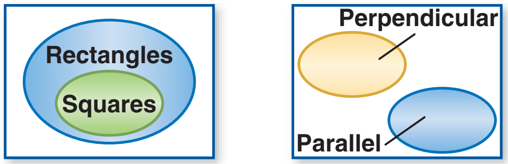
Determine the truth value of each conditional statement. If *true*, explain your reasoning. If *false*, give a counterexample. (Lesson 2-3)



12. If ∠1 and ∠2 form a linear pair, they are supplementary angles.

13. If ∠1 and ∠4 form a linear pair, they are congruent angles.

Use the Venn diagrams below to determine the truth value of each conditional. Explain your reasoning. (Lesson 2-3)



14. If a polygon is a square, then it is a rectangle.

15. If two lines are perpendicular, then they cannot be parallel.

16. **FOOTBALL** The Indianapolis Colts played the Chicago Bears in the 2007 Super Bowl. Determine whether the stated conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning. (Lesson 2-4)

Given: The Super Bowl winner has the highest score at the end of the game. The Colts had a score of 29 and the Bears had a score of 17.
Conclusion: The Colts won the Super Bowl.

17. **MULTIPLE CHOICE** Determine which statement follows logically from the given statements. (Lesson 2-4)

(1) If you are a junior in high school, then you are at least 16 years old.

(2) If you are at least 16 years old, then you are old enough to drive.

A If you are old enough to drive, then you are a junior in high school.

B If you are not old enough to drive, then you are a sophomore in high school.

C If you are a junior in high school, then you are old enough to drive.

D No valid conclusion possible.

Determine whether each statement is *always*, *sometimes*, or *never* true. Explain your reasoning. (Lesson 2-5)

18. Points J, K, L, and N are noncollinear and lie in the same plane M.

19. There is exactly one line through points R and S.

20. Line a contains only point Q.

LESSON 2-6 Algebraic Proof

Then

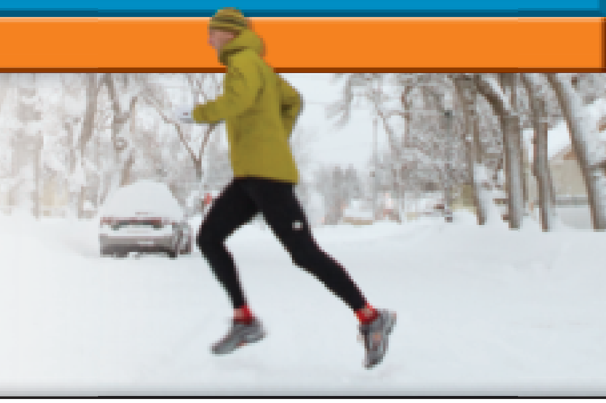
- You used postulates about points, lines, and planes to write paragraph proofs.

Now

1. Use algebra to write two-column proofs.
2. Use properties of equality to write geometric proofs.

Why?

- The Fahrenheit scale sets the freezing and boiling points of water at 32° and 212°, respectively, while the Celsius scale sets them at 0° and 100°. You can use an algebraic proof to show that if these scales are related by the formula $C = \frac{5}{9}(F - 32)$, then they are also related by the formula $F = \frac{9}{5}C + 32$.



NewVocabulary

algebraic proof
two-column proof
formal proof

Common Core State Standards

Content Standards
Preparation for G.CO.9
Prove theorems about lines and angles.

Mathematical Practices
3 Construct viable arguments and critique the reasoning of others.

1 Algebraic Proof

Algebra is a system with sets of numbers, operations, and properties that allow you to perform algebraic operations. The following table summarizes several properties of real numbers that you studied in algebra.

KeyConcept Properties of Real Numbers

The following properties are true for any real numbers *a*, *b*, and *c*.

Addition Property of Equality	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	If $a = b$, then $a - c = b - c$.
Multiplication Property of Equality	If $a = b$, then $a \cdot c = b \cdot c$.
Division Property of Equality	If $a = b$ and $c \neq 0$, then, $\frac{a}{c} = \frac{b}{c}$.
Reflexive Property of Equality	$a = a$
Symmetric Property of Equality	If $a = b$, then $b = a$.
Transitive Property of Equality	If $a = b$ and $b = c$, then $a = c$.
Substitution Property of Equality	If $a = b$, then *a* may be replaced by *b* in any equation or expression.
Distributive Property	$a(b + c) = ab + ac$

An **algebraic proof** is a proof that is made up of a series of algebraic statements. The properties of equality provide justification for many statements in algebraic proofs.

Example 1 Justify Each Step When Solving an Equation

Prove that if $-5(x + 4) = 70$, then $x = -18$. Write a justification for each step.

$-5(x + 4) = 70$	Original equation or Given
$-5x + (-5)4 = 70$	Distributive Property
$-5x - 20 = 70$	Substitution Property of Equality
$-5x - 20 + 20 = 70 + 20$	Addition Property of Equality
$-5x = 90$	Substitution Property of Equality
$\frac{-5x}{-5} = \frac{90}{-5}$	Division Property of Equality
$x = -18$	Substitution Property of Equality

GuidedPractice

State the property that justifies each statement.

1A. If $4 + (-5) = -1$, then $x + 4 + (-5) = x - 1$.

1B. If $5 = y$, then $y = 5$.

1C. Prove that if $2x - 13 = -5$, then $x = 4$. Write a justification for each step.

Example 1 is a proof of the conditional statement *If* $-5(x + 4) = 70$, *then* $x = -18$. Notice that the column on the left is a step-by-step process that leads to a solution. The column on the right contains the reason for each statement.

StudyTip

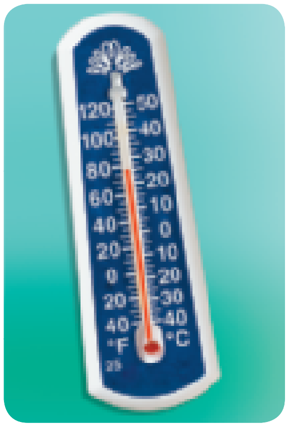
CCSS Arguments An *algorithm* is a series of steps for carrying out a procedure or solving a problem. Proofs can be considered a type of algorithm because they go step by step.

In geometry, a similar format is used to prove conjectures and theorems. A **two-column proof** or **formal proof** contains *statements* and *reasons* organized in two columns.

PT

Real-World Example 2 Write an Algebraic Proof

SCIENCE If the formula to convert a Fahrenheit temperature to a Celsius temperature is $C = \frac{5}{9}(F - 32)$, then the formula to convert a Celsius temperature to a Fahrenheit temperature is $F = \frac{9}{5}C + 32$. Write a two-column proof to verify this conjecture.



Begin by stating what is given and what you are to prove.

Given: $C = \frac{5}{9}(F - 32)$

Prove: $F = \frac{9}{5}C + 32$

Proof:

Statements	Reasons
1. $C = \frac{5}{9}(F - 32)$	**1.** Given
2. $\frac{9}{5}C = \frac{9}{5} \cdot \frac{5}{9}(F - 32)$	**2.** Multiplication Property of Equality
3. $\frac{9}{5}C = F - 32$	**3.** Substitution Property of Equality
4. $\frac{9}{5}C + 32 = F - 32 + 32$	**4.** Addition Property of Equality
5. $\frac{9}{5}C + 32 = F$	**5.** Substitution Property of Equality
6. $F = \frac{9}{5}C + 32$	**6.** Symmetric Property of Equality

StudyTip

Mental Math If your teacher permits you to do so, some steps may be eliminated by performing mental calculations. For example, steps 2 and 4 in Example 2 could be omitted. Then the reason for statement 3 would be Multiplication Property of Equality and the reason for statement 5 would be Addition Property of Equality.

GuidedPractice

Write a two-column proof to verify that each conjecture is true.

2A. If $\frac{5x + 1}{2} - 8 = 0$, then $x = 3$.

2B. **PHYSICS** If the distance d moved by an object with initial velocity u and final velocity v in time t is given by $d = t \cdot \frac{u + v}{2}$, then $u = \frac{2d}{t} - v$.

2 Geometric Proof

Since geometry also uses variables, numbers, and operations, many of the properties of equality used in algebra are also true in geometry. For example, segment measures and angle measures are real numbers, so properties from algebra can be used to discuss their relationships as shown in the table below.

Property	Segments	Angles
Reflexive	$AB = AB$	$m\angle 1 = m\angle 1$
Symmetric	If $AB = CD$, then $CD = AB$.	If $m\angle 1 = m\angle 2$, then $m\angle 2 = m\angle 1$.
Transitive	If $AB = CD$ and $CD = EF$, then $AB = EF$.	If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $m\angle 1 = m\angle 3$.

StudyTip

Commutative and Associative Properties Throughout this text we shall assume that if a, b, and c are real numbers, then the following properties are true.

Commutative Property of Addition
$a + b = b + a$

Commutative Property of Multiplication
$a \cdot b = b \cdot a$

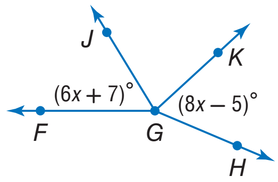
Associative Property of Addition
$(a + b) + c = a + (b + c)$

Associative Property of Multiplication
$(a \cdot b) \cdot c = a \cdot (b \cdot c)$

These properties can be used to write geometric proofs.

Example 3 Write a Geometric Proof

If $\angle FGJ \cong \angle JGK$ and $\angle JGK \cong \angle KGH$, then $x = 6$. Write a two-column proof to verify this conjecture.



Given: $\angle FGJ \cong \angle JGK$, $\angle JGK \cong \angle KGH$, $m\angle FGJ = 6x + 7$, $m\angle KGH = 8x - 5$

Prove: $x = 6$

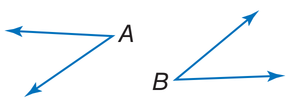
Proof:

Statements	Reasons
1. $m\angle FGH = 6x + 7$, $m\angle KGH = 8x - 5$; $\angle FGJ \cong \angle JGK$; $\angle JGK \cong \angle KGH$	1. Given
2. $m\angle FGJ = m\angle JGK$; $m\angle JGK = m\angle KGH$	2. Definition of congruent angles
3. $m\angle FGJ = m\angle KGH$	3. Transitive Property of Equality
4. $6x + 7 = 8x - 5$	4. Substitution Property of Equality
5. $6x + 7 + 5 = 8x - 5 + 5$	5. Addition Property of Equality
6. $6x + 12 = 8x$	6. Substitution Property of Equality
7. $6x + 12 - 6x = 8x - 6x$	7. Subtraction Property of Equality
8. $12 = 2x$	8. Substitution Property of Equality
9. $\frac{12}{2} = \frac{2x}{2}$	9. Division Property of Equality
10. $6 = x$	10. Substitution Property of Equality
11. $x = 6$	11. Symmetric Property of Equality

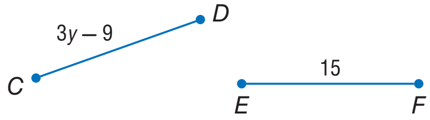
GuidedPractice

Write a two-column proof to verify each conjecture.

3A. If $\angle A \cong \angle B$ and $m\angle A = 37$, then $m\angle B = 37$.



3B. If $\overline{CD} \cong \overline{EF}$, then $y = 8$.



Check Your Understanding

● = Step-by-Step Solutions begin on page R14.

Example 1 **State the property that justifies each statement.**

1. If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $m\angle 1 = m\angle 3$.

2. $XY = XY$

3. If $5 = x$, then $x = 5$.

4. If $2x + 5 = 11$, then $2x = 6$.

Example 2 **5.** Complete the following proof.

Given: $\frac{y + 2}{3} = 3$

Prove: $y = 7$

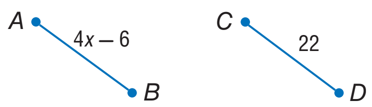
Proof:

Statements	Reasons
a. ___?___	**a.** Given
b. $3\left(\frac{y+2}{3}\right) = 3(3)$	**b.** ___?___
c. ___?___	**c.** ___?___
d. $y = 7$	**d.** Subtraction Property

Examples 2–3 **PROOF** **Write a two-column proof to verify each conjecture.**

6. If $-4(x - 3) + 5x = 24$, then $x = 12$.

7. If $\overline{AB} \cong \overline{CD}$, then $x = 7$.



8. CCSS **ARGUMENTS** Mai-Lin measures her heart rate whenever she exercises and tries to make sure that she is staying in her target heart rate zone. The American Heart Association suggests a target heart rate of $T = 0.75(220 - a)$, where T is a person's target heart rate and a is his or her age.

a. Prove that given a person's target heart rate, you can calculate his or her age using the formula $a = 220 - \frac{T}{0.75}$.

b. If Mai-Lin's target heart rate is 153, then how old is she? What property justifies your calculation?

Practice and Problem Solving

Extra Practice is on page R2.

Example 1 **State the property that justifies each statement.**

9. If $a + 10 = 20$, then $a = 10$.

10. If $\frac{x}{3} = -15$, then $x = -45$.

11 If $4x - 5 = x + 12$, then $4x = x + 17$.

12. If $\frac{1}{5}BC = \frac{1}{5}DE$, then $BC = DE$.

State the property that justifies each statement.

13. If $5(x + 7) = -3$, then $5x + 35 = -3$.

14. If $m\angle 1 = 25$ and $m\angle 2 = 25$, then $m\angle 1 = m\angle 2$.

15. If $AB = BC$ and $BC = CD$, then $AB = CD$.

16. If $3\left(x - \frac{2}{3}\right) = 4$, then $3x - 2 = 4$.

Example 2 CCSS **ARGUMENTS** **Complete each proof.**

17. Given: $\frac{8 - 3x}{4} = 32$

Prove: $x = -40$

Proof:

Statements	Reasons
a. $\frac{8 - 3x}{4} = 32$	**a.** Given
b. $4\left(\frac{8 - 3x}{4}\right) = 4(32)$	**b.** ___?___
c. $8 - 3x = 128$	**c.** ___?___
d. ___?___	**d.** Subtraction Property
e. $x = -40$	**e.** ___?___

18. Given: $\frac{1}{5}x + 3 = 2x - 24$

Prove: $x = 15$

Proof:

Statements	Reasons
a. ___?___	**a.** Given
b. ___?___	**b.** Multiplication Property
c. $x + 15 = 10x - 120$	**c.** ___?___
d. ___?___	**d.** Subtraction Property
e. $135 = 9x$	**e.** ___?___
f. ___?___	**f.** Division Property
g. ___?___	**g.** Symmetric Property

Example 3 **PROOF** **Write a two-column proof to verify each conjecture.**

19. If $-\frac{1}{3}n = 12$, then $n = -36$.

20. If $-3r + \frac{1}{2} = 4$, then $r = -\frac{7}{6}$.

21 **SCIENCE** Acceleration a in feet per second squared, distance traveled d in feet, velocity v in feet per second, and time t in seconds are related in the formula $d = vt + \frac{1}{2}at^2$.

a. Prove that if the values for distance, velocity, and time are known, then the acceleration of an object can be calculated using the formula $a = \frac{2d - 2vt}{t^2}$.

b. If an object travels 2850 feet in 30 seconds with an initial velocity of 50 feet per second, what is the acceleration of the object? What property justifies your calculation?

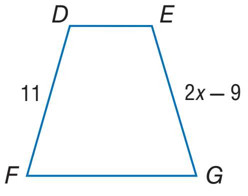
22. CCSS **ARGUMENTS** The Ideal Gas Law is given by the formula $PV = nRT$, where $P =$ pressure in atmospheres, $V =$ volume in liters, $n =$ the amount of gas in moles, R is a constant value, and $T =$ temperature in degrees Kelvin.

a. Prove that if the pressure, volume, and amount of the gas are known, then the formula $T = \frac{PV}{nR}$ gives the temperature of the gas.

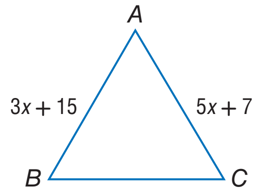
b. If you have 1 mole of oxygen with a volume of 25 liters at a pressure of 1 atmosphere, what is the temperature of the gas? The value of R is 0.0821. What property justifies your calculation?

PROOF **Write a two-column proof.**

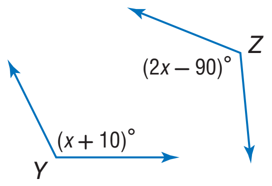
23. If $\overline{DF} \cong \overline{EG}$, then $x = 10$.



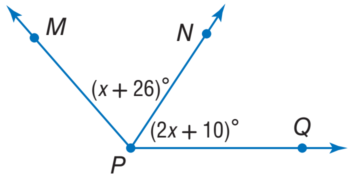
24. If $\overline{AB} \cong \overline{AC}$, then $x = 4$.



25. If $\angle Y \cong \angle Z$, then $x = 100$.



26. If $\angle MPN \cong \angle QPN$, then $x = 16$.

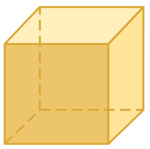


27. **ELECTRICITY** The voltage V of a circuit can be calculated using the formula $V = \frac{P}{I}$, where P is the power and I is the current of the circuit.

a. Write a proof to show that when the power is constant, the voltage is halved when the current is doubled.

b. Write a proof to show that when the current is constant, the voltage is doubled when the power is doubled.

28. **MULTIPLE REPRESENTATIONS** Consider a cube with a side length of s.



s units

a. Concrete Sketch or build a model of cubes with side lengths of 2, 4, 8, and 16 units.

b. Tabular Find the volume of each cube. Organize your results into a table like the one shown.

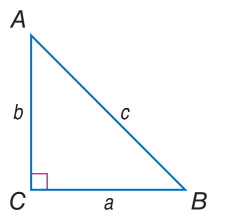
Side Lenth (s)	Volume (V)
2	
4	
8	
16	

c. Verbal Use your table to make a conjecture about the change in volume when the side length of a cube is doubled. Express your conjecture in words.

d. Analytical Write your conjecture as an algebraic equation.

e. Logical Write a proof of your conjecture. Be sure to write the *Given* and *Prove* statements at the beginning of your proof.

29. **PYTHAGOREAN THEOREM** The Pythagorean Theorem states that in a right triangle ABC, the sum of the squares of the measures of the lengths of the legs, a and b, equals the square of the measure of the hypotenuse c, or $a^2 + b^2 = c^2$. Write a two-column proof to verify that $a = \sqrt{c^2 - b^2}$. Use the Square Root Property of Equality, which states that if $a^2 = b^2$, then $a = \pm\sqrt{b^2}$.

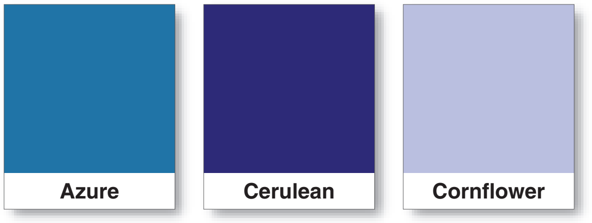


An *equivalence relation* is any relationship that satisfies the Reflexive, Symmetric, and Transitive Properties. For real numbers, equality is one type of equivalence relation. Determine whether each relation is an equivalence relation. Explain your reasoning.

30. "has the same birthday as," for the set of all human beings

31 "is taller than," for the set of all human beings

32. "is bluer than" for all the paint colors with blue in them



33. $\neq$, for the set of real numbers

34. $\geq$, for the set of real numbers

35. $\approx$, for the set of real numbers

H.O.T. Problems Use Higher-Order Thinking Skills

36. **OPEN ENDED** Give one real-world *example* and one real-world *non-example* of the Symmetric, Transitive, and Substitution properties.

37. CCSS **SENSE-MAKING** Point P is located on $\overline{AB}$. The length of $\overline{AP}$ is $2x + 3$, and the length of $\overline{PB}$ is $\frac{3x + 1}{2}$. Segment AB is 10.5 units long. Draw a diagram of this situation, and prove that point P is located two thirds of the way between point A and point B.

REASONING **Classify each statement below as *sometimes*, *always*, or *never* true. Explain your reasoning.**

38. If a and b are real numbers and $a + b = 0$, then $a = -b$.

39. If a and b are real numbers and $a^2 = b$, then $a = \sqrt{b}$.

40. **CHALLENGE** Ayana makes a conjecture that the sum of two odd integers is an even integer.

a. List information that supports this conjecture. Then explain why the information you listed does not prove that this conjecture is true.

b. Two odd integers can be represented by the expressions $2n - 1$ and $2m - 1$, where n and m are both integers. Give information that supports this statement.

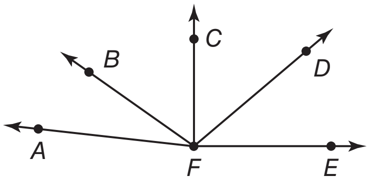
c. If a number is even, then it is a multiple of what number? Explain in words how you could use the expressions in part **a** and your answer to part **b** to prove Ayana's conjecture.

d. Write an algebraic proof that the sum of two odd integers is an even integer.

41. **WRITING IN MATH** Why is it useful to have different formats that can be used when writing a proof?

Standardized Test Practice

42. In the diagram, $m\angle CFE = 90$ and $\angle AFB \cong \angle CFD$. Which of the following conclusions does not have to be true?



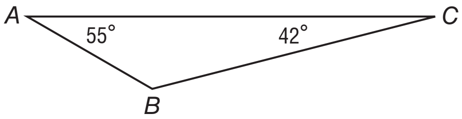
A $m\angle BFD = m\angle BFD$

B $\overline{BF}$ bisects $\angle AFD$.

C $m\angle CFD = m\angle AFB$

D $\angle CFE$ is a right angle.

43. SHORT RESPONSE Find the measure of $\angle B$ when $m\angle A = 55$ and $m\angle C = 42$.



44. ALGEBRA Kendra's walk-a-thon supporters have pledged \$30 plus \$7.50 for each mile she walks. Rebecca's supporters have pledged \$45 plus \$3.75 for each mile she walks. After how many miles will Kendra and Rebecca have raised the same amount of money?

F 10

G 8

H 5

J 4

45. SAT/ACT When 17 is added to $4m$, the result is $15z$. Which of the following equations represents the statement above?

A $17 + 15z = 4m$

B $(4m)(15z) = 17$

C $4m - 15z = 17$

D $17(4m) = 15z$

E $4m + 17 = 15z$

Spiral Review

Determine whether the following statements are *always*, *sometimes*, or *never* true. Explain. (Lesson 2-5)

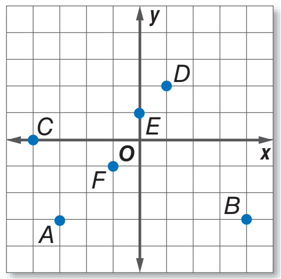
46. Four points will lie in one plane.

47. Two obtuse angles will be supplementary.

48. Planes $\mathcal{P}$ and $\mathcal{Q}$ intersect in line m. Line m lies in both plane $\mathcal{P}$ and plane $\mathcal{Q}$.

49. ADVERTISING An ad for Speedy Delivery Service says *When it has to be there fast, it has to be Speedy*. Catalina needs to send a package fast. Does it follow that she should use Speedy? Explain. (Lesson 2-4)

Write the ordered pair for each point shown. (Lesson 0-7)



50. A

51. B

52. C

53. D

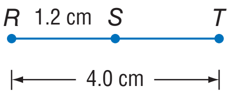
54. E

55. F

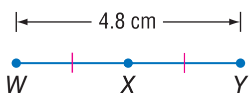
Skills Review

Find the measurement of each segment. Assume that each figure is not drawn to scale.

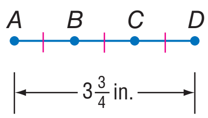
56. $\overline{ST}$



57. $\overline{WX}$



58. $\overline{BC}$



LESSON 2-7 Proving Segment Relationships

Then

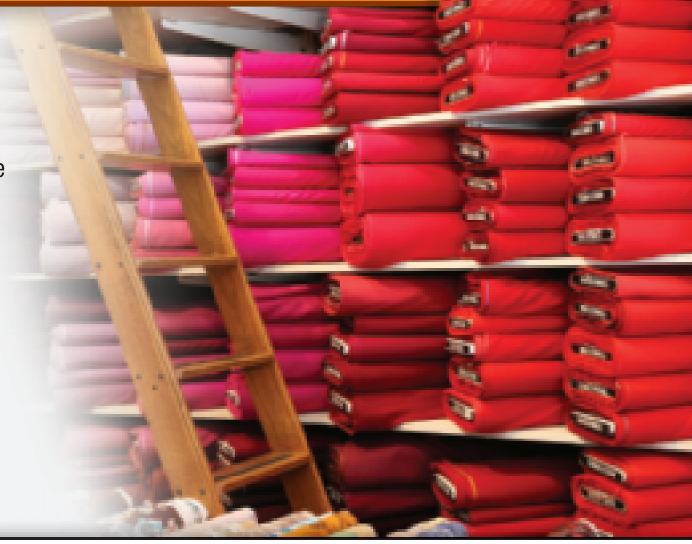
- You wrote algebraic and two-column proofs.

Now

- **1** Write proofs involving segment addition.
- **2** Write proofs involving segment congruence.

Why?

- Emma works at a fabric store after school. She measures a length of fabric by holding the straight edge of the fabric against a yardstick. To measure lengths such as 39 inches, which is longer than the yardstick, she marks a length of 36 inches. From the end of that mark, she measures an additional length of 3 inches. This ensures that the total length of fabric is $36 + 3$ inches or 39 inches.



CCSS Common Core State Standards

Content Standards

G.CO.9 Prove theorems about lines and angles.

G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).

Mathematical Practices

2 Reason abstractly and quantitatively.

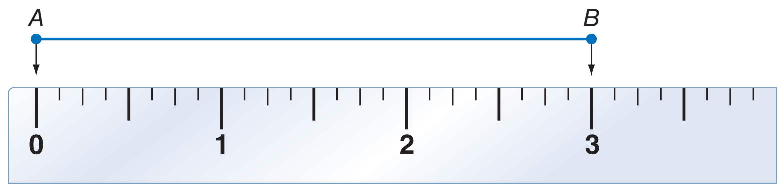
3 Construct viable arguments and critique the reasoning of others.

1 Ruler Postulate In Lesson 1-2, you measured segments with a ruler by matching the mark for zero with one endpoint and then finding the number on the ruler that corresponded to the other endpoint. This illustrates the Ruler Postulate.

Postulate 2.8 Ruler Postulate

Words The points on any line or line segment can be put into one-to-one correspondence with real numbers.

Symbols Given any two points A and B on a line, if A corresponds to zero, then B corresponds to a positive real number.

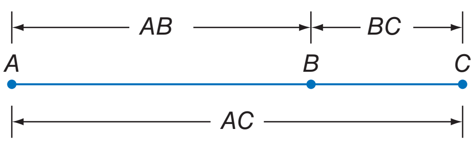


In Lesson 1-2, you also learned about what it means for a point to be *between* two other points. This relationship can be expressed as the Segment Addition Postulate.

Postulate 2.9 Segment Addition Postulate

Words If A, B, and C are collinear, then point B is between A and C if and only if $AB + BC = AC$.

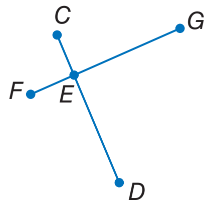
Symbols



The Segment Addition Postulate is used as a justification in many geometric proofs.

Example 1 Use the Segment Addition Postulate

Prove that if $\overline{CE} \cong \overline{FE}$ and $\overline{ED} \cong \overline{EG}$ then $\overline{CD} \cong \overline{FG}$.



Given: $\overline{CE} \cong \overline{FE}$; $\overline{ED} \cong \overline{EG}$

Prove: $\overline{CD} \cong \overline{FG}$

Proof:

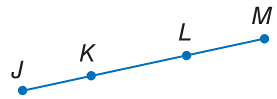
Statements	Reasons
1. $\overline{CE} \cong \overline{FE}$; $\overline{ED} \cong \overline{EG}$	**1.** Given
2. $CE = FE$; $ED = EG$	**2.** Definition of congruence
3. $CE + ED = CD$	**3.** Segment Addition Postulate
4. $FE + EG = CD$	**4.** Substitution (Steps 2 & 3)
5. $FE + EG = FG$	**5.** Segment Addition Postulate
6. $CD = FG$	**6.** Substitution (Steps 4 & 5)
7. $\overline{CD} \cong \overline{FG}$	**7.** Definition of congruence

ReadingMath

Substitution Property The Substitution Property of Equality is often just written as *Substitution.*

GuidedPractice

Copy and complete the proof.



1. Given: $\overline{JL} \cong \overline{KM}$

Prove: $\overline{JK} \cong \overline{LM}$

Proof:

Statements	Reasons
a. $\overline{JL} \cong \overline{KM}$	**a.** Given
b. $JL = KM$	**b.** ___?___
c. $JK + KL =$ ___?___ ; $KL + LM =$ ___?___	**c.** Segment Addition Postulate
d. $JK + KL = KL + LM$	**d.** ___?___
e. $JK + KL - KL = KL + LM - KL$	**e.** Subtraction Property of Equality
f. ___?___	**f.** Substitution
g. $\overline{JK} \cong \overline{LM}$	**g.** Definition of congruence

2 Segment Congruence

In Lesson 2-6, you saw that segment measures are reflexive, symmetric, and transitive. Since segments with the same measure are congruent, congruence of segments is also reflexive, symmetric, and transitive.

VocabularyLink

Symmetric

Everyday Use balanced or proportional

Math Use If $a = b$, then $b = a$.

Theorem 2.2 Properties of Segment Congruence

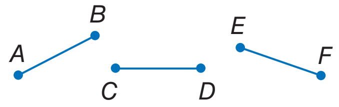
Reflexive Property of Congruence	$\overline{AB} \cong \overline{AB}$
Symmetric Property of Congruence	If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.
Transitive Property of Congruence	If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

You will prove the Symmetric and Reflexive Properties in Exercises 6 and 7, respectively.

Proof Transitive Property of Congruence

Given: $\overline{AB} \cong \overline{CD}$; $\overline{CD} \cong \overline{EF}$

Prove: $\overline{AB} \cong \overline{EF}$



Paragraph Proof:

Since $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, $AB = CD$ and $CD = EF$ by the definition of congruent segments. By the Transitive Property of Equality, $AB = EF$. Thus, $\overline{AB} \cong \overline{EF}$ by the definition of congruence.

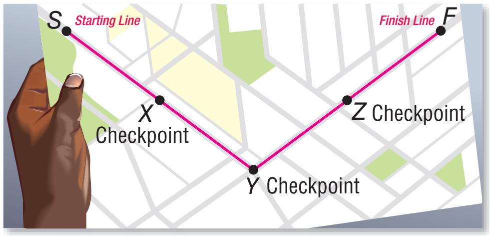
Real-WorldLink

According to a recent poll, 70% of teens who volunteer began doing so before age 12. Others said they would volunteer if given more opportunities to do so.

Source: Youth Service America

Real-World Example 2 Proof Using Segment Congruence

VOLUNTEERING The route for a charity fitness run is shown. Checkpoints *X* and *Z* are the midpoints between the starting line and Checkpoint *Y* and Checkpoint *Y* and the finish line *F*, respectively. If Checkpoint *Y* is the same distance from Checkpoints *X* and *Z*, prove that the route from Checkpoint *Z* to the finish line is congruent to the route from the starting line to Checkpoint *X*.



Given: X is the midpoint of $\overline{SY}$. Z is the midpoint of $\overline{YF}$. $XY = YZ$

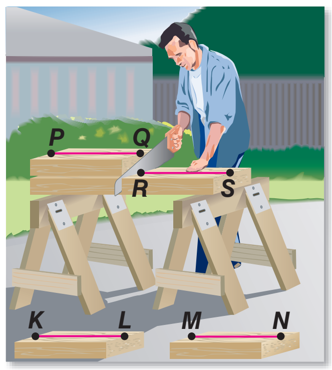
Prove: $\overline{ZF} \cong \overline{SX}$

Two-Column Proof:

Statements	Reasons
1. X is the midpoint of $\overline{SY}$. Z is the midpoint of $\overline{YF}$. $XY = YZ$	**1.** Given
2. $\overline{SX} \cong \overline{XY}$; $\overline{YZ} \cong \overline{ZF}$	**2.** Definition of midpoint
3. $\overline{XY} \cong \overline{YZ}$	**3.** Definition of congruence
4. $\overline{SX} \cong \overline{YZ}$	**4.** Transitive Property of Congruence
5. $\overline{SX} \cong \overline{ZF}$	**5.** Transitive Property of Congruence
6. $\overline{ZF} \cong \overline{SX}$	**6.** Symmetric Property of Congruence

GuidedPractice

2. CARPENTRY A carpenter cuts a 2″ × 4″ board to a desired length. He then uses this board as a pattern to cut a second board congruent to the first. Similarly, he uses the second board to cut a third board and the third board to cut a fourth board. Prove that the last board cut has the same measure as the first.



Check Your Understanding

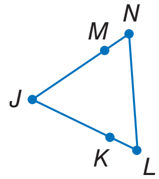
● = Step-by-Step Solutions begin on page R14.

Example 1

1. CCSS **ARGUMENTS** Copy and complete the proof.

Given: $\overline{LK} \cong \overline{NM}$, $\overline{KJ} \cong \overline{MJ}$

Prove: $\overline{LJ} \cong \overline{NJ}$



Proof:

Statements	Reasons
a. $\overline{LK} \cong \overline{NM}$, $\overline{KJ} \cong \overline{MJ}$	a. ___?___
b. ___?___	b. Def. of congruent segments
c. $LK + KJ = NM + MJ$	c. ___?___
d. ___?___	d. Segment Addition Postulate
e. $LJ = NJ$	e. ___?___
f. $\overline{LJ} \cong \overline{NJ}$	f. ___?___

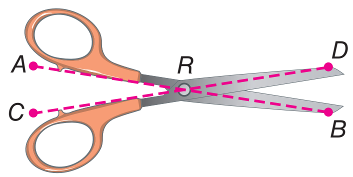
Example 2

2. **PROOF** Prove the following.

Given: $\overline{WX} \cong \overline{YZ}$

Prove: $\overline{WY} \cong \overline{XZ}$

3 **SCISSORS** Refer to the diagram shown. $\overline{AR}$ is congruent to $\overline{CR}$. $\overline{DR}$ is congruent to $\overline{BR}$. Prove that $AR + DR = CR + BR$.



Practice and Problem Solving

Extra Practice is on page R2.

Example 1

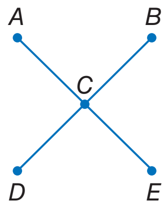
4. CCSS **ARGUMENTS** Copy and complete the proof.

Given: C is the midpoint of $\overline{AE}$.

C is the midpoint of $\overline{BD}$.

$\overline{AE} \cong \overline{BD}$

Prove: $\overline{AC} \cong \overline{CD}$

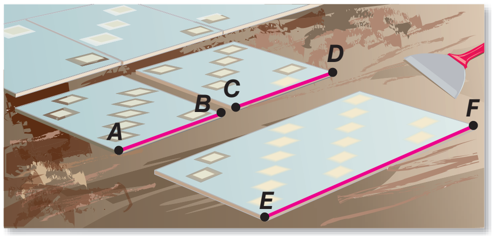


Proof:

Statements	Reasons
a. ___?___	a. Given
b. $AC = CE$, $BC = CD$	b. ___?___
c. $AE = BD$	c. ___?___
d. ___?___	d. Segment Addition Postulate
e. $AC + CE = BC + CD$	e. ___?___
f. $AC + AC = CD + CD$	f. ___?___
g. ___?___	g. Simplify.
h. ___?___	h. Division Property
i. $\overline{AC} \cong \overline{CD}$	i. ___?___

Example 2

5. TILING A tile setter cuts a piece of tile to a desired length. He then uses this tile as a pattern to cut a second tile congruent to the first. He uses the first two tiles to cut a third tile whose length is the sum of the measures of the first two tiles. Prove that the measure of the third tile is twice the measure of the first tile.



CCSS ARGUMENTS **Prove each theorem.**

6. Symmetric Property of Congruence (Theorem 2.2)

7. Reflexive Property of Congruence (Theorem 2.2)

8. TRAVEL Four cities in New York are connected by Interstate 90: Buffalo, Utica, Albany, and Syracuse. Buffalo is the farthest west.

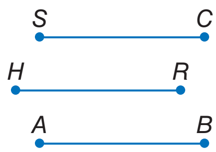
- Albany is 126 miles from Syracuse and 263 miles from Buffalo.
- Buffalo is 137 miles from Syracuse and 184 miles from Utica.

a. Draw a diagram to represent the locations of the cities in relation to each other and the distances between each city. Assume that Interstate 90 is straight.

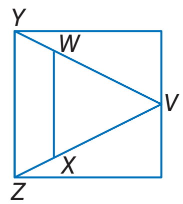
b. Write a paragraph proof to support your conclusion.

PROOF **Prove the following.**

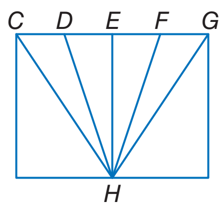
9. If $\overline{SC} \cong \overline{HR}$ and $\overline{HR} \cong \overline{AB}$, then $\overline{SC} \cong \overline{AB}$.



10. If $\overline{VZ} \cong \overline{VY}$ and $\overline{WY} \cong \overline{XZ}$, then $\overline{VW} \cong \overline{VX}$.



11. If E is the midpoint of $\overline{DF}$ and $\overline{CD} \cong \overline{FG}$, then $\overline{CE} \cong \overline{EG}$.

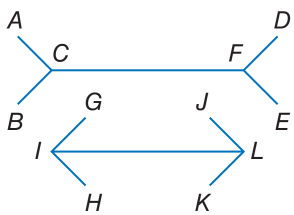


12. If B is the midpoint of $\overline{AC}$, D is the midpoint of $\overline{CE}$, and $\overline{AB} \cong \overline{DE}$, then $AE = 4AB$.

13. OPTICAL ILLUSION $\overline{AC} \cong \overline{GI}$, $\overline{FE} \cong \overline{LK}$, and $AC + CF + FE = GI + IL + LK$.

a. Prove that $\overline{CF} \cong \overline{IL}$.

b. Justify your proof using measurement. Explain your method.

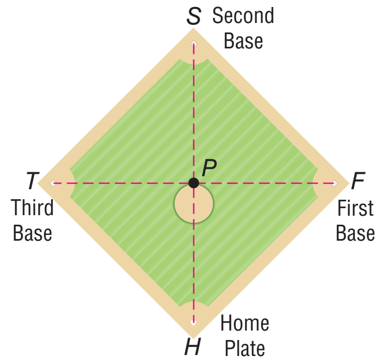


14. CONSTRUCTION Construct a segment that is twice as long as $\overline{PQ}$. Explain how the Segment Addition Postulate can be used to justify your construction.

15. BASEBALL Use the diagram of a baseball diamond shown.

a. On the diagram, $\overline{SH} \cong \overline{TF}$. P is the midpoint of $\overline{SH}$ and $\overline{TF}$. Using a two-column proof, prove that $\overline{SP} \cong \overline{TP}$.

b. The distance from home plate to second base is 127.3 feet. What is the distance from first base to second base?



16. MULTIPLE REPRESENTATIONS A is the midpoint of $\overline{PQ}$, B is the midpoint of $\overline{PA}$, and C is the midpoint of $\overline{PB}$.

a. Geometric Make a sketch to represent this situation.

b. Algebraic Make a conjecture as to the algebraic relationship between PC and PQ.

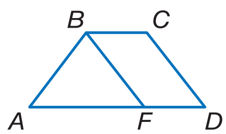
c. Geometric Copy segment $\overline{PQ}$ from your sketch. Then construct points B and C on $\overline{PQ}$. Explain how you can use your construction to support your conjecture.

d. Concrete Use a ruler to draw a segment congruent to $\overline{PQ}$ from your sketch and to draw points B and C on $\overline{PQ}$. Use your drawing to support your conjecture.

e. Logical Prove your conjecture.

H.O.T. Problems Use Higher-Order Thinking Skills

17. CCSS CRITIQUE In the diagram, $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{BF}$. Examine the conclusions made by Leslie and Shantice. Is either of them correct?



Leslie	Shantice
Since $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{BF}$, then $\overline{AB} \cong \overline{AF}$ by the Transitive Property of Congruence	Since $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{BF}$, then $\overline{AB} \cong \overline{BF}$ by the Reflexive Property of Congruence.

18. CHALLENGE $ABCD$ is a square. Prove that $\overline{AC} \cong \overline{BD}$.

19. WRITING IN MATH Does there exist an Addition Property of Congruence? Explain.

20. REASONING Classify the following statement as *true* or *false*. If false, provide a counterexample.

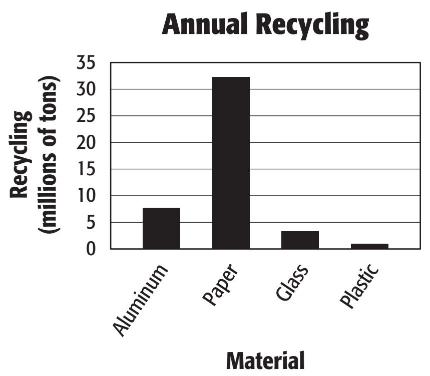
If A, B, C, D, and E are collinear with B between A and C, C between B and D, and D between C and E, and AC = BD = CE, then AB = BC = DE.

21. OPEN ENDED Draw a representation of the Segment Addition Postulate in which the segment is two inches long, contains four collinear points, and contains no congruent segments.

22. WRITING IN MATH Compare and contrast paragraph proofs and two-column proofs.

Standardized Test Practice

23. ALGEBRA The chart below shows annual recycling by material in the United States. About how many pounds of aluminum are recycled each year?



A 7.5 **C** 7,500,000

B 15,000 **D** 15,000,000,000

24. ALGEBRA Which expression is equivalent to $\frac{12x^{-4}}{4x^{-8}}$?

F $\frac{1}{3x^4}$ **H** $8x^2$

G $3x^4$ **J** $\frac{x^4}{3}$

25. SHORT RESPONSE The measures of two complementary angles are in the ratio 4 : 1. What is the measure of the smaller angle?

26. SAT/ACT Julie can word process 40 words per minute. How many minutes will it take Julie to word process 200 words?

A 0.5 **D** 10

B 2 **E** 12

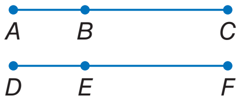
C 5

Spiral Review

27. PROOF Write a two-column proof. (Lesson 2-6)

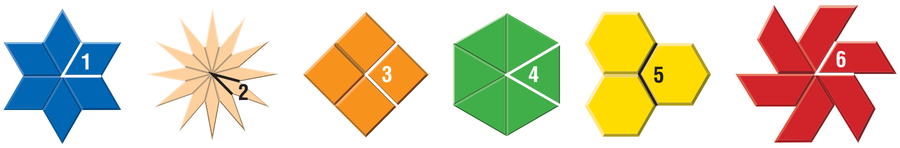
Given: $AC = DF$
$AB = DE$

Prove: $BC = EF$



28. MODELS Brian is using six squares of cardboard to form a rectangular prism. What geometric figure do the pieces of cardboard represent, and how many lines will be formed by their intersections? (Lesson 2-5)

29. PATTERN BLOCKS Pattern blocks can be arranged to fit in a circular pattern without leaving spaces. Remember that the measurement around a full circle is 360°. Determine the degree measure of the numbered angles shown below. (Lesson 1-4)



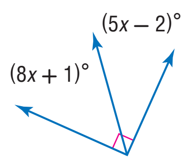
Simplify. (Lesson 0-9)

30. $\sqrt{48}$ **31.** $\sqrt{162}$ **32.** $\sqrt{25a^6b^4}$ **33.** $\sqrt{45xy^8}$

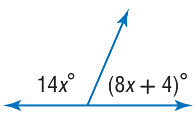
Skills Review

ALGEBRA **Find** x.

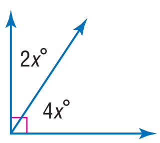
34.



35.



36.



LESSON

2-8 Proving Angle Relationships

Then

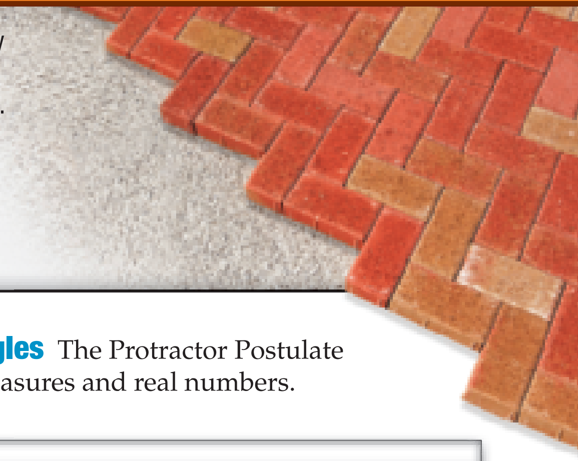
- You identified and used special pairs of angles.

Now

1. Write proofs involving supplementary and complementary angles.
2. Write proofs involving congruent and right angles.

Why?

- Jamal's school is building a walkway that will include bricks with the names of graduates from each class. All of the bricks are rectangular, so when the bricks are laid, all of the angles form linear pairs.



Common Core State Standards

Content Standards

G.CO.9 Prove theorems about lines and angles.

Mathematical Practices

3 Construct viable arguments and critique the reasoning of others.

6 Attend to precision.

FOTOG/Tetra Images/Photolibrary

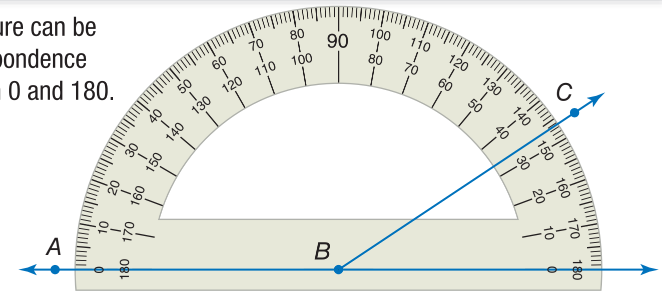
1 Supplementary and Complementary Angles

The Protractor Postulate illustrates the relationship between angle measures and real numbers.

Postulate 2.10 Protractor Postulate

Words Given any angle, the measure can be put into one-to-one correspondence with real numbers between 0 and 180.

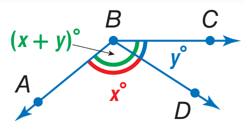
Example If $\overrightarrow{BA}$ is placed along the protractor at 0°, then the measure of $\angle ABC$ corresponds to a positive real number.



In Lesson 2-7, you learned about the Segment Addition Postulate. A similar relationship exists between the measures of angles.

Postulate 2.11 Angle Addition Postulate

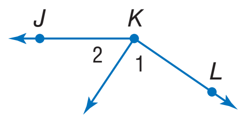
D is in the interior of $\angle ABC$ if and only if $m\angle ABD + m\angle DBC = m\angle ABC$.



Example 1 Use the Angle Addition Postulate

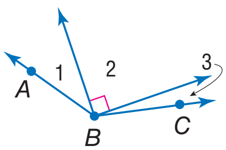
Find $m\angle 1$ if $m\angle 2 = 56$ and $m\angle JKL = 145$.

$m\angle 1 + m\angle 2 = m\angle JKL$	Angle Addition Postulate
$m\angle 1 + 56 = 145$	$m\angle 2 = 56$ $m\angle JKL = 145$
$m\angle 1 + 56 - 56 = 145 - 56$	Subtraction Property of Equality
$m\angle 1 = 89$	Substitution



Guided Practice

1. If $m\angle 1 = 23$ and $m\angle ABC = 131$, find the measure of $\angle 3$. Justify each step.



The Angle Addition Postulate can be used with other angle relationships to provide additional theorems relating to angles.

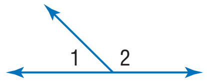
StudyTip

Linear Pair Theorem The Supplement Theorem may also be known as the *Linear Pair Theorem.*

Theorems

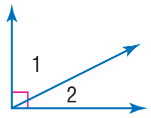
2.3 Supplement Theorem If two angles form a linear pair, then they are supplementary angles.

Example $m\angle 1 + m\angle 2 = 180$



2.4 Complement Theorem If the noncommon sides of two adjacent angles form a right angle, then the angles are complementary angles.

Example $m\angle 1 + m\angle 2 = 90$

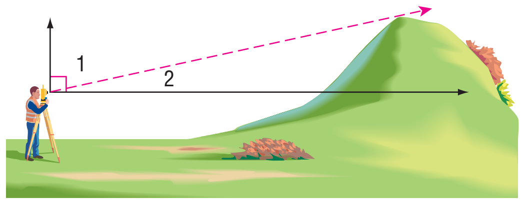


You will prove Theorems 2.3 and 2.4 in Exercises 16 and 17, respectively.

Real-World Example 2 Use Supplement or Complement

SURVEYING Using a transit, a surveyor sights the top of a hill and records an angle measure of about 73°. What is the measure of the angle the top of the hill makes with the horizon? Justify each step.

Understand Make a sketch of the situation. The surveyor is measuring the angle of his line of sight below the vertical. Draw a vertical ray and a horizontal ray from the point where the surveyor is sighting the hill, and label the angles formed. We know that the vertical and horizontal rays form a right angle.



Plan Since ∠1, and ∠2 form a right angle, you can use the Complement Theorem.

Solve

$m\angle 1 + m\angle 2 = 90$	Complement Theorem
$73 + m\angle 2 = 90$	$m\angle 1 = 73$
$73 + m\angle 2 - 73 = 90 - 73$	Subtraction Property of Equality
$m\angle 2 = 17$	Substitution

The top of the hill makes a 17° angle with the horizon.

Check Since we know that the sum of the angles should be 90, check your math. The sum of 17 and 73 is 90. ✓

ReviewVocabulary

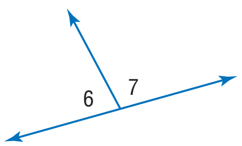
supplementary angles two angles with measures that add to 180

complementary angles two angles with measures that add to 90

linear pair a pair of adjacent angles with noncommon sides that are opposite rays

GuidedPractice

2. ∠6 and ∠7 form linear pair. If $m\angle 6 = 3x + 32$ and $m\angle 7 = 5x + 12$, find x, $m\angle 6$, and $m\angle 7$. Justify each step.



2 Congruent Angles

The properties of algebra that applied to the congruence of segments and the equality of their measures also hold true for the congruence of angles and the equality of their measures.

Theorem 2.5 Properties of Angle Congruence

Reflexive Property of Congruence

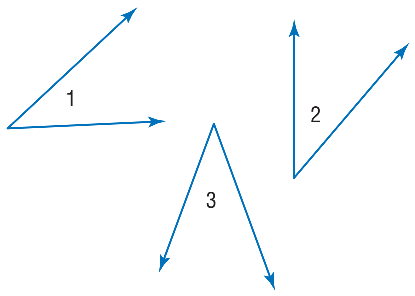
$\angle 1 \cong \angle 1$

Symmetric Property of Congruence

If $\angle 1 \cong \angle 2$, then $\angle 2 \cong \angle 1$.

Transitive Property of Congruence

If $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, then $\angle 1 \cong \angle 3$.

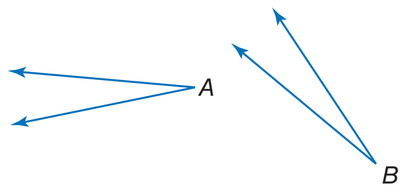


You will prove the Reflexive and Transitive Properties of Congruence in Exercises 18 and 19, respectively.

Proof Symmetric Property of Congruence

Given: $\angle A \cong \angle B$

Prove: $\angle B \cong \angle A$



Paragraph Proof:

We are given $\angle A \cong \angle B$. By the definition of congruent angles, $m\angle A = m\angle B$. Using the Symmetric Property of Equality, $m\angle B = m\angle A$. Thus, $\angle B \cong \angle A$ by the definition of congruent angles.

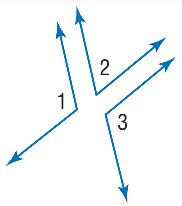
Algebraic properties can be applied to prove theorems for congruence relationships involving supplementary and complementary angles.

Theorems

2.6 Congruent Supplements Theorem
Angles supplementary to the same angle or to congruent angles are congruent.

Abbreviation *$\angle$s suppl. to same $\angle$ or $\cong$ $\angle$s are $\cong$.*

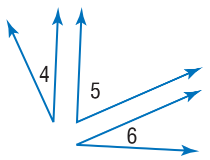
Example If $m\angle 1 + m\angle 2 = 180$ and $m\angle 2 + m\angle 3 = 180$, then $\angle 1 \cong \angle 3$.



2.7 Congruent Complements Theorem
Angles complementary to the same angle or to congruent angles are congruent.

Abbreviation *$\angle$s compl. to same $\angle$ or $\cong$ $\angle$s are $\cong$.*

Example If $m\angle 4 + m\angle 5 = 90$ and $m\angle 5 + m\angle 6 = 90$, then $\angle 4 \cong \angle 6$.



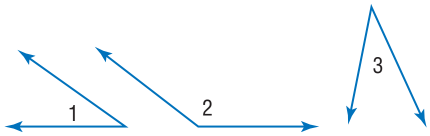
You will prove one case of Theorem 2.6 in Exercise 6.

ReadingMath
Abbreviations and Symbols
The notation $\angle$s means *angles*.

Proof One Case of the Congruent Supplements Theorem

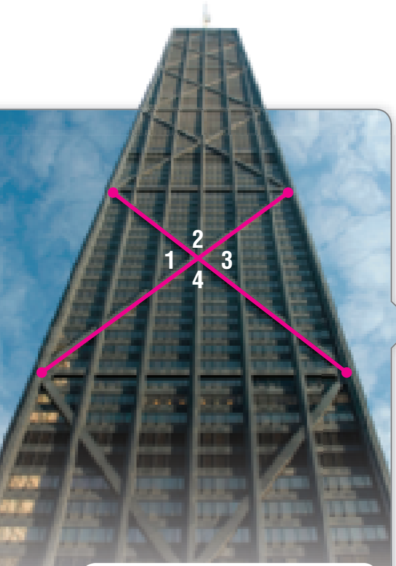
Given: $\angle 1$ and $\angle 2$ are supplementary.
$\angle 2$ and $\angle 3$ are supplementary.

Prove: $\angle 1 \cong \angle 3$



Proof:

Statements	Reasons
1. $\angle 1$ and $\angle 2$ are supplementary. $\angle 2$ and $\angle 3$ are supplementary.	**1.** Given
2. $m\angle 1 + m\angle 2 = 180$; $m\angle 2 + m\angle 3 = 180$	**2.** Definition of supplementary angles
3. $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$	**3.** Substitution
4. $m\angle 2 = m\angle 2$	**4.** Reflexive Property
5. $m\angle 1 = m\angle 3$	**5.** Subtraction Property
6. $\angle 1 \cong \angle 3$	**6.** Definition of congruent angles



Real-WorldLink

The 100-story John Hancock Building uses huge X-braces in its design. These diagonals are connected to the exterior columns, making it possible for strong wind forces to be carried from the braces to the exterior columns and back.

Source: PBS

Example 3 Proofs Using Congruent Comp. or Suppl. Theorems

PT

Prove that vertical angles 2 and 4 in the photo at the left are congruent.

Given: $\angle 2$ and $\angle 4$ are vertical angles.

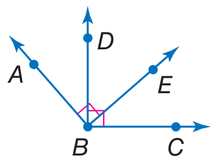
Prove: $\angle 2 \cong \angle 4$

Proof:

Statements	Reasons
1. $\angle 2$ and $\angle 4$ are vertical angles.	**1.** Given
2. $\angle 2$ and $\angle 4$ are nonadjacent angles formed by intersecting lines.	**2.** Definition of vertical angles
3. $\angle 2$ and $\angle 3$ from a linear pair. $\angle 3$ and $\angle 4$ form a linear pair.	**3.** Definition of a linear pair
4. $\angle 2$ and $\angle 3$ are supplementary. $\angle 3$ and $\angle 4$ are supplementary.	**4.** Supplement Theorem
5. $\angle 2 \cong \angle 4$	**5.** $\measuredangle$ suppl. to same $\angle$ or $\cong$ $\measuredangle$ are $\cong$.

GuidedPractice

3. In the figure, $\angle ABE$ and $\angle DBC$ are right angles. Prove that $\angle ABD \cong \angle EBC$.



ReviewVocabulary

Vertical Angles two nonadjacent angles formed by intersecting lines

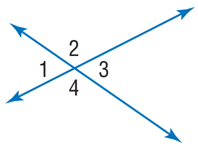
Note that in Example 3, $\angle 1$ and $\angle 3$ are vertical angles. The conclusion in the example supports the following Vertical Angles Theorem.

Theorem 2.8 Vertical Angles Theorem

If two angles are vertical angles, then they are congruent.

Abbreviation *Vert.* $\measuredangle$ *are* $\cong$.

Example $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$

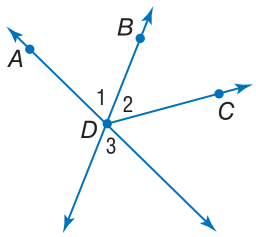


You will prove Theorem 2.8 in Exercise 28.

Yannis Emmanuel Mavromatakis/Alamy

Example 4 Use Vertical Angles

Prove that if $\overrightarrow{DB}$ bisects $\angle ADC$, then $\angle 2 \cong \angle 3$.



Given: $\overrightarrow{DB}$ bisects $\angle ADC$.

Prove: $\angle 2 \cong \angle 3$

Proof:

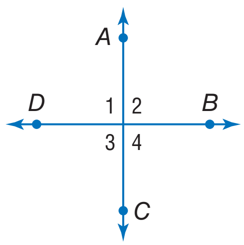
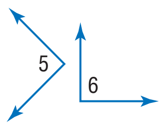
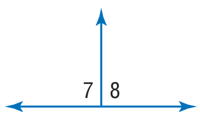
Statements	Reasons
1. $\overrightarrow{DB}$ bisects $\angle ADC$.	**1.** Given
2. $\angle 1 \cong \angle 2$	**2.** Definition of angle bisector
3. $\angle 1$ and $\angle 3$ are vertical angles.	**3.** Definition of vertical angles
4. $\angle 3 \cong \angle 1$	**4.** Vert. ∠s are ≅.
5. $\angle 3 \cong \angle 2$	**5.** Transitive Property of Congruence
6. $\angle 2 \cong \angle 3$	**6.** Symmetric Property of Congruence

GuidedPractice

4. If $\angle 3$ and $\angle 4$ are vertical angles, $m\angle 3 = 6x + 2$, and $m\angle 4 = 8x - 14$, find $m\angle 3$ and $m\angle 4$. Justify each step.

The theorems in this lesson can be used to prove the following right angle theorems.

Theorems Right Angle Theorems

Theorem	Example
2.9 Perpendicular lines intersect to form four right angles. **Example** If $\overleftrightarrow{AC} \perp \overleftrightarrow{DB}$, then $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ are rt. ∠s.	
2.10 All right angles are congruent. **Example** If $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ are rt. ∠s, then $\angle 1 \cong \angle 2 \cong \angle 3 \cong \angle 4$.	
2.11 Perpendicular lines form congruent adjacent angles. **Example** If $\overleftrightarrow{AC} \perp \overleftrightarrow{DB}$, then $\angle 1 \cong \angle 2$, $\angle 2 \cong \angle 4$, $\angle 3 \cong \angle 4$, and $\angle 1 \cong \angle 3$.	
2.12 If two angles are congruent and supplementary, then each angle is a right angle. **Example** If $\angle 5 \cong \angle 6$ and $\angle 5$ is suppl. to $\angle 6$, then $\angle 5$ and $\angle 6$ are rt. ∠s.	
2.13 If two congruent angles form a linear pair, then they are right angles. **Example** If $\angle 7$ and $\angle 8$ form a linear pair, then $\angle 7$ and $\angle 8$ are rt. ∠s.	

You will prove Theorems 2.9–2.13 in Exercises 22–26.

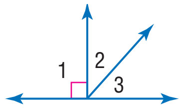
ReadingMath

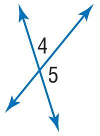
Perpendicular Recall from Lesson 1-5 that the symbol $\perp$ means *is perpendicular to.*

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Example 1 **Find the measure of each numbered angle, and name the theorems that justify your work.**

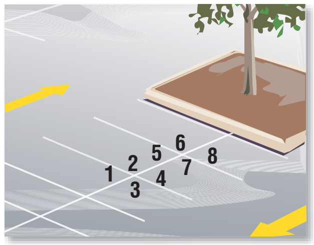




1. $m\angle 2 = 26$
2. $m\angle 2 = x$, $m\angle 3 = x - 16$
3. $m\angle 4 = 2x$, $m\angle 5 = x + 9$
4. $m\angle 4 = 3(x - 1)$, $m\angle 5 = x + 7$

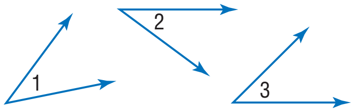
Example 2

5. **PARKING** Refer to the diagram of the parking lot at the right. Given that $\angle 2 \cong \angle 6$, prove that $\angle 4 \cong \angle 8$.



Example 3

6. **PROOF** **Copy and complete the proof of one case of Theorem 2.6.**



Given: $\angle 1$ and $\angle 3$ are complementary.
$\angle 2$ and $\angle 3$ are complementary.

Prove: $\angle 1 \cong \angle 2$

Proof:

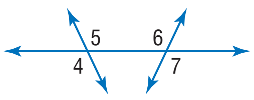
Statements	Reasons
a. $\angle 1$ and $\angle 3$ are complementary. $\angle 2$ and $\angle 3$ are complementary.	**a.** ___?___
b. $m\angle 1 + m\angle 3 = 90$; $m\angle 2 + m\angle 3 = 90$	**b.** ___?___
c. $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$	**c.** ___?___
d. ___?___	**d.** Reflexive Property
e. $m\angle 1 = m\angle 2$	**e.** ___?___
f. $\angle 1 \cong \angle 2$	**f.** ___?___

Example 4

7. CCSS **ARGUMENTS** Write a two-column proof.

Given: $\angle 4 \cong \angle 7$

Prove: $\angle 5 \cong \angle 6$

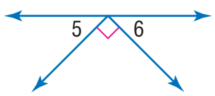


Practice and Problem Solving

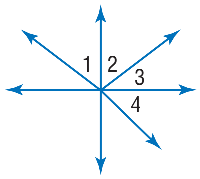
Extra Practice is on page R2.

Examples 1–3 **Find the measure of each numbered angle, and name the theorems used that justify your work.**

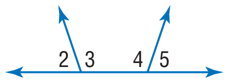
8. $m\angle 5 = m\angle 6$



9. $\angle 2$ and $\angle 3$ are complementary. $\angle 1 \cong \angle 4$ and $m\angle 2 = 28$

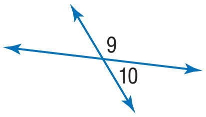


10. $\angle 2$ and $\angle 4$ and $\angle 4$ and $\angle 5$ are supplementary. $m\angle 4 = 105$

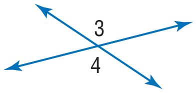


Find the measure of each numbered angle and name the theorems used that justify your work.

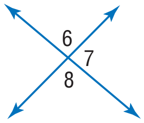
11. $m\angle 9 = 3x + 12$
$m\angle 10 = x - 24$



12. $m\angle 3 = 2x + 23$
$m\angle 4 = 5x - 112$



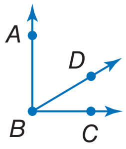
13. $m\angle 6 = 2x - 21$
$m\angle 7 = 3x - 34$



Example 4 **PROOF** **Write a two-column proof.**

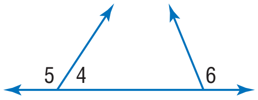
14. Given: $\angle ABC$ is a right angle.

Prove: $\angle ABD$ and $\angle CBD$ are complementary.



15. Given: $\angle 5 \cong \angle 6$

Prove: $\angle 4$ and $\angle 6$ are supplementary.



Write a proof for each theorem.

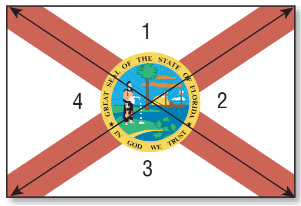
16. Supplement Theorem

17. Complement Theorem

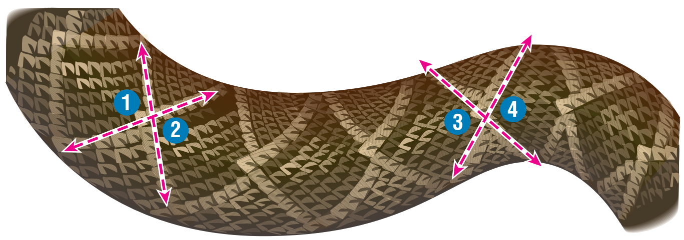
18. Reflexive Property of Angle Congruence

19. Transitive Property of Angle Congruence

20. **FLAGS** Refer to the Florida state flag at the right. Prove that the sum of the four angle measures is 360.



21. **CCSS ARGUMENTS** The diamondback rattlesnake is a pit viper with a diamond pattern on its back. An enlargement of a skin is shown below. If $\angle 1 \cong \angle 4$, prove that $\angle 2 \cong \angle 3$.



PROOF **Use the figure to write a proof of each theorem.**

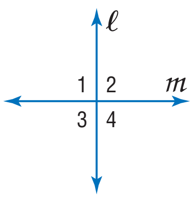
22. Theorem 2.9

23. Theorem 2.10

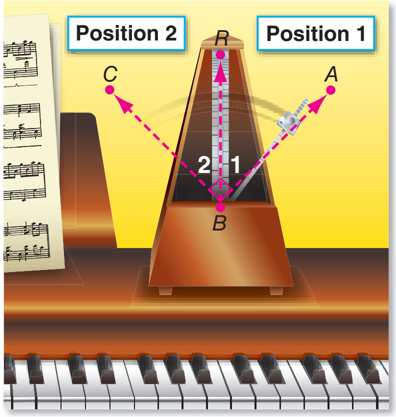
24. Theorem 2.11

25. Theorem 2.12

26. Theorem 2.13



27. **ARGUMENTS** To mark a specific tempo, the weight on the pendulum of a metronome is adjusted so that it swings at a specific rate. Suppose $\angle ABC$ in the photo is a right angle. If $m\angle 1 = 45$, write a paragraph proof to show that $\overrightarrow{BR}$ bisects $\angle ABC$.



28. **PROOF** Write a proof of Theorem 2.8.

29. **GEOGRAPHY** Utah, Colorado, Arizona, and New Mexico all share a common point on their borders called Four Corners. This is the only place where four states meet in a single point. If $\angle 2$ is a right angle, prove that lines ℓ and m are perpendicular.

30. **MULTIPLE REPRESENTATIONS** In this problem, you will explore angle relationships.

a. **Geometric** Draw a right angle ABC. Place point D in the interior of this angle and draw $\overrightarrow{BD}$. Draw $\overrightarrow{KL}$ and construct $\angle JKL$ congruent to $\angle ABD$.

b. **Verbal** Make a conjecture as to the relationship between $\angle JKL$ and $\angle DBC$.

c. **Logical** Prove your conjecture.

H.O.T. Problems Use Higher-Order Thinking Skills

31. **OPEN ENDED** Draw an angle WXZ such that $m\angle WXZ = 45$. Construct $\angle YXZ$ congruent to $\angle WXZ$. Make a conjecture as to the measure of $\angle WXY$, and then prove your conjecture.

32. **WRITING IN MATH** Write the steps that you would use to complete the proof below.

Given: $\overline{BC} \cong \overline{CD}$, $AB = \frac{1}{2}BD$

Prove: $\overline{AB} \cong \overline{CD}$

33. **CHALLENGE** In this lesson, one case of the Congruent Supplements Theorem was proven. In Exercise 6, you proved the same case for the Congruent Complements Theorem. Explain why there is another case for each of these theorems. Then write a proof of this second case for each theorem.

34. **REASONING** Determine whether the following statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

If one of the angles formed by two intersecting lines is acute, then the other three angles formed are also acute.

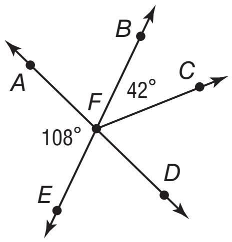
35. **WRITING IN MATH** Explain how you can use your protractor to quickly find the measure of the supplement of an angle.

Standardized Test Practice

36. GRIDDED RESPONSE What is the mode of this set of data?

4, 3, −2, 1 , 4, 0, 1, 4

37. Find the measure of $\angle CFD$.



A 66°

B 72°

C 108°

D 138°

38. ALGEBRA Simplify.

$$4(3x - 2)(2x + 4) + 3x^2 + 5x - 6$$

F $9x^2 + 3x - 14$

G $9x^2 + 13x - 14$

H $27x^2 + 37x - 38$

J $27x^2 + 27x - 26$

39. SAT/ACT On a coordinate grid where each unit represents 1 mile, Isabel's house is located at (3, 0) and a mall is located at (0, 4). What is the distance between Isabel's house and the mall?

A 3 miles

B 5 miles

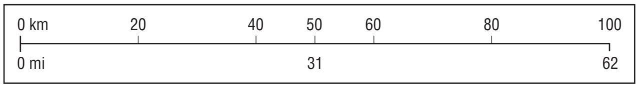
C 12 miles

D 13 miles

E 25 miles

Spiral Review

40. MAPS On a U.S. map, there is a scale that lists kilometers on the top and miles on the bottom.



Suppose $\overline{AB}$ and $\overline{CD}$ are segments on this map. If $AB = 100$ kilometers and $CD = 62$ miles, is $\overline{AB} \cong \overline{CD}$? Explain. (Lesson 2-7)

State the property that justifies each statement. (Lesson 2-6)

41. If $y + 7 = 5$, then $y = -2$.

42. If $MN = PQ$, then $PQ = MN$.

43. If $a - b = x$ and $b = 3$, then $a - 3 = x$.

44. If $x(y + z) = 4$, then $xy + xz = 4$.

Determine the truth value of the following statement for each set of conditions.
If you have a fever, then you are sick. (Lesson 2-3)

45. You do not have a fever, and you are sick.

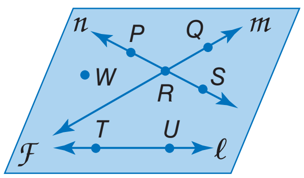
46. You have a fever, and you are not sick.

47. You do not have a fever, and you are not sick.

48. You have a fever, and you are sick.

Skills Review

Refer to the figure.



49. Name a line that contains point P.

50. Name the intersection of lines n and m.

51. Name a point not contained in lines ℓ, m, or n.

52. What is another name for line n?

53. Does line ℓ intersect line m or line n? Explain.

CHAPTER 2

Study Guide and Review

Study Guide

KeyConcepts

Inductive Reasoning and Logic (Lessons 2-1 and 2-2)

- Inductive reasoning: a conjecture is reached based on observations of a previous pattern
- Counterexample: an example that proves a conjecture is false
- Negation of statement p: *not* p
- Conjunction: a compound statement formed with the word *and*
- Disjunction: a compound statement formed with the word *or*

Conditional Statements (Lesson 2-3)

- An if-then statement is written in the form if p, then q in which p is the hypothesis and q is the conclusion.

statement	$p \rightarrow q$
converse	$q \rightarrow p$
inverse	not $p \rightarrow$ not q
contrapositive	not $q \rightarrow$ not p

Deductive Reasoning (Lesson 2-4)

- Law of Detachment: If $p \rightarrow q$ is true and p is true, then q is also true.
- Law of Syllogism: If $p \rightarrow q$ and $q \rightarrow r$ are true, then $p \rightarrow r$ is also true.

Proof (Lessons 2-5 through 2-8)

Step 1 List the given information and draw a diagram, if possible.

Step 2 State what is to be proved.

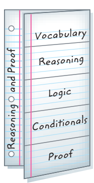
Step 3 Create a deductive argument.

Step 4 Justify each statement with a reason.

Step 5 State what you have proved.

FOLDABLES StudyOrganizer

Be sure the Key Concepts are noted in your Foldable.



KeyVocabulary

algebraic proof (p. 136)
axiom (p. 127)
compound statement (p. 99)
conclusion (p. 107)
conditional statement (p. 107)
conjecture (p. 91)
conjunction (p. 99)
contrapositive (p. 109)
converse (p. 109)
counterexample (p. 94)
deductive argument (p. 129)
deductive reasoning (p. 117)
disjunction (p. 100)
formal proof (p. 137)
hypothesis (p. 107)
if-then statement (p. 107)
inductive reasoning (p. 91)
informal proof (p. 129)
inverse (p. 109)
logically equivalent (p. 110)
negation (p. 99)
paragraph proof (p. 129)
postulate (p. 127)
proof (p. 128)
related conditionals (p. 109)
statement (p. 99)
theorem (p. 129)
truth table (p. 101)
truth value (p. 99)
two-column proof (p. 137)

VocabularyCheck

State whether each sentence is *true* or *false*. If *false*, replace the underlined term to make a true sentence.

1. A postulate is a statement that requires proof.
2. The first part of an if-then statement is the conjecture.
3. Deductive reasoning uses the laws of mathematics to reach logical conclusions from given statements.
4. The contrapositive is formed by negating the hypothesis and conclusion of a conditional.
5. A conjunction is formed by joining two or more statements with the word *and*.
6. A theorem is a statement that is accepted as true without proof.
7. The converse is formed by exchanging the hypothesis and conclusion of a conditional.
8. To show that a conjecture is false, you would provide a disjunction.
9. The inverse of a statement p would be written in the form *not* p.
10. In a two-column proof, the properties that justify each step are called reasons.

Lesson-by-Lesson Review

2-1 Inductive Reasoning and Conjecture

Determine whether each conjecture is *true* or *false*. If false, give a counterexample.

11. If ∠1 and ∠2 are supplementary angles, then ∠1 and ∠2 form a linear pair.

12. If $W(-3, 2)$, $X(-3, 7)$, $Y(6, 7)$, $Z(6, 2)$, then quadrilateral $WXYZ$ is a rectangle.

13. **PARKS** Jacinto enjoys hiking with his dog in the forest at his local park. While on vacation in Smoky Mountain National Park in Tennessee, he was disappointed that dogs were not allowed on most hiking trails. Make a conjecture about why his local park and the national park have differing rules with regard to pets.

Example 1

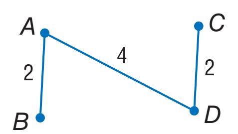
Determine whether each conjecture is *true* or *false*. If false, give a counterexample.

a. $c = d, d = c$ **is an example of a property of real numbers.**

$c = d, d = c$ is an example of the Symmetric Property of real numbers, so the conjecture is true.

b. **If** $AB + CD = AD$, **then** B **and** C **are between** A **and** D.

This conjecture is false. In the figure below, $AB + CD = AD$, but B and C are not between A and D.



2-2 Logic

Use the following statements to write a compound statement for each conjunction or disjunction. Then find its truth value. Explain.

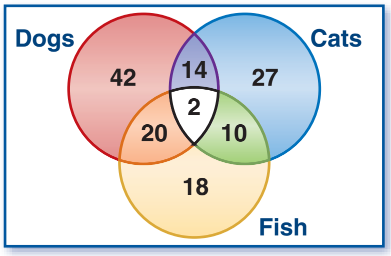
p: A plane contains at least three noncollinear points.

q: A square yard is equivalent to three square feet.

r: The sum of the measures of two complementary angles is 180.

14. $\sim q \vee r$ **15.** $p \wedge \sim r$ **16.** $\sim p \vee q$

17. **PETS** The Venn diagram shows the results of a pet store survey to determine the pets customers owned.



a. How many customers had only fish?

b. How many had only cats and dogs?

c. How many had dogs as well as fish?

Example 2

Use the following statements to write a compound statement for each conjunction or disjunction. Then find its truth value. Explain.

p: x^2 is a nonnegative number.

q: Adjacent angles lie in the same plane.

r: A negative number is not a real number.

a. $\sim q \wedge r$

$\sim q \wedge r$: Adjacent angles do not lie in the same plane, and a negative number is not a real number.

Since both $\sim q$ and r are false, $\sim q \wedge r$ is false.

b. p or r

p or r: x^2 is a nonnegative number, or a negative number is not a real number.

p or r is true because p is true. It does not matter that r is false.

Study Guide and Review *Continued*

2-3 Conditional Statements

Determine the truth value of each conditional statement. If *true*, explain your reasoning. If *false*, give a counterexample.

18. If you square an integer, then the result is a positive integer.

19. If a hexagon has eight sides, then all of its angles will be obtuse.

20. Write the converse, inverse, and contrapositive of the following true conditional. Then, determine whether each related conditional is *true* or *false*. If a statement is false, find a counterexample.

If two angles are congruent, then they have the same degree measure.

Example 3

Write the *converse*, *inverse*, and *contrapositive* of the following true conditional.

If a figure is a square, then it is a parallelogram.

Converse: If a figure is a parallelogram, then it is a square.

Inverse: If a figure is not a square, then it is not a parallelogram.

Contrapositive: If a figure is not a parallelogram, then it is not a square.

2-4 Deductive Reasoning

Draw a valid conclusion from the given statements, if possible. Then state whether your conclusion was drawn using the Law of Detachment or the Law of Syllogism. If no valid conclusion can be drawn, write *no valid conclusion* and explain your reasoning.

21. Given: If a quadrilateral has diagonals that bisect each other, then it is a parallelogram.

The diagonals of quadrilateral *PQRS* bisect each other.

22. Given: If Liana struggles in science class, then she will receive tutoring.

If Liana stays after school on Thursday, then she will receive tutoring.

23. **EARTHQUAKES** Determine whether the stated conclusion is valid based on the given information. If not, write *invalid*. Explain.

Given: If an earthquake measures a 7.0 or higher on the Richter scale, then it is considered a major earthquake that could cause serious damage. The 1906 San Francisco earthquake measured 8.0 on the Richter scale.

Conclusion: The 1906 San Francisco earthquake was a major earthquake that caused serious damage.

Example 4

Use the Law of Syllogism to determine whether a valid conclusion can be reached from the following statements.

(1) If the measure of an angle is greater than 90, then it is an obtuse angle.

(2) If an angle is an obtuse angle, then it is not a right angle.

p: the measure of an angle is greater than 90

q: the angle is an obtuse angle

r: the angle is not a right angle

Statement (1): $p \to q$

Statement (2): $q \to r$

Since the given statements are true, use the Law of Syllogism to conclude that $p \to r$. That is, *If the measure of an angle is greater than 90, then it is not a right angle.*

2-5 Postulates and Paragraph Proofs

Determine whether each statement is *always, sometimes,* or *never* true. Explain.

24. Two planes intersect at a point.

25. Three points are contained in more than one plane.

26. If line *m* lies in plane $\mathcal{X}$ and line *m* contains a point *Q*, then point *Q* lies in plane $\mathcal{X}$.

27. If two angles are complementary, then they form a right angle.

28. **NETWORKING** Six people are introduced at a business convention. If each person shakes hands with each of the others, how many handshakes will be exchanged? Include a model to support your reasoning.

Example 5

Determine whether each statement is *always, sometimes,* or *never* true. Explain.

a. ***If points X, Y, and Z lie in plane $\mathcal{R}$, then they are not collinear.***

Sometimes; the fact that *X, Y,* and *Z* are contained in plane $\mathcal{R}$ has no bearing on whether those points are collinear or not.

b. **For any two points *A* and *B*, there is exactly one line that contains them.**

Always; according to Postulate 2-1, there is exactly one line through any two points.

2-6 Algebraic Proof

State the property that justifies each statement.

29. If $7(x - 3) = 35$, then $35 = 7(x - 3)$.

30. If $2x + 19 = 27$, then $2x = 8$.

31. $5(3x + 1) = 15x + 5$

32. $7x - 2 = 7x - 2$

33. If $12 = 2x + 8$ and $2x + 8 = 3y$, then $12 = 3y$.

34. Copy and complete the following proof.

Given: $6(x - 4) = 42$

Prove: $x = 11$

Statements	Reasons
a. $6(x - 4) = 42$	**a.** ?
b. $6x - 24 = 42$	**b.** ?
c. $6x = 66$	**c.** ?
d. $x = 11$	**d.** ?

35. Write a two-column proof to show that if $PQ = RS$, $PQ = 5x + 9$, and $RS = x - 31$, then $x = -10$.

36. **GRADES** Jerome received the same quarter grade as Paula. Paula received the same quarter grade as Heath. Which property would show that Jerome and Heath received the same grade?

Example 6

Write a two-column proof.

Given: $\frac{5x - 3}{6} = 2x + 1$

Prove: $x = -\frac{9}{7}$

Proof:

Statements	Reasons
1. $\frac{5x - 3}{6} = 2x + 1$	**1.** Given
2. $5x - 3 = 6(2x + 1)$	**2.** Multiplication Property of Equality
3. $5x - 3 = 12x + 6$	**3.** Distributive Property of Equality
4. $-3 = 7x + 6$	**4.** Subtraction Property of Equality
5. $-9 = 7x$	**5.** Subtraction Property of Equality
6. $-\frac{9}{7} = x$	**6.** Division Property of Equality
7. $x = -\frac{9}{7}$	**7.** Symmetric Property of Equality

2-7 Proving Segment Relationships

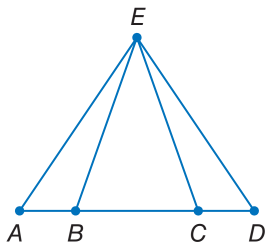
Write a two-column proof.

37. **Given:** X is the midpoint of $\overline{WY}$ and $\overline{VZ}$.

Prove: $VW = ZY$

38. **Given:** $AB = DC$

Prove: $AC = DB$



39. **GEOGRAPHY** Leandro is planning to drive from Kansas City to Minneapolis along Interstate 35. The map he is using gives the distance from Kansas City to Des Moines as 194 miles and from Des Moines to Minneapolis as 243 miles. What allows him to conclude that the distance he will be driving is 437 miles from Kansas City to Minneapolis? Assume that Interstate 35 forms a straight line.

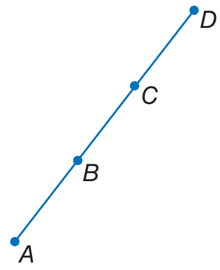
Example 7

Write a two-column proof.

Given: B is the midpoint of $\overline{AC}$.

C is the midpoint of $\overline{BD}$.

Prove: $\overline{AB} \cong \overline{CD}$



Proof:

Statements	Reasons
1. B is the midpoint of $\overline{AC}$.	**1.** Given
2. $\overline{AB} \cong \overline{BC}$	**2.** Definition of midpoint
3. C is the midpoint of $\overline{BD}$.	**3.** Given
4. $\overline{BC} \cong \overline{CD}$	**4.** Definition of midpoint
5. $\overline{AB} \cong \overline{CD}$	**5.** Transitive Property of Equality

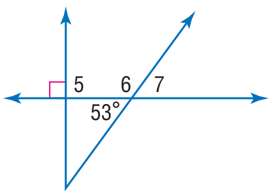
2-8 Proving Angle Relationships

Find the measure of each angle.

40. $\angle 5$

41. $\angle 6$

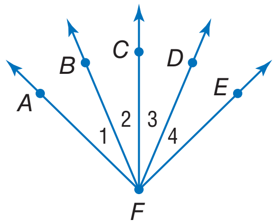
42. $\angle 7$



43. **PROOF** Write a two-column proof.

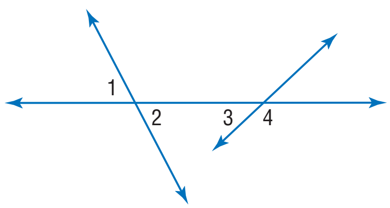
Given: $\angle 1 \cong \angle 4$, $\angle 2 \cong \angle 3$

Prove: $\angle AFC \cong \angle EFC$



Example 8

Find the measure of each numbered angle if $m\angle 1 = 72$ and $m\angle 3 = 26$.



$m\angle 2 = 72$, since $\angle 1$ and $\angle 2$ are vertical angles.

$\angle 3$ and $\angle 4$ form a linear pair and must be supplementary angles.

$26 + m\angle 4 = 180$ Definition of supplementary angles

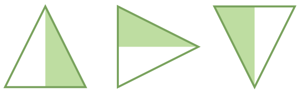
$m\angle 4 = 154$ Subtract 26 from each side.

CHAPTER 2 Practice Test

Write a conjecture that describes the pattern in each sequence. Then use your conjecture to find the next item in the sequence.

1. 15, 30, 45, 60

2.



Use the following statements to write a compound statement for each conjunction or disjunction. Then find its truth value.

***p*: 5 < −3**

***q*: All vertical angles are congruent.**

***r*: If 4*x* = 36, then *x* = 9.**

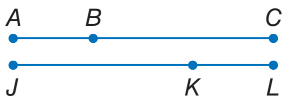
3. p and q

4. $(p \vee q) \wedge r$

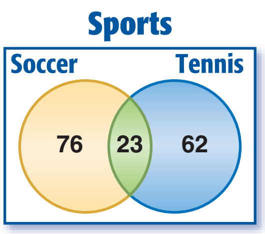
5. **PROOF** Write a paragraph proof.

Given: $\overline{JK} \cong \overline{CB}$, $\overline{KL} \cong \overline{AB}$

Prove: $\overline{JL} \cong \overline{AC}$



6. **SPORTS** Refer to the Venn diagram that represents the sports students chose to play at South High School last year.



a. Describe the sports that the students in the nonintersecting portion of the tennis region chose.

b. How many students played soccer and tennis?

7. Determine whether the stated conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning.

Given: If a lawyer passes the bar exam, then he or she can practice law. Candice passed the bar exam.

Conclusion: Candice can practice law.

8. **PROOF** Copy and complete the following proof.

Given: $3(x - 4) = 2x + 7$

Prove: $x = 19$

Proof:

Statements	Reasons
a. $3(x - 4) = 2x + 7$	**a.** Given
b. $3x - 12 = 2x + 7$	**b.** ___?___
c. ___?___	**c.** Subtraction Property
d. $x = 19$	**d.** ___?___

Determine whether each statement is *always*, *sometimes*, or *never* true.

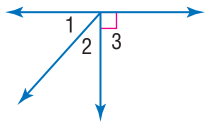
9. Two angles that are supplementary form a linear pair.

10. If *B* is between *A* and *C*, then $AC + AB = BC$.

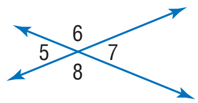
11. If two lines intersect to form congruent adjacent angles, then the lines are perpendicular.

Find the measure of each numbered angle, and name the theorems that justify your work.

12. $m\angle 1 = x$, $m\angle 2 = x - 6$



13. $m\angle 7 = 2x + 15$, $m\angle 8 = 3x$



Write each statement in if-then form.

14. An acute angle measures less than 90.

15. Two perpendicular lines intersect to form right angles.

16. **MULTIPLE CHOICE** If a triangle has one obtuse angle, then it is an obtuse triangle.

Which of the following statements is the contrapositive of the conditional above?

A If a triangle is not obtuse, then it has one obtuse angle.

B If a triangle does not have one obtuse angle, then it is not an obtuse triangle.

C If a triangle is not obtuse, then it does not have one obtuse angle.

D If a triangle is obtuse, then it has one obtuse angle.

CHAPTER 2

Preparing for Standardized Tests

Logical Reasoning

Solving geometry problems frequently requires the use of logical reasoning. You can use the fundamentals of logical reasoning to help you solve problems on standardized tests.

Lauren Nicole/Photographer's Choice RF/Getty Images

Strategies for Using Logical Reasoning

Step 1

Read the problem to determine what information you are given and what you need to find out in order to answer the question.

Step 2

Determine if you can apply one of the principles of logical reasoning to the problem.

- **Counterexample**: A counterexample contradicts a statement that is known to be true.

  Identify any answer choices that contradict the problem statement and eliminate them.

- **Postulates:** A postulate is a statement that describes a fundamental relationship in geometry.

  Determine if you can apply a postulate to draw a logical conclusion.

Step 3

If you cannot reach a conclusion using only the principles in Step 2, determine if one of the tools below would be helpful.

- **Patterns:** Look for a pattern to make a conjecture.
- **Truth Tables:** Use a truth table to organize the truth values of the statement provided in the problem.
- **Venn Diagrams:** Use a Venn Diagram to clearly represent the relationships between members of groups.
- **Proofs:** Use deductive and inductive reasoning to reach a conclusion in the form of a proof.

Step 4

If you still cannot reach a conclusion using the tools in Step 3, make a **conjecture**, or educated guess, about which answer choice is most reasonable. Then mark the problem so that you can return to it if you have extra time at the end of the exam.

Standardized Test Example

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

In a school of 292 students, 94 participate in sports, 122 participate in academic clubs, and 31 participate in both. How many students at the school do not participate in sports or academic clubs?

A 95 **C** 122

B 107 **D** 138

Read the problem carefully. There are no clear counterexamples, and a postulate cannot be used to draw a logical conclusion. Therefore, consider the tools that you can use to organize the information.

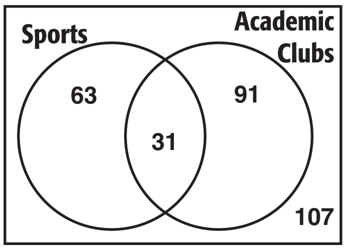
A Venn diagram can be used to show the intersection of two sets. Make a Venn diagram with the information provided in the problem statement.

Determine how many students participate in only sports or academic clubs.

Only sports: $94 - 31 = 63$

Only academic clubs: $122 - 31 = 91$

School Participation



Use the information to calculate the number of students who do not participate in either sports or academic clubs.

$292 - 63 - 91 - 31 = 107$

There are 107 students who do not participate in either sports or academic clubs. The correct answer is B.

Exercises

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Determine the truth of the following statement. If the statement is false, give a counterexample.

The product of two even numbers is even.

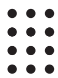
A false; $8 \times 4 = 32$

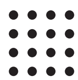
B false; $7 \times 6 = 42$

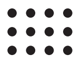
C false; $3 \times 10 = 30$

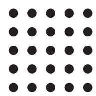
D true

2. Find the next item in the pattern.

F 

H 

G 

J 

CHAPTER 2 Standardized Test Practice

Cumulative, Chapters 1 through 2

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Which conjunction is true for statements p and q below?

p: There are four letters in MATH.

q: There are two vowels in MATH.

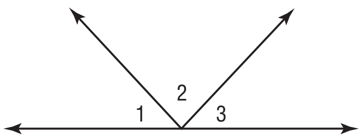
A $\sim p \wedge \sim q$

B $p \wedge q$

C $p \wedge \sim q$

D $\sim p \wedge q$

2. In the diagram below, $\angle 1 \cong \angle 3$.



Which of the following conclusions does not have to be true?

F $m\angle 1 - m\angle 2 + m\angle 3 = 90$

G $m\angle 1 + m\angle 2 + m\angle 3 = 180$

H $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$

J $m\angle 2 - m\angle 1 = m\angle 2 - m\angle 3$

3. *Two supplementary angles always form a linear pair.*

Which of the following best describes a *counterexample* to the assertion above?

A two acute angles

B two nonadjacent angles

C two obtuse angles

D two right angles

> **Test-Taking Tip**
>
> **Question 3** A *counterexample* is an example used to show that a given statement is not always true.

4. Determine which statement follows logically from the given statements.

If it rains today, the game will be cancelled.

Cancelled games are made up on Saturdays.

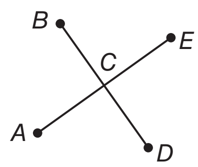
F If a game is cancelled, it was because of rain.

G If it rains today, the game will be made up on Saturday.

H Some cancelled games are not made up on Saturdays.

J If it does not rain today, the game will not be made up on Saturday.

5. In the diagram, $\overline{BD}$ intersects $\overline{AE}$ at C. Which of the following conclusions does *not* have to be true?



A $\angle ACB \cong \angle ECD$

B $\angle ACB$ and $\angle ACD$ form a linear pair.

C $\angle BCE$ and $\angle ACD$ are vertical angles.

D $\angle BCE$ and $\angle ECD$ are complementary angles.

6. A farmer needs to make a 1000-square-foot rectangular enclosure for her cows. She wants to save money by purchasing the least amount of fencing possible to enclose the area. What whole-number dimensions will require the least amount of fencing?

F 8 ft by 125 ft

G 10 ft by 100 ft

H 20 ft by 50 ft

J 25 ft by 40 ft

Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

7. Points A, B, C, and D are collinear, with point B between points A and C and point C between points B and D. Complete the statement.

$AB + \underline{\ \ ?\ \ } = AD$

8. GRIDDED RESPONSE Suppose line m contains points D, E, and F. If $DE = 12$ millimeters, $EF = 15$ millimeters, and point D is between points E and F, what is the length of $\overline{DF}$? Express your answer in millimeters.

9. Use the proof to answer the question.

Given: $\angle A$ is the complement of $\angle B$.
$m\angle B = 46$

Prove: $m\angle A = 44$

Proof:

Statements	Reasons
1. A is the complement of $\angle B$; $m\angle B = 46$.	**1.** Given
2. $m\angle A + m\angle B = 90$	**2.** Def. of comp. angles
3. $m\angle A + 46 = 90$	**3.** Substitution Prop.
4. $m\angle A + 46 - 46 = 90 - 46$	**4.** $\underline{\ \ ?\ \ }$
5. $m\angle A = 44$	**5.** Substitution Prop.

What reason can be given to justify Statement 4?

10. Write the contrapositive of the statement.

If an angle measures greater than 90°, then it is obtuse.

11. GRIDDED RESPONSE Point E is the midpoint of $\overline{DF}$. If $DE = 8x - 3$ and $EF = 3x + 7$, what is x?

Extended Response

Record your answers on a sheet of paper. Show your work.

12. Consider the pattern.

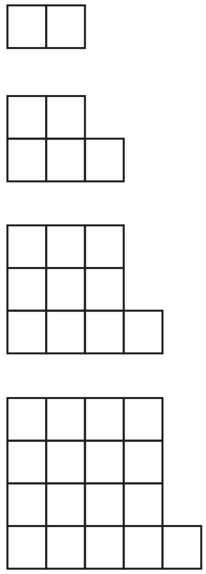


Figure 1

Figure 2

Figure 3

Figure 4

a. Make a conjecture about the number of squares in each figure.

b. Write an algebraic expression that can be used to find the number of squares in the n^{th} figure in the pattern.

c. How many squares will be needed to make the 6^{th} figure of the pattern?

Need ExtraHelp?

If you missed Question...	1	2	3	4	5	6	7	8	9	10	11	12
Go to Lesson...	2-2	2-8	2-1	2-4	2-7	1-6	1-2	1-2	2-8	2-3	1-3	2-1