Student Handbook

This **Student Handbook** can help you answer these questions.

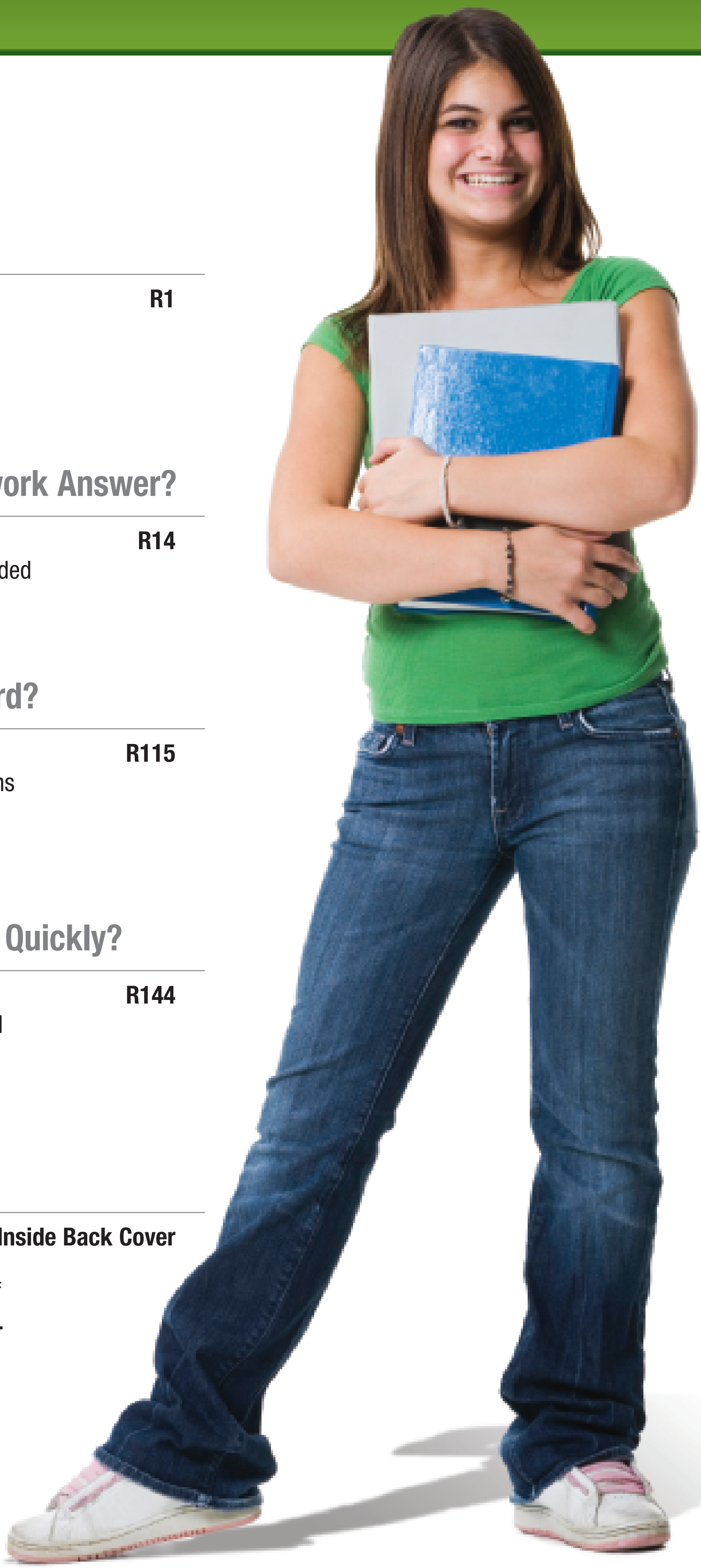
What if I Need More Practice?

What if I Need to Check a Homework Answer?

What if I Forget a Vocabulary Word?

What if I Need to Find Something Quickly?

What if I Forget a Formula?



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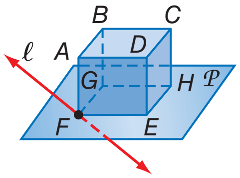
Extra Practice

CHAPTER 1 Tools of Geometry

Refer to the figure. (Lesson 1-1)



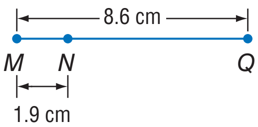
1. How many planes are shown in this figure?

2. Name the intersection of plane *ADE* with plane 𝒫.

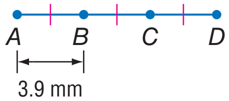
3. Name two noncoplanar lines that do not intersect.

Find the measurement of each segment. Assume that each figure is not drawn to scale. (Lesson 1-2)

4. $\overline{NQ}$



5. $\overline{BD}$



6. Find *x* and *AB* if *B* is between *A* and *C*, $AB = 3x$, $BC = 14$, and $AC = 41$. (Lesson 1-2)

7. **PLAYGROUND** The pivot or midpoint of a seesaw on a playground is 13.5 feet from the swing set. If the swing set is 8 feet from the edge of the seesaw when the seesaw is level, how long is the seesaw? (Lesson 1-3)

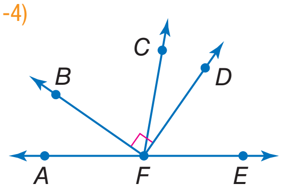
8. Find the coordinates of *M* if *N*(1.5, 2.5) is the midpoint of $\overline{MP}$ and *P* has coordinates (6, 9) (Lesson 1-3)

Copy the diagram shown, and extend each ray. Classify each angle as *right*, *acute*, or *obtuse*. Then use a protractor to measure the angle to the nearest degree. (Lesson 1-4)

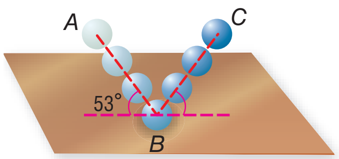
9. $\angle AFC$

10. $\angle DFB$

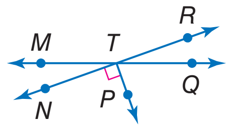
11. $\angle CFD$



12. **SPORTS** When a ball is bounced on a smooth surface, the angles formed by the path of the ball are congruent. If the path of a ball forms a 53° angle with the floor, find $m\angle ABC$. (Lesson 1-5)



Determine whether each statement can be assumed from the figure. Explain. (Lesson 1-5)

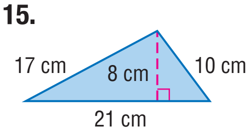


13. $\angle RTQ$ and $\angle MTN$ are vertical angles.

14. $\overrightarrow{PT}$ is perpendicular to $\overleftrightarrow{TM}$

Find the perimeter or circumference and area of each figure. Round to the nearest tenth, if necessary. (Lesson 1-6)

15.



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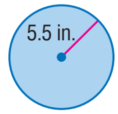
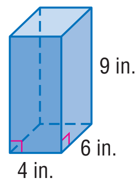


17. **PLANNING** Tim is responsible for renting tables for an awards banquet. He needs 20 tables, and he can choose between circular tables with a diameter of 5.5 feet and square tables with a side length of 5 feet. Which option should Tim choose so that the tables cover the *smallest* area? (Lesson 1-6)

Find the surface area and volume of each solid to the nearest tenth. (Lesson 1-7)

18.



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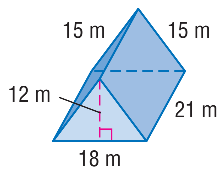
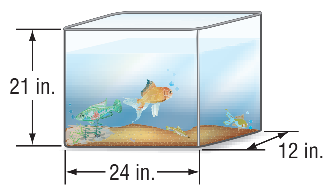


20. **FISH** Sarah has a fish tank with the dimensions shown. (Lesson 1-7)

   a. What is the surface area of the fish tank?

   b. If Sarah fills the tank to a depth of 17 inches, what will be the volume of the water in the tank?



Extra Practice

1. Determine whether the statement below is *true* or *false*. If false, provide a counterexample. (Lesson 2-1)

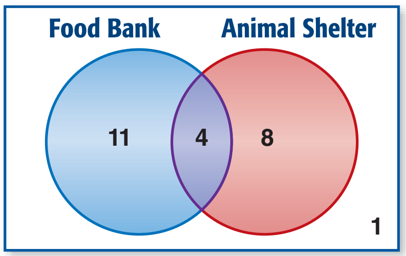
If $\overline{AB}$ is perpendicular to $\overleftrightarrow{CD}$, then exactly two right angles are formed.

Use the following statements to write a compound statement for each conjunction or disjunction. Then find its truth value. Explain your reasoning. (Lesson 2-2)
p: A prism has two bases.
q: A pyramid has two bases.
r: A sphere has no bases.

2. $p \wedge r$ **3.** $q \vee \sim r$

COMMUNITY SERVICE **Refer to the Venn diagram that represents the organizations served by the students in a service club.** (Lesson 2-2)

Community Service



4. How many students volunteer in both organizations?

5. How many students volunteer for only one organization?

6. How many students are in the club?

Determine the truth value of each conditional statement. If *true*, explain your reasoning. If *false*, give a counterexample. (Lesson 2-3)

7. If a number is divisible by 6, then it is divisible by 3.

8. If $x^4 = 16$, then $x = 2$.

9. **GRADES** Draw a valid conclusion from the statement below, if possible. Then state whether your conclusion was drawn using the Law of Detachment or the Law of Syllogism. If no valid conclusion can be drawn, write *no valid conclusion* and explain your reasoning. (Lesson 2-4)

If Amy scores above a 90 on her chemistry test, she will make an A. If she makes an A, her parents will take her to the theater. Amy scored a 93 on her chemistry test.

Determine whether each conclusion is based on *inductive* or *deductive* reasoning. (Lesson 2-4)

10. Andrew's mom makes spaghetti for dinner on Mondays. Today is Monday. He concludes that his mom will make spaghetti for dinner.

11. If Beth turns in her report early, she receives extra credit. Beth turned in her report early. She concludes that she will receive extra credit.

Determine whether each statement is *always*, *sometimes*, or *never* true. Explain you reasoning. (Lesson 2-5)

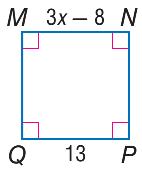
12. If planes $\mathcal{A}$ and $\mathcal{B}$ intersect, then their intersection is a line.

13. If point A lies in plane $\mathcal{P}$, then $\overleftrightarrow{AB}$ lies in plane $\mathcal{P}$.

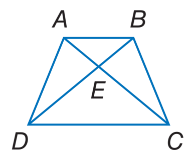
14. Lines p and q intersect in points M and N.

15. **PHYSICS** The formula for pressure under water is $P = \rho g h$, where ρ is the density of water, g is gravity, and h is the height of the water. Given the water pressure, prove that you can calculate the height of the water using $h = \frac{P}{\rho g}$. (Lesson 2-6)

16. **PROOF** Write a two-column proof to verify that if $\overline{MN} \cong \overline{QP}$, then $x = 7$. (Lesson 2-6)

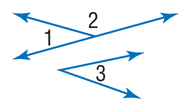


17. **PROOF** Prove the following. (Lesson 2-7)
Given: $\overline{AC} \cong \overline{BD}$
$\overline{EC} \cong \overline{ED}$
Prove: $\overline{AE} \cong \overline{BE}$

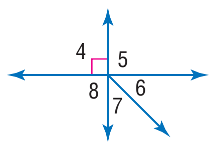


Find the measure of each numbered angle, and name the theorems that justify your work. (Lesson 2-8)

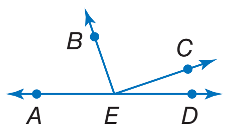
18. $\angle 2$ and $\angle 3$ are supplementary; $m\angle 2 = 149$



19. $\angle 6 \cong \angle 7$



20. **PROOF** Given that $\angle BEC$ is a right angle, prove that $\angle AEB$ and $\angle CED$ are complementary. (Lesson 2-8)



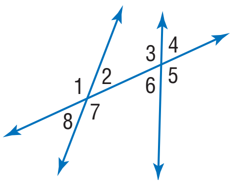
Classify the relationship between each pair of angles as *alternate interior*, *alternate exterior*, *corresponding*, or *consecutive interior* angles. (Lesson 3-1)

1. $\angle 5$ and $\angle 7$

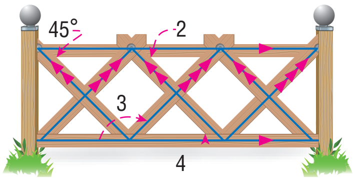
2. $\angle 2$ and $\angle 6$

3. $\angle 6$ and $\angle 7$

4. $\angle 4$ and $\angle 8$

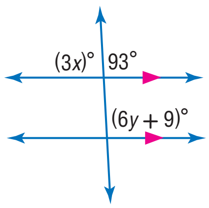


5. FENCES Find the measures of $\angle 2$, $\angle 3$, and $\angle 4$ of the fence shown. (Lesson 3-2)



Find the value of the variables in each figure. Explain your reasoning. (Lesson 3-2)

6.



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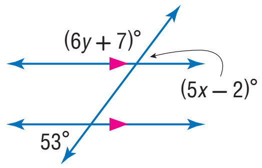


8. JOBS Katie mows lawns in the summer to earn extra money. She started with 3 lawns, and she now mows 12 lawns in her fourth summer. (Lesson 3-3)

a. Graph the line that models the growth of Katie's business over time.

b. What is the slope of the graph? What does it represent?

c. Assuming that the business continues to grow at the same rate, how many lawns should Katie plan to mow during her sixth summer?

Determine whether $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are *parallel*, *perpendicular*, or *neither*. Graph each line to verify your answer. (Lesson 3-3)

9. $A(-4, -2)$, $B(0, 4)$, $C(-2, -4)$, $D(4, 5)$

10. $A(0, -3)$, $B(3, 9)$, $C(-4, 0)$, $D(8, 3)$

Graph the line that satisfies each condition. (Lesson 3-3)

11. slope $= -\frac{1}{2}$, passes through $W(4, 5)$

12. passes through $M(-3, -5)$, perpendicular to $\overleftrightarrow{NP}$ with $N(-3, 2)$ and $P(9, 4)$

Write an equation in point-slope form of the line having the given slope that contains the given point. Then graph the line. (Lesson 3-4)

13. $m = -\frac{4}{5}, (10, -14)$

14. $m = 5, (-1, -3)$

Write an equation of the line through each pair of points in slope-intercept form. (Lesson 3-4)

15. $(4, 0)$ and $(-3, -7)$

16. $(-1, 8)$ and $(5, -4)$

17. MOVIES Wade watches movies online through an online movie rental company. His current plan costs \$7.99 per month with an additional charge of \$2.39 per movie. He is considering switching to a plan that costs \$14.99 per month with an additional charge of \$1.49 per movie. (Lesson 3-4)

a. Write an equation to represent the total monthly cost for each plan.

b. Graph the equations.

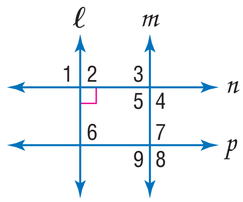
c. If Wade watches an average of 10 movies per month, should he keep his current plan or change to the other plan? Explain.

Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer. (Lesson 3-5)

18. $m\angle 4 + m\angle 7 = 180$

19. $\angle 1 \cong \angle 4$

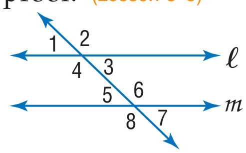
20. $m\angle 6 = 90$



21. PROOF Write a two-column proof. (Lesson 3-5)

Given: $\angle 1$ and $\angle 8$ are supplementary.

Prove: $\ell \parallel m$



Find the distance from P to ℓ. (Lesson 3-6)

22. Line ℓ contains points $(0, 3)$ and $(-4, -9)$. Point P has coordinates $(-6, -5)$.

23. Line ℓ contains points $(-5, 6)$ and $(1, -6)$. Point P has coordinates $(6, 4)$.

Find the distance between each pair of parallel lines with the given equations. (Lesson 3-6)

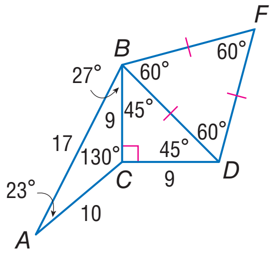
24. $y = -4x - 6$
$y = -4x + 11$

25. $y = \frac{1}{2}x + \frac{7}{2}$
$y = \frac{1}{2}x + 1$

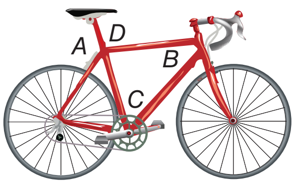
CHAPTER 4 Congruent Triangles

Classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*, and as *equilateral*, *isosceles*, or *scalene*. (Lesson 4-1)

1. $\triangle ABC$
2. $\triangle BCD$
3. $\triangle BDF$

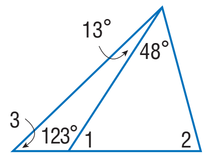


4. **BICYCLING** In the bicycle shown, $m\angle C = 60$ and $m\angle D = 105$. Find $m\angle B$. (Lesson 4-2)



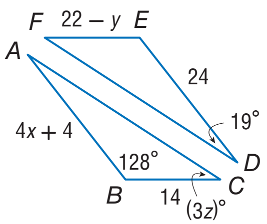
Find each measure. (Lesson 4-2)

5. $m\angle 1$
6. $m\angle 2$
7. $m\angle 3$



In the figure, $\triangle ABC \cong \triangle DEF$. Find each value. (Lesson 4-3)

8. x
9. y
10. z



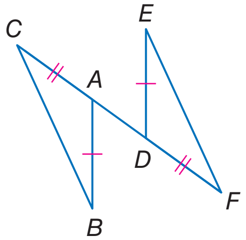
Determine whether $\triangle PQR \cong \triangle XYZ$. Explain. (Lesson 4-4)

11. $P(-4, 2)$, $Q(2, 2)$, $R(2, 8)$; $X(-1, -3)$, $Y(5, -3)$, $Z(5, 4)$
12. $P(-2, 4)$, $Q(-7, 3)$, $R(0, 9)$; $X(3, 6)$, $Y(2, 1)$, $Z(8, 8)$

13. **PROOF** Write a two-column proof. (Lesson 4-4)

    Given: $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, $\overline{AB} \parallel \overline{DE}$

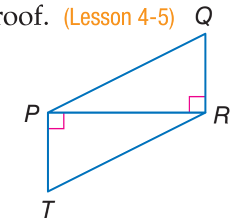
    Prove: $\triangle ABC \cong \triangle DEF$



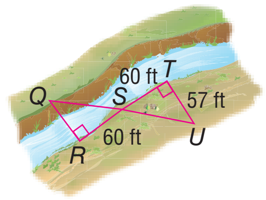
14. **PROOF** Write a paragraph proof. (Lesson 4-5)

    Given: $m\angle PRQ = 90$
    $m\angle RPT = 90$
    $\angle Q \cong \angle T$

    Prove: $\triangle QRP \cong \triangle TPR$

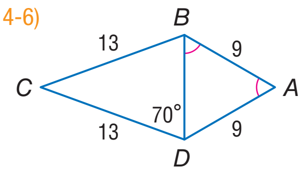


15. **NATURE** What is the approximate width of the creek shown below? Explain your reasoning. (Lesson 4-5)

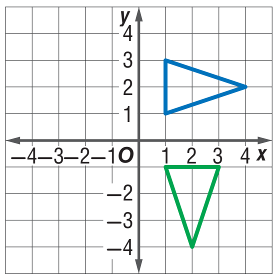


Find each measure. (Lesson 4-6)

16. $m\angle BDA$
17. BD
18. $m\angle BCD$



19. Identify the type of congruence transformation shown as a *reflection*, *translation*, or *rotation*. (Lesson 4-7)

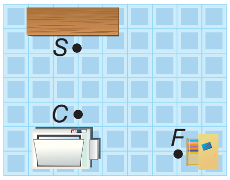


Graph each pair of triangles with the given vertices. Then, identify the transformation, and verify that it is a congruence transformation. (Lesson 4-7)

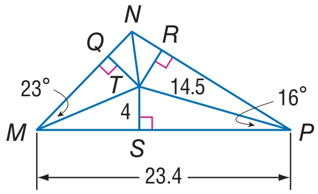
20. $A(2, 2)$, $B(4, -1)$, $C(1, -2)$; $M(-4, -1)$, $N(-1, -2)$, $P(-2, 2)$
21. $A(-1, -1)$, $B(-1, -4)$, $C(2, -4)$; $M(-1, 1)$, $N(-4, 1)$, $P(-4, 4)$

22. **CAMPUS** The gym at Alex's school is located 70 feet south and 90 feet west of the main building. The vocational building is located 120 feet south and 200 feet east of the main building. Show that the triangle formed by these three buildings is scalene. Explain your reasoning. (Lesson 4-8)

1. **OFFICE DESIGN** The copy machine C, filing cabinet F, and supply shelves S are positioned in an office as shown. Copy the diagram, and find the location for the center of a work table so that it is the same distance from all three points. (Lesson 5-1)



Point T is the incenter of $\triangle MNP$. Find each measure below. Round to the nearest tenth, if necessary. (Lesson 5-1)

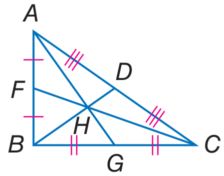


2. QT

3. MT

4. $m\angle PNT$

In $\triangle ABC$, $CH = 70\frac{2}{3}$, $AG = 85$, and $DH = 20\frac{1}{3}$. Find each length. (Lesson 5-2)



5. FH

6. BD

7. AH

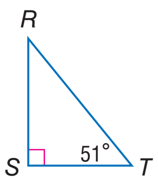
Find the indicated point of concurrency for each triangle with the given vertices. (Lesson 5-2)

8. centroid; $A(-4, -5)$, $B(-1, 2)$, $C(2, -4)$

9. orthocenter; $M(-9, -2)$, $N(1, 8)$, $P(9, -8)$

List the angles and sides of each triangle in order from smallest to largest. (Lesson 5-3)

10.



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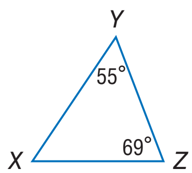
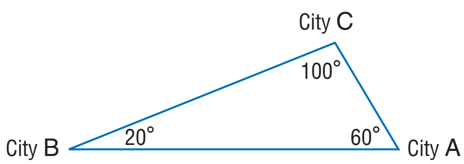


12. **MAPS** Three cities lie in a triangle as shown. Which two cities are the farthest apart? (Lesson 5-3)



Write an indirect proof of each statement. (Lesson 5-4)

13. If $-7 + 6x < 11$, then $x < 3$.

14. If $5 - 2x < -13$, then $x > 9$.

15. **JOBS** Nate worked for 6 hours today with two breaks. Use indirect reasoning to show that he worked for longer than two hours without a break at some point during his shift. (Lesson 5-4)

16. Write an indirect proof of the statement below. (Lesson 5-4)

A triangle can have at most one obtuse angle.

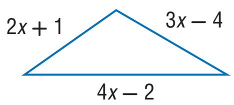
17. Is it possible to form a triangle with side lengths 3 centimeters, 8 centimeters, and 11 centimeters? If not, explain why not. (Lesson 5-5)

Find the range for the measure of the third side of a triangle given the measures of two sides. (Lesson 5-5)

18. 9 ft, 16 ft

19. 1.7 m, 2.9 m

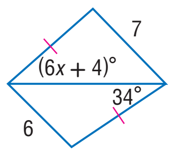
20. Determine the possible values of x. (Lesson 5-5)



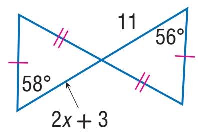
21. **SNOWSHOEING** Two groups of friends leave the same cabin to go snowshoeing. Group A goes 2.5 miles due north and then turns 79° east of north and travels an additional 2.5 miles. Group B goes 2.5 miles due south and then turns 86° east of south and travels an additional 2.5 miles. Who is now closer to the cabin? Use a diagram to explain your reasoning. (Lesson 5-6)

Find the range of values containing x. (Lesson 5-6)

22.



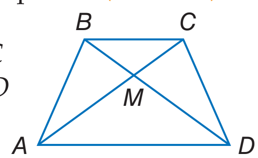
23.



24. **PROOF** Write a two-column proof. (Lesson 5-6)

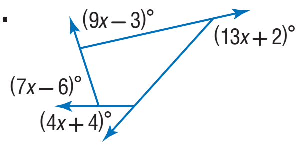
Given: $\overline{AB} \cong \overline{CD}$,
$m\angle CDB > m\angle BAC$
$m\angle BDA > m\angle CAD$

Prove: $AC > DB$

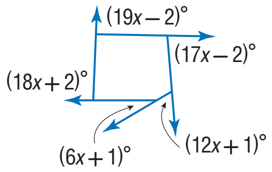


Find the value of x in each diagram. (Lesson 6-1)

1.



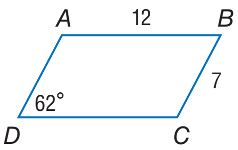
2.



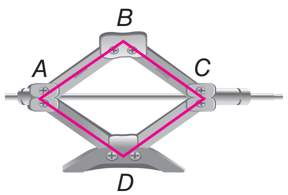
Use parallelogram *ABCD* to find each measure. (Lesson 6-2)

3. $m\angle A$

4. AD

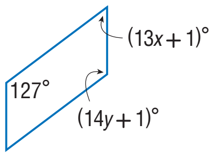


5. AUTO REPAIR A scissor jack is used to lift part of a car to make repairs. *ABCD* is a parallelogram. As the jack is raised, $m\angle A$ and $m\angle C$ increase. Explain what must happen to $m\angle B$ and $m\angle D$. (Lesson 6-2)

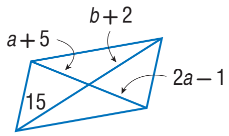


Find the value of each variable in each parallelogram. (Lesson 6-2)

6.

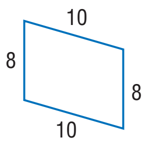


7.

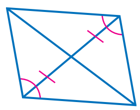


Determine whether each quadrilateral is a parallelogram. Justify your answer. (Lesson 6-3)

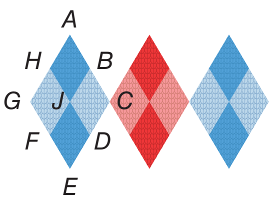
8.



9.

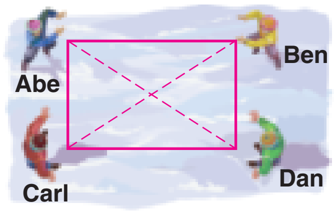


10. DESIGN Valerie is using the design shown for the front of a sweater. If all of the blocks of color are congruent parallelograms, write a two-column proof to show that *ACEG* is a parallelogram. (Lesson 6-3)

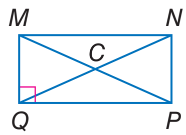


11. Graph the quadrilateral with vertices $A(0, 3)$, $B(10, 4)$, $C(6, -1)$, and $D(-4, -2)$. Determine whether *ABCD* is a parallelogram. Justify your answer. (Lesson 6-3)

12. SNOW Four boys are throwing snowballs, such that their positions are the vertices of a rectangle as shown. If Carl is 100 feet from Ben, and Abe and Ben throw snowballs along the diagonals at the same time and the snowballs collide in mid air, how far had the snowballs traveled at the time of collision? (Lesson 6-4)



Quadrilateral *MNPQ* is a rectangle. (Lesson 6-4)



13. If $m\angle MNQ = 3x + 3$ and $m\angle QNP = 10x - 4$, find $m\angle MQN$.

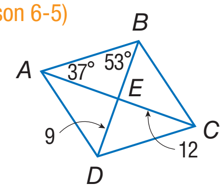
14. If $MC = 4.5x - 2.5$ and $NC = 4x + 1.5$, find QN.

15. PROOF Write a proof to show that the four smaller triangles formed by the diagonals of a rectangle are isosceles. (Lesson 6-4)

Quadrilateral *ABCD* is a rhombus. Find each value or measure. (Lesson 6-5)

16. $m\angle DBC$

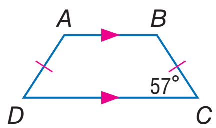
17. DC



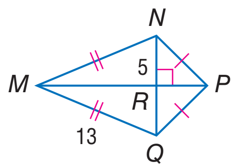
18. Determine whether ▱*WXYZ* with verticies $W(-2, 0)$, $X(1, 1)$, $Y(2, -2)$, $Z(-1, -3)$ is a *rhombus*, a *rectangle*, or a *square*. List all that apply. Explain. (Lesson 6-5)

Find each measure. (Lesson 6-6)

19. $m\angle A$



20. MR

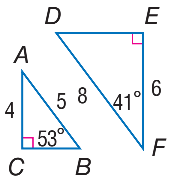


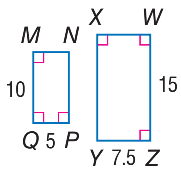
Find the measures of the angles or sides in each triangle. (Lesson 7-1)

1. The ratios of the measures of the three angles is 4 : 5 : 11.
2. The ratio of the three sides is 5 : 3 : 6 and the perimeter is 98 centimeters.

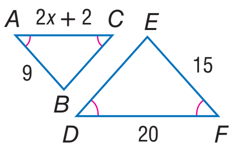
3. **FAMILY** Shawn surveyed his homeroom class to find out how many of his classmates have at least 1 sibling. Of the 32 people in his homeroom, 25 have at least one sibling. (Lesson 7-1)
   a. If there are 470 students in Shawn's school, write a proportion that could be used to predict the number of students in the school who have at least one sibling.
   b. What is the number of students in Shawn's school with at least one sibling?

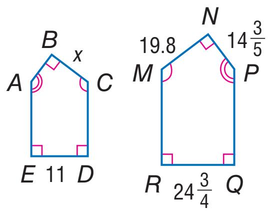
Determine whether each pair of figures is similar. If so, write the similarity statement and scale factor. If not, explain your reasoning. (Lesson 7-2)

4. 

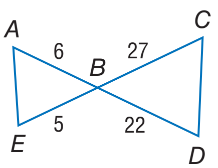
5. 

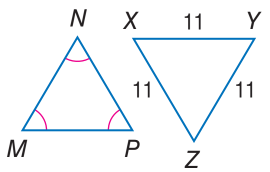
Each pair of polygons is similar. Find the value of *x*. (Lesson 7-2)

6. 

7. 

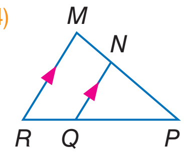
Determine whether the triangles are similar. If so, write a similarity statement. Explain your reasoning. (Lesson 7-3)

8. 

9. 

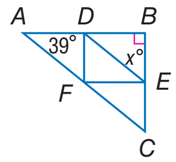
10. **HEIGHT** When Rachel stands next to her cousin, Rachel's shadow is 2 feet long and her cousin's shadow is 1 foot long. If Rachel is 5 feet 6 inches tall, how tall is her cousin? (Lesson 7-3)

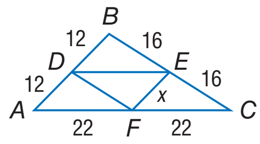
Refer to the figure shown. (Lesson 7-4)



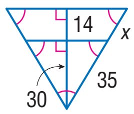
11. If $PN = 4$, $NM = 1$, and $PQ = 5$, find PR.
12. If $PR = 13$, $PQ = 9$, and $NM = 3$, find PN.

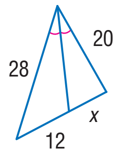
$\overline{DE}$, $\overline{EF}$, and $\overline{FD}$ are midsegments of $\triangle ABC$. Find the value of x. (Lesson 7-4)

13. 

14. 

Find *x*. (Lesson 7-5)

15. 

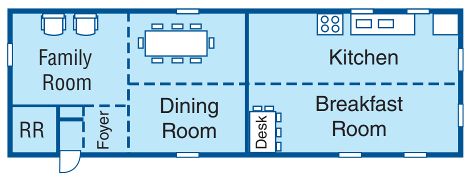
16. 

17. **FOOSBALL** Jason wants to determine if his foosball table is a dilation of his school's soccer field. The dimension of the table are 30 inches by $55\frac{1}{2}$ inches, and the dimensions of the field are 60 yards by 110 yards. Is the table a dilation? Explain. (Lesson 7-6)

Graph the original figure and its dilated image. Then verify that the dilation is a similarity transformation. (Lesson 7-6)

18. $A(0, -2)$, $B(-1, 3)$, $C(3, -2)$; $X(0, -6)$, $Y(-3, 9)$, $Z(9, -6)$
19. $D(-6, 6)$, $E(6, 2)$, $F(-2, -4)$; $M(-3, 3)$, $N(3, 1)$, $P(-1, -2)$

20. **ARCHITECTURE** The blueprint below is a scale drawing of a proposed structure. (Lesson 7-7)



Scale: 1 in. = 25 ft

   a. What are the approximate dimensions of the family room? Round to the nearest 0.5 foot, if necessary.
   b. What are the approximate dimensions of the dining room? Round to the nearest 0.5 foot, if necessary.

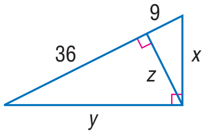
Find the geometric mean between each pair of numbers. (Lesson 8-1)

1. 7 and 12

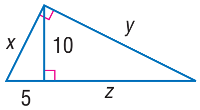
2. 8 and 36

Find x, y, and z. (Lesson 8-1)

3.

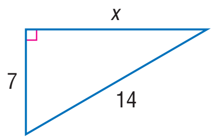


4.

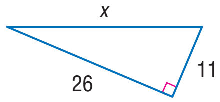


Find x. (Lesson 8-2)

5.



6.



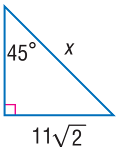
Determine whether each set of numbers can be the measures of the sides of a triangle. If so, classify the triangle as *acute*, *obtuse*, or *right*. Justify your answer. (Lesson 8-2)

7. 24, 32, 41

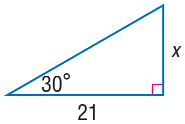
8. 17.5, 60, 62.5

Find x. (Lesson 8-3)

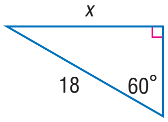
9.



10.



11.



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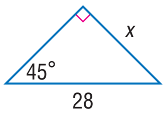
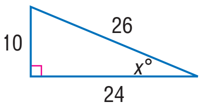
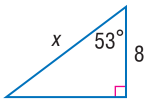


Find x. Round to the nearest tenth, if necessary. (Lesson 8-4)

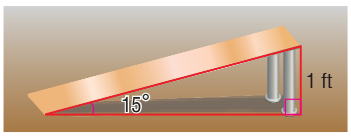
13.



14.



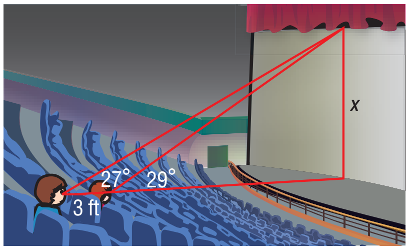
15. SKATEBOARDING Lindsey is building a skateboard ramp. She wants the ramp to be 1 foot tall at the end and she wants it to make a 15° angle with the ground. What length of board should she buy for the ramp itself? Round to the nearest foot. (Lesson 8-4)



16. BUILDINGS Kara is standing about 50 feet from the base of her apartment building, looking up at it with an angle of elevation of 75°. What is the approximate height of Kara's building? (Lesson 8-5)

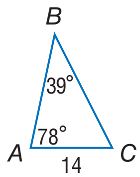
17. ROLLER COASTERS Evan is looking down the hill of a roller coaster from a height of 75 feet with an angle of depression of about 70°. What is the approximate horizontal distance from the top of the hill to the bottom of the hill? (Lesson 8-5)

18. MOVIES Kim is sitting in the row behind her friend Somi at the movies. Kim is looking at the screen with an angle of elevation of about 27° and Somi's angle of elevation is about 29°. If there are 3 feet between each row of seats, about how tall is the movie screen? (Lesson 8-5)

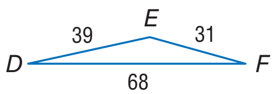


Solve each triangle. Round angle measures to the nearest degree and side measures to the nearest tenth. (Lesson 8-6)

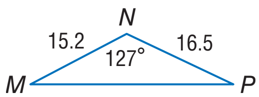
19.



20.



21.



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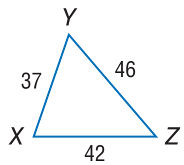
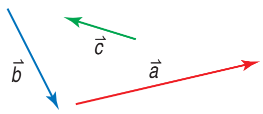


Copy the vectors to find each sum or difference. (Lesson 8-7)

23. $\vec{a} - \vec{b}$

24. $\vec{a} + \vec{c}$

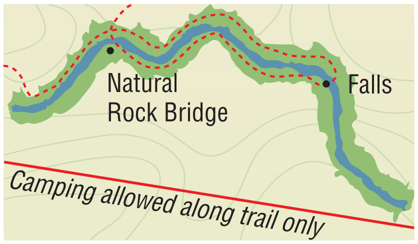


25. SWIMMING Kendall is swimming due south at 4.5 feet per second. The current of the river is moving with a velocity of 2 feet per second due east. Find Kendall's resultant speed and direction. (Lesson 8-7)

Extra Practice

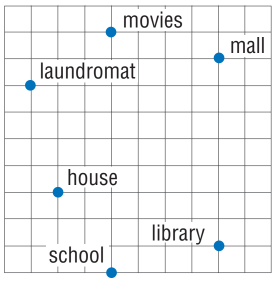
Graph each figure and its image under the given reflection. (Lesson 9-1)

1. $\triangle ABC$ with vertices $A(-1, -4)$, $B(-5, 3)$, and $C(0, 5)$ in the line $y = x$
2. quadrilateral $WXYZ$ with vertices $W(-3, -2)$, $X(-4, 1)$, $Y(1, 4)$, and $Z(2, -2)$ in the y-axis
3. **CAMPING** Jin and Tom plan to hike to the rock bridge one day, return to camp, and then hike to the falls on the next day. Where along the trail should they place their camp in order to minimize the distance they must hike? (Lesson 9-1)



Graph each figure and its image along the given vector. (Lesson 9-2)

4. $\triangle XYZ$ with vertices $X(-2, -1)$, $Y(1, 3)$, and $Z(4, -2)$; $\langle -4, 2\rangle$
5. trapezoid $MNPQ$ with vertices $M(-4, -3)$, $N(-2, 2)$, $P(1, 2)$, and $Q(3, -3)$; $\langle 3, -1\rangle$
6. **MAPS** Caleb's house and several places he visits are shown on the grid. (Lesson 9-2)



   a. If Caleb leaves his house and goes one block west and four blocks north, what is his new location?
   b. Write a translation vector that will take Caleb from the library to the movies.

Graph each figure and its image after the specified rotation about the origin. (Lesson 9-3)

7. $\triangle PQR$ with vertices $P(-1, -2)$, $Q(-5, -4)$, and $R(-3, -6)$; 90°
8. parallelogram $WXYZ$ with vertices $W(-3, 3)$, $X(-2, 7)$, $Y(4, 5)$ and $Z(3, 1)$; 180°
9. **CLOCKS** Anna looks at a clock at 11:05. When she looks at the clock for a second time during the same hour, the minute hand has rotated 270°. At what time does Anna look at the clock for the second time? (Lesson 9-3)

Graph each figure with the given vertices and its image after the indicated glide reflection. (Lesson 9-4)

10. $\triangle DEF$: $D(-5, 1)$, $E(-3, 5)$, $F(0, 3)$
    Translation: along $\langle -4, 3\rangle$; Reflection: in x-axis
11. $\triangle MNP$: $M(2, 5)$, $N(6, 2)$, $P(8, 6)$
    Translation: along $\langle 2, 4\rangle$; Reflection: in $y = x$

Graph each figure with the given vertices and its image after the indicated composition of transformations. (Lesson 9-4)

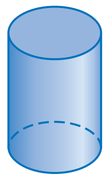
12. $\overline{XY}$: $X(7, 9)$ and $Y(2, 1)$
    Rotation: 90°; Translation: $\langle -5, -2\rangle$
13. $\overline{AB}$: $A(-4, -6)$ and $B(-2, 5)$
    Reflection: in x-axis; Rotation: 270°

ALPHABET Determine whether each letter below has *line* symmetry, *rotational* symmetry, *both*, or *neither*. Draw all lines of symmetry and state their number. Then determine the center of rotational symmetry and state the order and magnitude of rotational symmetry. (Lesson 9-5)

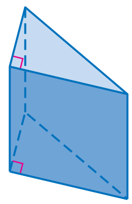
14. B
15. X

State whether the figure has *plane* symmetry, *axis* symmetry, *both*, or *neither*. (Lesson 9-5)

16.



17.



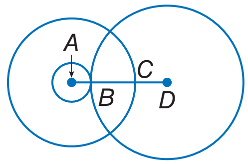
18. **NATURE** The diameter of a snowflake is 2 millimeters. If the diameter appears to be 3 centimeters when viewed under a microscope, what magnification setting (scale factor) was used? (Lesson 9-6)

Find the image of each polygon with the given vertices after a dilation centered at the origin with the given scale factor. (Lesson 9-6)

19. $A(-5, -4)$, $B(-2, -3)$, $C(-1, -6)$, $D(-4, -8)$; $k = \frac{1}{2}$
20. $X(2, 4)$, $Y(4, 0)$, $Z(5, 5)$; $k = 1.5$

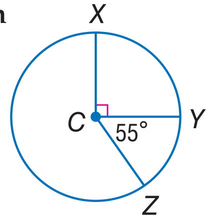
The diameter of the smaller circle centered at *A* is 3 inches, and the diameter of the larger circle centered at *A* is 9 inches. The diameter of ⊙*D* is 11 inches. Find each measure. (Lesson 10-1)

1. *BC*
2. *CD*



3. **DECORATIONS** To decorate for homecoming, Brittany estimates that she will need to purchase enough streamers to go around the school's circular fountain twice. If the diameter of the fountain is 88 inches, about how many feet of streamers should she buy? (Lesson 10-1)

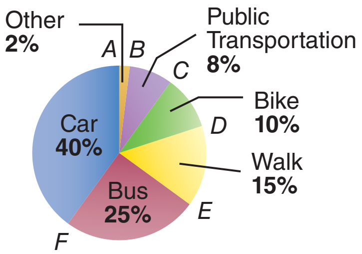
Use ⊙*C* to find the length of each arc. Round to the nearest hundredth. (Lesson 10-2)



4. $\widehat{XY}$ if the radius is 5 feet
5. $\widehat{YZ}$ if the diameter is 8 meters

6. **TRANSPORTATION** The graph shows the results of a survey in which students at a high school were asked how they get to school. (Lesson 10-2)

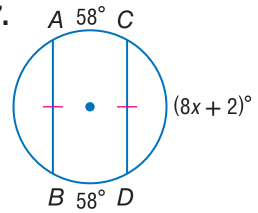
How Students Get to School



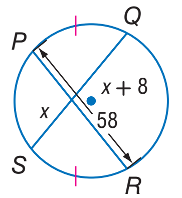
a. Find $m\widehat{CD}$.
b. Find $m\widehat{BC}$.

Find the value of *x*. (Lesson 10-3)

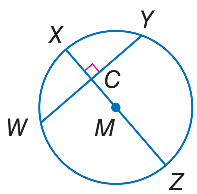
7.



8.



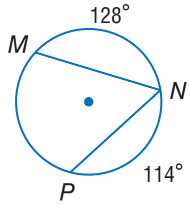
In ⊙*M*, *MZ* = 12 and *WY* = 20. Find each measure. Round to the nearest hundredth. (Lesson 10-3)



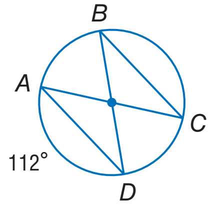
9. *CM*
10. *XC*

Find each measure. (Lesson 10-4)

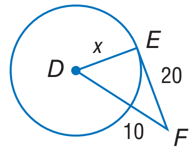
11. $m\angle N$



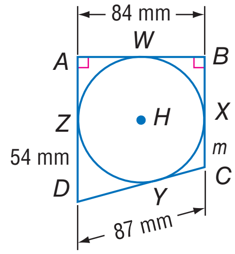
12. $m\angle B$



13. Find *x*. Assume that segments that appear to be tangent are tangent. (Lesson 10-5)

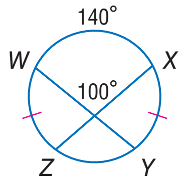


14. Quadrilateral *ABCD* is circumscribed about ⊙*H*. Find *m*. (Lesson 10-5)

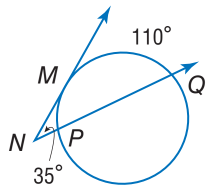


Find each measure. Assume that segments that appear to be tangent are tangent. (Lesson 10-6)

15. $m\widehat{XYZ}$

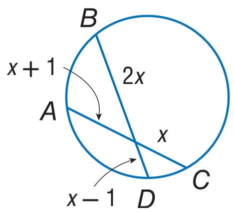


16. $\widehat{MP}$

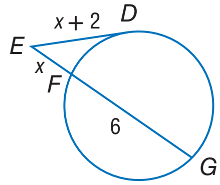


Find *x* to the nearest tenth. Assume that segments that appear to be tangent are tangent. (Lesson 10-7)

17.



18.

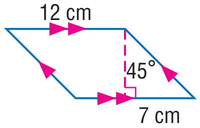


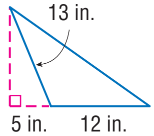
19. **CELL PHONES** A cell phone tower covers a circular area with a radius of 15 miles. (Lesson 10-8)
   a. If the tower is located at the origin, write an equation for this circular area of coverage.
   b. Will a person 11 miles west and 12 miles south of the tower have coverage? Explain.

20. Write an equation of a circle that contains points *A*(−1, 5), *B*(−5, 9), and *C*(−9, 5). Then graph the equation. (Lesson 10-8)

Extra Practice

Find the perimeter and area of each figure. Round to the nearest tenth if necessary. (Lesson 11-1)

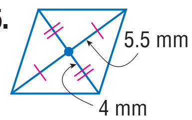
1.


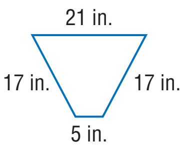
2.


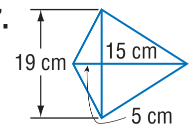
3. The height of a parallelogram is three times its base. If the area of the parallelogram is 108 square meters, find its base and height. (Lesson 11-1)

4. The height of a triangle is three feet less than its base. If the area of the triangle is 275 square feet, find its base and height. (Lesson 11-1)

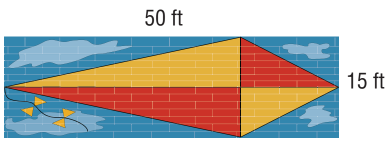
Find the area of each trapezoid, rhombus, or kite. (Lesson 11-2)

5.


6.


7.


8. MODELS Joni is designing a mural for the side of a building. The wall is 15 feet high and 50 feet long. If she covers the wall with a kite as shown, what is the area of the kite? (Lesson 11-2)



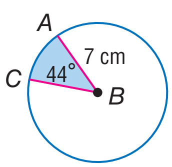
9. A trapezoid has a height of 12 inches, a base length of 9 inches, and an area of 150 square inches. What is the length of the other base? (Lesson 11-2)

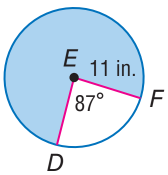
Find the indicated measure. Round to the nearest tenth. (Lesson 11-3)

10. The area of a circle is 201 square meters. Find the radius.

11. Find the diameter of a circle with an area of 79 square feet.

Find the area of each shaded sector. Round to the nearest tenth if necessary. (Lesson 11-3)

12.


13.


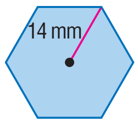
14. GRAPHS Len created a circle graph using the survey results shown in the table. (Lesson 11-3)

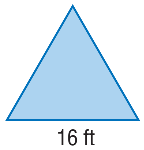
Preferred Type of Exercise	
Treadmill	62%
Stationary Bike	8%
Swimming	7%
Aerobics	12%
Other	11%

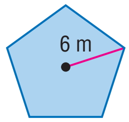
a. What is the angle measure of the sector representing swimming?

b. If the graph has a 3-inch diameter, what is the area of the sector representing treadmill?

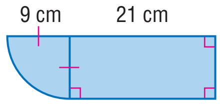
Find the area of each regular polygon. Round to the nearest tenth if necessary. (Lesson 11-4)

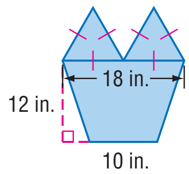
15.


16.


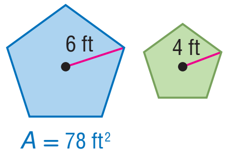
17.


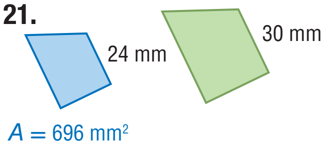
Find the area of each figure. Round to the nearest tenth if necessary. (Lesson 11-4)

18.


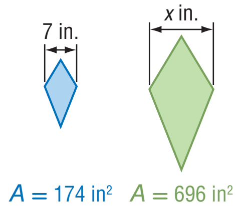
19.


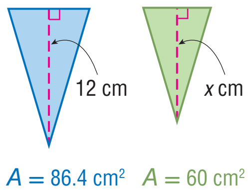
For each pair of similar figures, find the area of the green figure. (Lesson 11-5)

20.


21.


For each pair of similar figures, use the given areas to find the scale factor from the blue to the green figure. Then find *x*. (Lesson 11-5)

22.


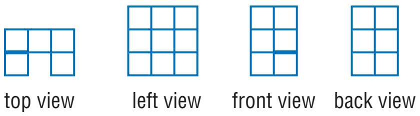
23.


24. MODELS Anna is making a model of her house. The area of the model's living space is 70 square inches. If the area of the actual living space is 1750 square feet, how many feet are represented by each inch of the model? (Lesson 11-5)

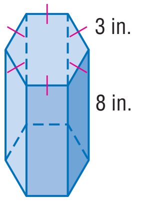
CHAPTER 12 Extending Surface Area and Volume

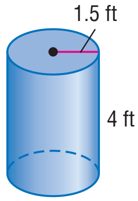
Use isometric dot paper to sketch each prism. (Lesson 12-1)

1. triangular prism 4 units high, with two sides of the base that are 3 units long and 4 units long
2. rectangular prism 1 unit high, 5 units wide, and 3 units long
3. Use isometric dot paper and the orthographic drawing to sketch the solid. (Lesson 12-1)



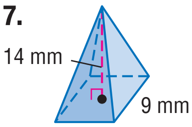
Find the lateral and surface area of each solid. Round to the nearest tenth. (Lesson 12-2)

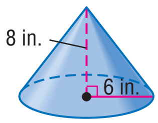
4. 

5. 

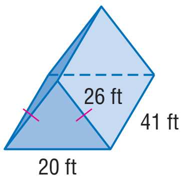
6. **MANUFACTURING** An office has recycling barrels that are cylindrical with cardboard sides and plastic lids and bases. Each barrel is 3 feet tall with a diameter of 30 inches. How many square feet of cardboard are used to make each barrel? (Lesson 12-2)

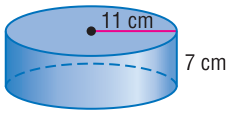
Find the lateral and surface area of each regular pyramid or cone. Round to the nearest tenth if necessary. (Lesson 12-3)

7. 

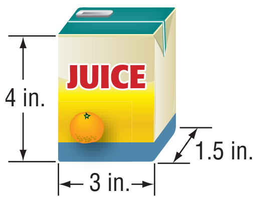
8. 

Find the volume of each solid. Round to the nearest tenth if necessary. (Lesson 12-4)

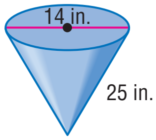
9. 

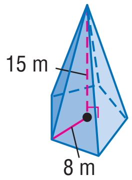
10. 

11. **ADVERTISING** A company advertises that their juice boxes contain 20% more juice than their competitor's. If the base dimensions of the boxes are the same, how much taller are the larger boxes? (Lesson 12-4)

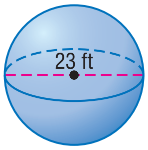


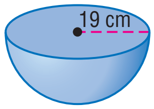
Find the volume of each solid. Round to the nearest tenth if necessary. (Lesson 12-5)

12. 

13. 

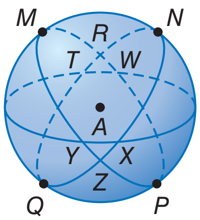
Find the surface area and volume of each sphere or hemisphere. Round to the nearest tenth. (Lesson 12-6)

14. 

15. 

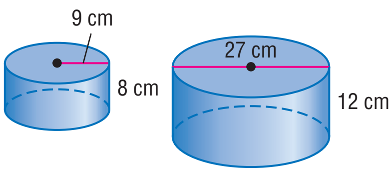
16. **SPORTS** The diameter of a tennis ball is 2.7 inches, and the diameter of a baseball is 2.9 inches. How many times as great is the volume of the baseball as the volume of the tennis ball? (Lesson 12-6)

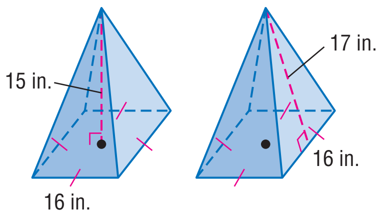
Name each of the following on sphere $\mathcal{A}$. (Lesson 12-7)



17. a triangle
18. two segments on the same great circle

Determine whether each pair of solids is *similar*, *congruent*, or *neither*. If the solids are similar, state the scale factor. (Lesson 12-8)

19. 

20. 

CHAPTER 13 Probability and Measurement

1. **FITNESS** Laura wants to go to a fitness class tomorrow. She can choose a 5:00 or a 7:30 class and spin or water aerobics. Represent the sample space for the situation by making an organized list, a table, and a tree diagram. (Lesson 13-1)

2. **SCHOOL UNIFORMS** Susan has a school uniform that consists of a polo shirt, an oxford shirt, a skirt, and a pair of pants. She also has a sweater than she can wear if she chooses. Draw a tree diagram to represent the sample space for Susan's uniform. (Lesson 13-1)

3. **CONSTRUCTION** Bert's family is building a house in a new neighborhood, and they must choose one option listed below for each feature. What is the number of possible outcomes for the situation? (Lesson 13-1)

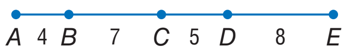
Feature	Options
floor plan	Elevation 1, Elevation 2
counters	formica, granite
cabinets	French antique glazed, oak, cherry
basement	unfinished, partially finished, finished
garage	none, one car, two car

4. **DIVING** At a swim meet, the order of the divers is randomly selected. If there are 12 divers, what is the probability that Danielle, Nora, and Li will dive first, second, and third, respectively? (Lesson 13-2)

5. **NUMBERS** Charlie's phone number is 555–3703. If he places each of the digits in a bowl and randomly selects one number at a time without replacement, what is the probability that he will choose his phone number? (Lesson 13-2)

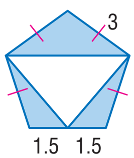
6. **RAFFLES** Participants in a raffle received tickets 1101 through 1125. If four winners are chosen, what is the probability that the winning tickets are 1103, 1111, 1118, and 1122? (Lesson 13-2)

7. Point X is chosen at random on $\overline{AE}$. Find the probability that X is on $\overline{CE}$. (Lesson 13-3)

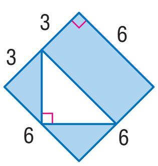


Find the probability that a point chosen at random lies in the shaded region. (Lesson 13-3)

8.



9.



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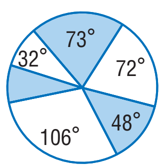


11. **FOOTBALL** Wes made 92% of his point after touchdown attempts last season. Design and conduct a simulation using a random number generator that can be used to estimate the probability that he will make his next point after touchdown attempt. (Lesson 13-4)

BASKETBALL For each field goal attempt in basketball, a player can earn 0, 2, or 3 points. The probability that a certain player will score 0 points on an attempt is 45%, 2 points is 40%, and 3 points is 15%. (Lesson 13-4)

12. Calculate the expected value for one attempt.

13. Design a simulation using a geometric probability model and estimate the player's average value per field goal attempt.

14. Compare the values for Exercises 12 and 13.

15. A die is rolled twice. What is the probability that the first number rolled is a 3 and the second number rolled is a 5? (Lesson 13-5)

16. Three cards are randomly chosen from a deck of 52 cards without replacement. What is the probability that they will all be red? (Lesson 13-5)

17. A spinner numbered 1 through 6 is spun. Find the probability that the number spun is a 3 given that it was less than 4. (Lesson 13-5)

BOOKS The table shows the number and type of books that Sarah owns. Find each probability. (Lesson 13-6)

Medium	Classic	Mystery	Biography
print	29	8	32
audio	3	6	10
electronic	8	3	43

18. A randomly chosen title is a print or audio book.

19. A randomly chosen title is not a biography.

20. **DOGS** The table shows the ages and genders of the dogs at an animal shelter. What is the probability that a randomly chosen dog is a female or over 5 years old? (Lesson 13-6)

Age	Male	Female
under 1 year	6	5
1–5 years	8	7
6–10 years	4	6
over 10 years	3	5

Selected Answers and Solutions

Go to Hotmath.com for step-by-step solutions of most odd-numbered exercises free of charge.

CHAPTER 0
Preparing for Geometry

Lesson 0-1

1. cm **3.** kg **5.** mL **7.** 10 **9.** 10,000 **11.** 0.18 **13.** 2.5 **15.** 24 **17.** 0.370 **19.** 4 **21.** 5 **23.** 16 **25.** 208 **27.** 9050

Lesson 0-2

1. 20 **3.** 12.1 **5.** 16 **7.** 12 **9.** 5.4 **11.** 22.47 **13.** 1.125 **15.** 5.4 **17.** 15 **19.** 367.9 g **21.** 735.8 g

Lesson 0-3

1. $\frac{1}{3}$ or 33% **3.** $\frac{2}{3}$ or 67% **5.** $\frac{1}{3}$ or 33% **7.** $\frac{13}{28}$ or about 46% **9.** $\frac{11}{14}$ or about 79% **11.** $\frac{9}{70}$ or about 13% **13.** $\frac{9}{28}$ or about 32% **15.** $\frac{1}{28}$ or about 3.6% **17.** $\frac{13}{28}$ or about 46% **19.** $\frac{13}{14}$ or about 93% **21.** $\frac{1}{10}$ or 10%; $\frac{1}{8}$ or 12.5% **23.** Sample answer: Assign each friend a different colored marble: red, blue, or green. Place all the marbles in a bag and without looking, select a marble from the bag. Whoever's marble is chosen gets to go first.

Lesson 0-4

1. 3 **3.** −2 **5.** −1 **7.** −26 **9.** 26 **11.** 15

Lesson 0-5

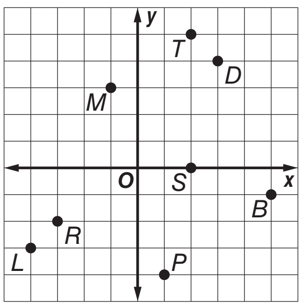
1. −8 **3.** 15 **5.** −72 **7.** $-\frac{15}{2}$ **9.** $\frac{7}{2}$ **11.** −15 **13.** −7 **15.** −7 **17.** −1 **19.** 60 **21.** −4 **23.** 4 **25.** 15 **27.** 21 **29.** −2 **31.** $-\frac{29}{2}$ **33.** −6 **35.** 1

Lesson 0-6

1. $\{x|x < 13\}$ **3.** $\{y|y < 5\}$ **5.** $\{t|t > -42\}$ **7.** $\{d|d \leq 4\}$ **9.** $\{k|k \geq -3\}$ **11.** $\{z|z < -2\}$ **13.** $\{m|m < 29\}$ **15.** $\{b|b \geq -16\}$ **17.** $\{z|z > -2\}$ **19.** $\{b|b \leq 10\}$ **21.** $\{q|q \geq 2\}$ **23.** $\left\{w|w \geq -\frac{7}{3}\right\}$

Lesson 0-7

1. (−2, 3) **3.** (2, 2) **5.** (−3, 1) **7.** (4, 1) **9.** (−1, −1) **11.** (3, 0) **13.** (2, −4) **15.** (−4, 2) **17.** none **19.** IV **21.** I **23.** III



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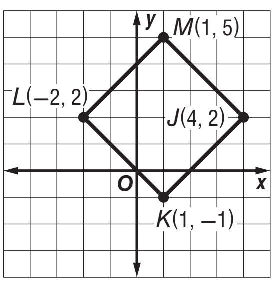

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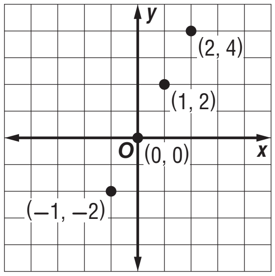

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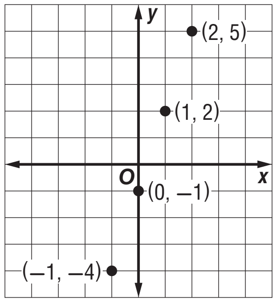


Lesson 0-8

1. (2, 0) **3.** no solution **5.** (2, −5) **7.** $\left(-\frac{4}{3}, 3\right)$ **9.** (4, 1) **11.** elimination, no solution **13.** elimination or substitution, (3, 0) **15.** elimination or substitution, (−6, 4)

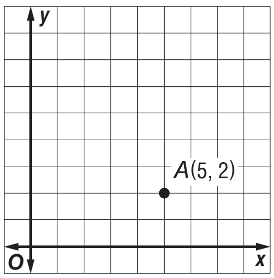
Lesson 0-9

1. $4\sqrt{2}$ **3.** $10\sqrt{5}$ **5.** 6 **7.** $7x|y^3|\sqrt{2x}$ **9.** $\frac{9}{7}$ **11.** $\frac{3\sqrt{14}}{4}$ **13.** $\frac{p\sqrt{30p}}{9}$ **15.** $\frac{20 + 8\sqrt{3}}{13}$ **17.** $\frac{\sqrt{3}}{4}$ **19.** $\frac{6\sqrt{5} + 3\sqrt{10}}{2}$

CHAPTER 1
Tools of Geometry

Chapter 1 Get Ready

1.



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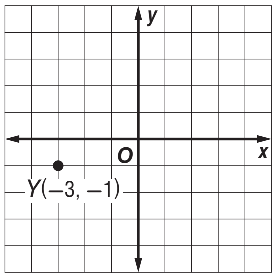
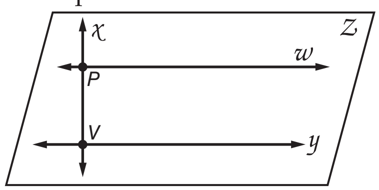


5. e5 **7.** $6\frac{29}{36}$ **9.** $5\frac{2}{15}$ **11.** 81 **13.** 153 **15.** 6

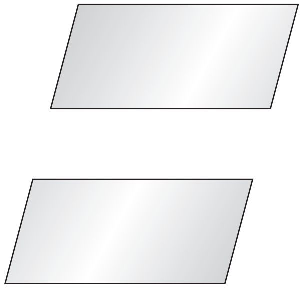
Lesson 1-1

1. Sample answer: m **3.** $\mathcal{B}$ **5.** plane **7.** Sample answer:

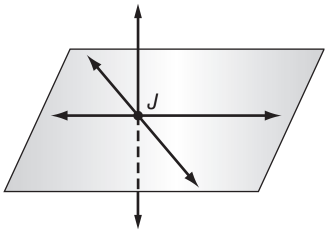


9. Sample answer: A, H, and B **11.** Yes; points B, D, and F lie in plane BDF. **13.** Sample answer: n and q **15.** $\mathcal{R}$ **17** Sample answer: Points A, B, and C are contained in plane $\mathcal{R}$. Since point P is not contained in plane $\mathcal{R}$, it is not coplanar with points A, B, and C. **19.** points A and P **21.** Yes; line n intersects line q when the lines are extended. **23.** intersecting lines

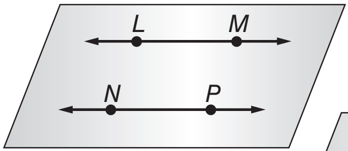
25. two planes intersecting in a line **27.** point
29. line **31.** intersecting planes
33. Sample answer:



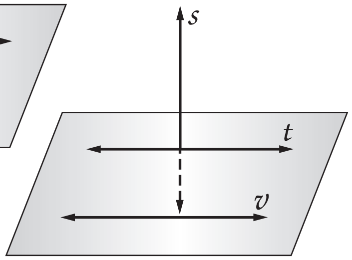
35. Sample answer:



37. Sample answer:



39. Sample answer:



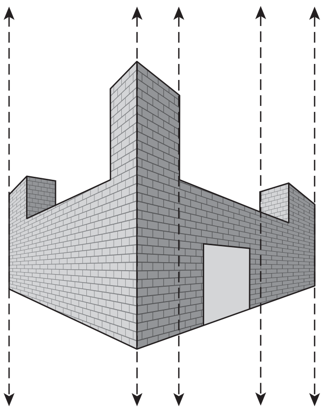
41. edges **43.** Sample answer: M and N

45 The planes appear to be parallel. Since they do not have any lines in common, they do not intersect.

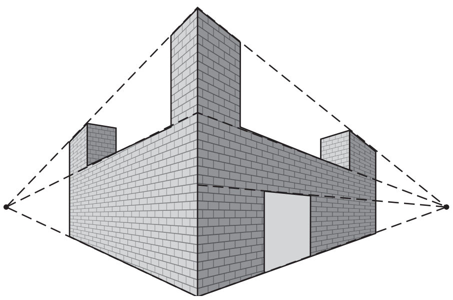
47. No; V does not lie in the same plane.

49 **a.** The intersection between the signs and the pole is represented by a point. **b.** The two planes intersect in a line.

51a. Sample answer:



51b.

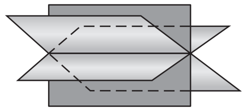


51c. Sample answer: They get closer together.
53. Sample answer: The airplanes are in different horizontal planes.

55 **a.** There are four ways to choose three points: *FGH*, *FGK*, *GHK*, and *FHK*. Only one way, *FGH*, has three points collinear. So, the probability is $\frac{1}{4}$.

b. There is exactly one plane through any three noncollinear points and infinitely many planes through three collinear points. Therefore, the probability that the three points chosen are coplanar is 1.

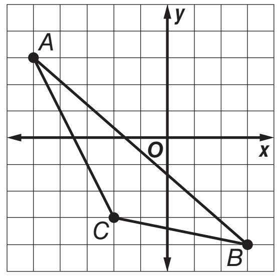
57. Sample answer:



59. 4 **61.** Sample answer: A table is a finite plane. It is not possible to have a real-life object that is an infinite plane because all real-life objects have boundaries.

63. H **65.** B **67.** $6\sqrt{7}$ **69.** $\frac{\sqrt{2}}{2}$ **71.** $\frac{|a^3|\sqrt{6}}{9}$
73. $4\sqrt{15} - 8\sqrt{3}$

75.



77. Sample answer: 424.5 g
79. Sample answer: 1.1 kg
81. > **83.** < **85.** >

Lesson 1-2

1. 5.7 cm or 57 mm **3.** $1\frac{7}{8}$ in. **5.** 3.8 in.

7

$BD = 4x$	Given
$12 = 4x$	Substitution
$\frac{12}{4} = \frac{4x}{4}$	Divide each side by 4.
$3 = x$	Simplify.

$BC = 2x$	Given
$= 2(3)$	$x = 3$
$= 6$	Simplify.

9. $\overline{AG} \cong \overline{FG}$, $\overline{BG} \cong \overline{EG}$, $\overline{CG} \cong \overline{DG}$ **11.** 38 mm
13. $\frac{15}{16}$ in. **15.** 1.1 cm **17.** 1.5 in. **19.** 4.2 cm **21.** $c = 18$; $YZ = 72$ **23.** $a = 4$; $YZ = 20$ **25.** $n = 4\frac{1}{3}$; $YZ = 1\frac{2}{3}$

27 Yes; $KJ = 4$ in. and $HL = 4$ in. Since the segments have the same measure, they are congruent.

29. no **31.** yes

33 Sample answer: All the segments that have one slash are congruent: $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DE} \cong \overline{DG} \cong \overline{BG} \cong \overline{CG}$, $\overline{AC} \cong \overline{EC}$. All segments with two slashes are congruent: $\overline{AH} \cong \overline{HG} \cong \overline{GF} \cong \overline{FE}$, $\overline{AG} \cong \overline{HF} \cong \overline{GE}$. All segments with three slashes are congruent: $\overline{BH} \cong \overline{DF}$.

35. Sample answer: $\overline{BD} \cong \overline{CE}$; $\overline{BD} \cong \overline{PQ}$; $\overline{YZ} \cong \overline{JK}$; $\overline{PQ} \cong \overline{RS}$; $\overline{GK} \cong \overline{KL}$ **37.** If point B is between points A and C, and you know AB and BC, add AB and BC to find AC. If you know AB and AC, subtract AB from

AC to find BC. **39.** $JK = 12$, $KL = 16$ **41.** Sample answer: Having a standard of measure is important so that there is a reference point against which other measures can be compared and evaluated. **43.** D **45.** D **47.** Sample answer: plane CDF **49.** points C, B, and F **51a.** about 2.1 s **51b.** about 9.7 in. **53.** $\{p \mid p > 9\}$ **55.** $\{x \mid x \leq -13\}$ **57.** 12 **59.** 5.5 **61.** $\sqrt{185}$

Lesson 1-3

1. 8

3

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance Formula
$= \sqrt{(2-4)^2 + (-3-9)^2}$	$(x_1, y_1) = (4, 9)$ and $(x_2, y_2) = (2, -3)$
$= \sqrt{(-2)^2 + (-12)^2}$	Subtract.
$= \sqrt{4 + 144}$ or $\sqrt{148}$	Simplify.

The distance between the time capsules is $\sqrt{148}$ or about 12.2 units.

5. $\sqrt{58}$ or about 7.6 units **7.** -3 **9.** $(4, -5.5)$

11 Let G be (x_1, y_1) and J be (x_2, y_2) in the Midpoint Formula.

$F\left(\frac{x_1 + 6}{2}, \frac{y_1 + (-2)}{2}\right) = F(1, 3.5)$ $(x_2, y_2) = (6, -2)$

Write two equations to find the coordinates of G.

$\frac{x_1 + 6}{2} = 1$	Midpoint Formula	$\frac{y_1 + (-2)}{2} = 3.5$
$x_1 + 6 = 2$	Multiply each side by 2.	$y_1 + (-2) = 7$
$x_1 = -4$	Simplify.	$y_1 = 9$

The coordinates of G are $(-4, 9)$.

13. 5 **15.** 9 **17.** 12 **19.** $\sqrt{89}$ or about 9.4 units **21.** $\sqrt{58}$ or about 7.6 units **23.** $\sqrt{208}$ or about 14.4 units **25.** $\sqrt{65}$ or about 8.1 units **27.** $\sqrt{53}$ or about 7.3 units **29.** $\sqrt{18}$ or about 4.2 units **31.** 4.5 mi **33.** 6 **35.** -4.5 **37.** 3

39

$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	Midpoint Formula
$= M\left(\frac{22 + 15}{2}, \frac{4 + 7}{2}\right)$	$(x_1, y_1) = (22, 4)$ and $(x_2, y_2) = (15, 7)$
$= M\left(\frac{37}{2}, \frac{11}{2}\right)$	Simplify.
$= M(18.5, 5.5)$	Simplify.

The coordinates of the midpoint are (18.5, 5.5).

41. $(-6.5, -3)$ **43.** $(-4.2, -10.4)$ **45.** $\left(-\frac{1}{2}, \frac{1}{2}\right)$ **47.** $A(1, 6)$ **49.** $C(16, -4)$ **51.** $C(-12, 13.25)$ **53.** 58 **55.** 4.5

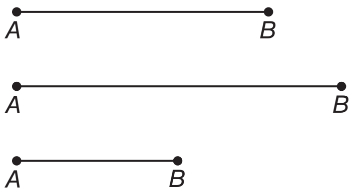
57 **a.** If the center of the court is the origin, the player in the first is half of the length of the court or 47 feet to the right and half of the width of the court or 25 feet down. Since you go to the right, the x-coordinate is positive, and since you go down, the y-coordinate is negative. The ordered pair is $(47, -25)$. **b.** The distance that the ball travels is the distance between (0, 0) and (47, −25).

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance Formula
$d = \sqrt{(47 - 0)^2 + (-25 - 0)^2}$	$(x_1, y_1) = (0, 0)$, $(x_2, y_2) = (47, -25)$
$d = \sqrt{2834}$	Simplify.
$d \approx 53.2$	Use a calculator.

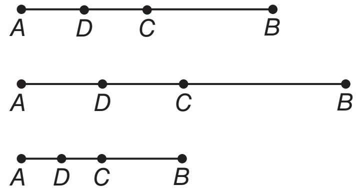
The distance between the two players is about 53.2 feet.

59. =AVERAGE(B2, D2) **61.** $(-5, 0)$, $(7, 0)$ **63.** $\left(-1\frac{1}{2}, -1\right)$ **65.** ± 5

67 **a.** Sample answer:



b. Sample answer:



c. Sample answer:

line	AB (cm)	AC (cm)	AD (cm)
1	4	2	1
2	6	3	1.5
3	3	1.5	0.75

d.

$AC = \frac{1}{2}AB$	Definition of midpoint
$AC = \frac{1}{2}x$	$AB = x$
$AD = \frac{1}{2}AC$	Definition of midpoint
$AD = \frac{1}{2}\left(\frac{1}{2}x\right)$	Substitution
$AD = \frac{1}{4}x$	Simplify.

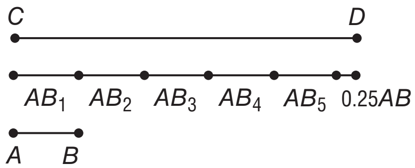
e. Look for a pattern.

Number of Midpoints	Length of Smallest Segment
1	$\frac{1}{2}x$
2	$\frac{1}{2} \cdot \frac{1}{2}x = \frac{1}{2(2)}x$
3	$\frac{1}{2} \cdot \frac{1}{2(2)}x = \frac{1}{2^3}x$
4	$\frac{1}{2} \cdot \frac{1}{2(3)}x = \frac{1}{2^4}x$
n	$\frac{1}{2^n}x$

Sample answer: If n midpoints are found, then the smallest segment will have a measure of $\frac{1}{2^n}x$.

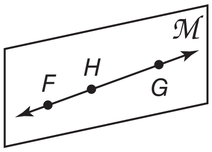
69. Sample answer: Sometimes; when the point (x_1, y_1) has coordinates (0, 0).

71. Sample answer:



Draw $\overline{AB}$. Next, draw a construction line and place point C on it. From point C, strike 6 arcs in succession of length AB. On the sixth $\overline{AB}$ length, perform a segment bisector two times to create a $\frac{1}{4}AB$ length. Label the endpoint D. **73.** C **75.** C **77.** $2\frac{1}{8}$ in.

79.



81. $4x + 38 \leq 75$; 9.25 lb or less **83.** 15.5 **85.** $2\frac{1}{3}$ **87.** 4

Lesson 1-4

1. U **3.** $\angle XYU$, $\angle UYX$ **5.** acute; 40 **7.** right; 90 **9.** 156 **11a.** 45; When joined together, the angles form a right angle, which measures 90. If the two angles that form this right angle are congruent, then the measure of each angle is $90 \div 2$ or 45. The angle of the cut is an acute angle. **11b.** The joint is the angle bisector of the frame angle. **13.** P **15.** M **17.** $\overrightarrow{NV}$, $\overline{NM}$ **19.** $\overrightarrow{RP}$, $\overrightarrow{RQ}$ **21.** $\angle TPQ$ **23.** $\angle TPN$, $\angle NPT$, $\angle TPM$, $\angle MPT$ **25.** $\angle 4$ **27.** S, Q

29 Sample answer: $\angle MPR$ and $\angle PRQ$ share points P and R.

31. 90, right **33.** 45, acute **35.** 135, obtuse

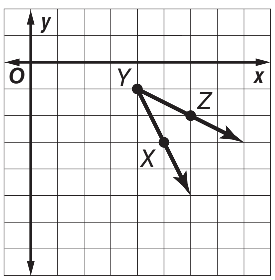
37

$m\angle ABE = m\angle EBF$	Definition of $\cong$ ∠s
$2n + 7 = 4n - 13$	Substitution
$7 = 2n - 13$	Subtract $2n$ from each side.
$20 = 2n$	Add 13 to each side.
$10 = n$	Divide each side by 2.

$m\angle ABE = 2n + 7$	Given
$= 2(10) + 7$	$n = 10$
$= 20 + 7$ or 27	Simplify.

39. 16 **41.** 47 **43a.** about 50 **43b.** about 140 **43c.** about 20 **43d.** 0

45. acute



47 **a.** $m\angle 1 \approx 110$; since $110 > 90$, the angle is obtuse. **b.** $m\angle 2 \approx 85$; since $85 < 90$, the angle is acute. **c.** about 15; If the original path of the light is extended, the measure of the angle the original path makes with the refracted path represents the number of degrees the path of the light changed. The sum of the measure of this angle and the measure of $\angle 3$ is 180. The measure of $\angle 3$ is $360 - (110 + 85)$ or 165, so the measure of the angle the original path makes with the refracted path is $180 - 165$ or 15.
49. The two angles formed are acute angles. Sample answer: With my compass at point A, I drew an arc in the interior of the angle. With the same compass setting, I drew an arc from point C that intersected the arc from point A. From the vertex, I drew $\overrightarrow{BD}$. I used the same compass setting to draw the intersecting arcs, so $\overrightarrow{BD}$ divides $\angle ABC$ so that the measurement of $\angle ABD$ and $\angle DBC$ are equal. Therefore, $\overrightarrow{BD}$ bisects $\angle ABC$. **51.** Sometimes; sample answer: For example, if you add an angle measure of 4 and an angle measure of 6, you will have an angle measure of 10, which is still acute. But if you add angles with measure of 50 and 60, you will have an obtuse angle with a measure of 110.
53. Sample answer: To measure an acute angle, you can fold the corner of the paper so that the edges meet. This would bisect the angle, allowing you to determine whether the angle was between 0° and 45° or between 45° and 90°. If the paper is folded two more times in the same manner and cut off this corner of the paper, the fold lines would form the increments of a homemade protractor that starts at 0° on one side and progresses in $90 \div 8$ or 11.25° increments, ending at the adjacent side, which would indicate 90°. You can estimate halfway between each fold line, which would give you an accuracy of $11.25° \div 2$ or about 6°. The actual measure of the angle shown is 52°. An estimate between 46° and 58° would be acceptable. **55.** Sample answer: Leticia's survey does not represent the entire student body because she did not take a random sample; she only took a sample of students from one major.
57. E **59.** 8.25 **61.** 15.81 **63.** 10.07 **65.** $x = 11$; $ST = 22$ **67.** 4.5 **69.** 56 **71.** 14.75 **73.** 24.8 **75.** $17\frac{1}{3}$

Lesson 1-5

1. $\angle ZVY$, $\angle WVU$ **3a.** vertical **3b.** 15

5 If $x \perp y$, then $m\angle 2 = 90$ and $m\angle 3 = 90$.

$m\angle 2 = 3a - 27$	Given
$90 = 3a - 27$	Substitution
$117 = 3a$	Add 27 to each side.
$39 = a$	Divide each side by 3.

$m\angle 3 = 2b + 14$	Given
$90 = 2b + 14$	Substitution
$76 = 2b$	Subtract 14 from each side.
$38 = b$	Divide each side by 3.

7. Yes: they share a common side and vertex, so they are adjacent. Since $m\angle EDB + m\angle BDA + m\angle ADC = 90$, $\angle EDB$ and $\angle BDA$ cannot be complementary or supplementary. **9.** Sample answer: $\angle BFC$, $\angle DFE$ **11.** $\angle FDG$, $\angle GDE$ **13.** Sample answer: $\angle CBF$, $\angle ABF$ **15.** $\angle GDE$ **17.** $\angle CAE$ **19.** 65

21

$2x + 25 = 3x - 10$	Vertical ∠s are $\cong$ and have equal measures.
$25 = x - 10$	Subtract $2x$ from each side.
$35 = x$	Add 10 to each side.

$3x - 10 + y = 180$	Def. of supplementary ∠s
$3(35) - 10 + y = 180$	Substitution
$105 - 10 + y = 180$	Multiply.
$95 + y = 180$	Simplify.
$y = 85$	Subtract 95 from each side.

23. $x = 48$; $y = 21$ **25.** $m\angle F = 63$; $m\angle E = 117$ **27.** 40

29 If $\angle KNM$ is a right angle, then $m\angle KNM = 90$.

$m\angle KNL + m\angle LNM = m\angle KNM$	Sum of parts = whole
$6x - 4 + 4x + 24 = 90$	Substitution
$10x + 20 = 90$	Combine like terms.
$10x = 70$	Subtract 20 from each side.
$x = 7$	Divide each side by 10.

Selected Answers and Solutions

31. 92 **33.** 53; 37 **35.** $a = 8$; $b = 54$ **37.** Yes; the angles form a linear pair. **39.** No; the measures of each angle are unknown. **41.** No; the angles are not adjacent. **43.** Sample answer: $\angle 1$ and $\angle 3$

45 $\angle 1$ and $\angle 3$ are vertical angles, so they are congruent; $m\angle 3 = m\angle 1 = 110$. $\angle 1$ and $\angle 4$ are a linear pair, so they are supplementary.

$m\angle 4 + m\angle 1 = 180$	Def. of supplementary ∡
$m\angle 4 + 110 = 180$	Substitution
$m\angle 4 = 70$	Subtract 110 from each side.

47. Sample answer: Yes; if the wing is not rotated at all, then all of the angles are right angles, which are neither acute nor obtuse. **49.** Yes; angles that are right or obtuse do not have complements because their measures are greater than or equal to 90. **51a.** Line a is perpendicular to plane $\mathcal{P}$. **51b.** Line m is in plane $\mathcal{P}$. **51c.** Any plane containing line a is perpendicular to plane $\mathcal{P}$. **53.** C **55.** J **57.** 125, obtuse **59.** 90, right **61.** $\left(-3\frac{1}{2}, 1\right)$ **63.** 81.5 cm **65.** $\overline{FG} \cong \overline{HJ} \cong \overline{JK} \cong \overline{FL}$, $\overline{GH} \cong \overline{LK}$; $\angle F \cong \angle J$, $\angle G \cong \angle H \cong \angle K \cong \angle L$ **67.** $\overline{WX} \cong \overline{XY} \cong \overline{YZ} \cong \overline{ZW}$; $\angle W \cong \angle Y$, $\angle X \cong \angle Z$

Lesson 1-6

1. pentagon; concave; irregular **3.** octagon; regular **5.** hexagon; irregular **7.** ≈ 40.2 cm; ≈ 128.7 cm^2 **9.** C **11.** triangle; convex; regular

13 The polygon has 8 sides, so it is an octagon. All of the lines containing the sides of the polygon will pass through the interior of the octagon, so it is concave. Since the polygon is not convex, it is irregular.

15. hendecagon; concave; irregular **17.** 7.8 m; ≈ 3.1 m^2 **19.** 26 in.; 42.3 in^2

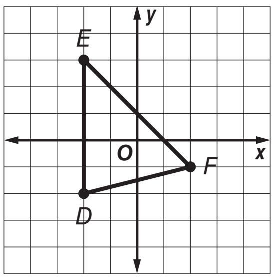
21

$c^2 = a^2 + b^2$	Pythagorean Theorem
$c^2 = 6.5^2 + 4.5^2$	$a = 6.5$, $b = 4.5$
$c^2 = 62.5$ or ≈ 7.9	Simplify.

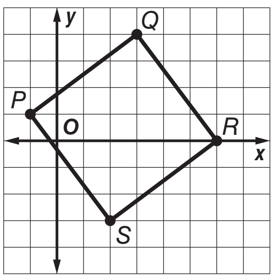
$P = a + b + c$	$A = \frac{1}{2}bh$
$\approx 6.5 + 4.5 + 7.9$	$= \frac{1}{2}(4.5)(6.5)$
≈ 18.9 cm	≈ 14.6 cm^2

23. ≈ 2.55 in.

25. triangle; $P = 5 + \sqrt{32} + \sqrt{17} \approx 14.78$ units; $A = 10$ units^2



27. quadrilateral or square; $P = 20$ units; $A = 25$ units^2



29a. 14 ft **29b.** 12 ft^2 **29c.** The perimeter doubles; the area quadruples. The perimeter of a rectangle with dimensions 6 ft and 8 ft is 28 ft, which is twice the perimeter of the original figure since 2 • 14 ft = 28 ft. The area of a rectangle with dimensions 6 ft and 8 ft is 48 ft^2, which is four times the area of the original figure, since 4 • 12 ft^2 = 48 ft^2. **29d.** The perimeter is halved; the area is divided by 4. The perimeter of a rectangle with dimensions 1.5 ft and 2 ft is 7 ft, which is half the perimeter of the original figure, since $\frac{1}{2}$ • 14 ft = 7 ft. The area of a rectangle with dimensions 1.5 ft and 2 ft is 3 ft^2, which is $\frac{1}{4}$ the area of the original figure, since $\frac{1}{4}$ • 12 ft^2 = 3 ft^2. **31.** 60 yd, 6 yd

33

$C = \pi d$	Circumference	$C = \pi d$
$= \pi(8)$		$= \pi(10)$
≈ 25.1	Simplify.	≈ 31.4

minimum circumference: 25.1 in.; maximum circumference: 31.4 in.

$A = \pi r^2$	Area of a circle
$= \pi(4)^2$	$r = 4$
≈ 50.3	Simplify.

$A = \pi r^2$	Circumference
$= \pi(5)^2$	$r = 5$
≈ 78.5	Simplify.

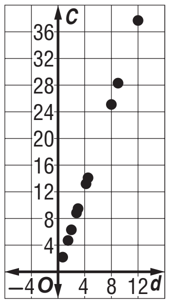
minimum area: 50.3 in^2; maximum area: 78.5 in^2

35. 21.2 m **37.** $2\pi\sqrt{32}$ or about 35.5 units **39.** $12\sqrt{6}$ or about 29.4 in. **41.** 108 in.; 729 in^2

43a–b. Sample answer:

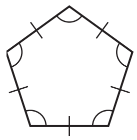
Object	d (cm)	C (cm)	$\frac{C}{d}$
1	3	9.4	3.13
2	9	28.3	3.14
3	4.2	13.2	3.14
4	12	37.7	3.14
5	4.5	14.1	3.13
6	2	6.3	3.15
7	8	25.1	3.14
8	0.7	2.2	3.14
9	1.5	4.7	3.13
10	2.8	8.8	3.14

43c. Sample answer:



43d. Sample answer: $C = 3.14d$; the equation represents a formula for approximating the circumference of a circle. The slope represents an approximation for pi. **45.** 290.93 units^2

47. Sample answer: The pentagon is convex, since no points of the lines drawn on the edges are in the interior. The pentagon is regular since all of the angles and sides were constructed with the same measurement, making them congruent to each other.



49. Sample answer: If a convex polygon is equiangular but not also equilateral, then it is not a regular polygon. Likewise, if a polygon is equiangular and equilateral, but not convex, then it is not a regular polygon. **51.** F **53.** C **55.** No; we do not know anything about these measures. **57.** Yes; they form a linear pair. **59.** elimination; $x = -3, y = -1$ **61.** substitution; $x = -4, y = -2.5$ **63.** 24 **65.** 169.6

Lesson 1-7

1. not a polyhedron; cylinder

3 $T = PH + 2B$ Surface area of a prism
$= (14)(3) + 2(12)$ $P = 14$ cm, $h = 3$ cm, $B = 12$ cm^2
$= 66$ cm^2 Simplify.

$V = BH$ Volume of a prism
$= (12)(3)$ $B = 12$ cm^2, $h = 3$ cm
$= 36$ cm^3 Simplify.

5a. ≈ 27.2 in^3 **5b.** 13.6π or about 42.7 in^2 **7.** pyramid; a polyhedron **9.** rectangular prism; a polyhedron **11.** cylinder; not a polyhedron **13.** not a polyhedron; cone **15.** not a polyhedron; sphere **17.** a polyhedron; pentagonal pyramid; base: *JHGFD*; faces: $\overline{JHGFD}$, △*JEH*, △*HEG*, △*GEF*, △*FED*, △*EDJ*; edges: $\overline{HG}$, $\overline{GF}$, $\overline{FD}$, $\overline{DJ}$, $\overline{JH}$, $\overline{EJ}$, $\overline{EH}$, $\overline{EG}$, $\overline{EF}$, $\overline{ED}$; vertices: *J*, *H*, *G*, *F*, *D*, *E* **19.** 121.5 m^2; 91.1 m^3

21 $T = PH + 2B$ $V = BH$
$= (24)(5) + 2(24)$ $= (24)(5)$
$= 168$ cm^2 $= 120$ cm^3

23. 150π or about 471.2 mm^2; 250π or about 785.4 mm^3

25 **a.** $V = \pi r^2 h$ Volume of a cylinder
$= \pi\left(7\frac{3}{4}\right)^2\left(11\frac{3}{4}\right)$ $r = 7\frac{3}{4}$ in., $h = 11\frac{3}{4}$ in.
≈ 2217.1 in^3 Simplify.

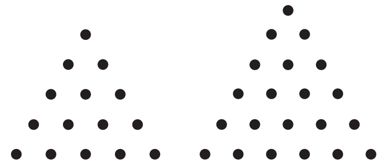
b. $T = 2\pi rh + 2\pi r^2$ Surface area of a cylinder
$= 2\pi\left(7\frac{3}{4}\right)\left(11\frac{3}{4}\right) + 2\pi\left(7\frac{3}{4}\right)^2$ $r = 7\frac{3}{4}$ in., $h = 11\frac{3}{4}$ in.
≈ 949.5 in^2 Simplify.

27. 3 in. **29.** 1212 in^2; 1776 in^3 **31a.** 96 in^2 **31b.** 113.1 in^2 **31c.** prism: 2 cans; cylinder: 3 cans **31d.** 2.18 in.; if the height is 10 in., then the surface area of the rectangular cake is 152 in^2. To find the radius of a cylindrical cake with the same height, solve the equation $152 = \pi r^2 + 20\pi r$. The solutions are $r = -22.18$ or $r = 2.18$. Using a radius of 2.18 in. gives surface area of about 152 in^2.

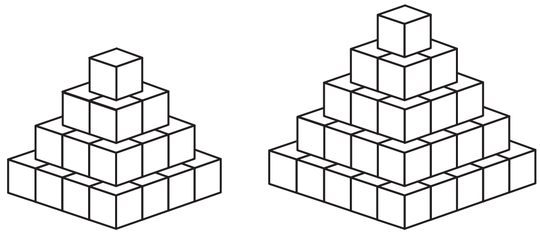
33 1 ft^3 = (12 in.)^3 = 1728 in^3
$4320 \text{ in}^3 \cdot \frac{1 \text{ ft}^3}{1728 \text{ in}^3} = 2.5 \text{ ft}^3$

35. The volume of the original prism is 4752 cm^3. The volume of the new prism is 38,016 cm^3. The volume increased by a factor of 8 when each dimension was doubled. **37.** Neither; sample answer: the surface area is twice the sum of the areas of the top, front, and left side of the prism or $2(5 \cdot 3 + 5 \cdot 4 + 3 \cdot 4)$, which is 94 in^2. **39a.** cone **39b.** cylinder **41.** 27 mm^3 **43.** 55.2 **45.** F **47.** quadrilateral; convex; regular **49.** dodecagon; concave; irregular **51.** 10 **53.** The intersection of a plane and a line not in the plane is a point. **55.** Two lines intersect in one point.

57.



59.



Chapter 1 Study Guide and Review

1. plane **3.** perpendicular **5.** point *P* **7.** point *W* **9.** line **11.** $x = 6$, $XP = 27$ **13.** yes **15.** 1.5 mi **17.** 10 **19.** (16, −6.5) **21.** (−27, 16) **23.** *G* **25.** $\overrightarrow{CA}$ and $\overrightarrow{CH}$ **27.** Sample answer: ∠*A* and ∠*B* are right, ∠*E* and ∠*C* are obtuse, and ∠*D* is acute. **29.** Sample answer: ∠*QWP* and ∠*XWV* **31.** 66 **33.** dodecagon, concave, irregular **35.** Option 1 = 12,000 ft^2, Option 2 = 12,100 ft^2, Option 3 ≈ 15,393.8 ft^2. Option 3 provides the greatest area. **37.** hexagonal prism. Bases: *ABCDEF* and *GHJKLM*; Faces: ▭*ABHG*, ▭*BCJH*, ▭*CDKJ*, ▭*DELK*, ▭*EFML*, ▭*FAGM*; Edges: $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EF}$, $\overline{FA}$, $\overline{GH}$, $\overline{HJ}$, $\overline{JK}$, $\overline{KL}$, $\overline{LM}$, $\overline{MG}$, $\overline{AG}$, $\overline{BH}$, $\overline{CJ}$, $\overline{DK}$, $\overline{EL}$, $\overline{FM}$; Vertices: *A*, *B*, *C*, *D*, *E*, *F*, *G*, *H*, *J*, *K*, *L*, *M* **39.** 384 in^2; 384 in^3 **41.** 72 m^2, 36 m^3 **43.** ≈ 23.6 in^2, ≈ 7.1 in^3

CHAPTER 2
Reasoning and Proof

Chapter 2 Get Ready

1. 31 **3.** 14 **5.** 12 **7.** $x^2 + 3$ **9.** −7 **11.** 10.8 **13.** $4x = 52$; \$13 **15.** ∠*CXD*, ∠*DXE* **17.** 38

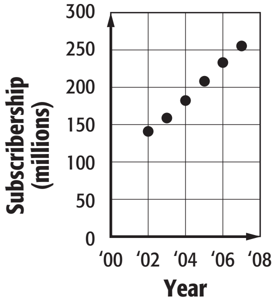
Lesson 2-1

1. Each cost is \$2.25 more than the previous cost; \$11.25. **3.** In each figure, the shading moves to the next point clockwise.

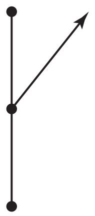
5 3, 3, 6, 9, 15
+3 +3 +6

Beginning with the third element, each element in the pattern is the sum of the previous two elements. So, the next element will be 15 + 9 or 24. **7.** The product of two even numbers is an even number. **9.** The set of points in a plane equidistant from point *A* is a circle.

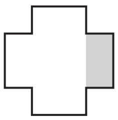
11a. **Wireless Subscribership by Year**

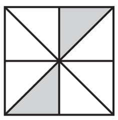


11b. Sample answer: About 372,000,000 Americans will have wireless subscriptions in 2012.

13 
If a ray intersects a segment at its midpoint and forms adjacent angles that are not right angles, then the ray is not perpendicular to the segment. **15.** Each element in the pattern is three more than the previous element; 18.

17. Each element has an additional two as part of the number; 22222. **19.** Each element is one half the previous element; $\frac{1}{16}$. **21.** Each percentage is 7% less than the previous percentage; 79%. **23.** Each meeting is two months after the previous meeting; July.

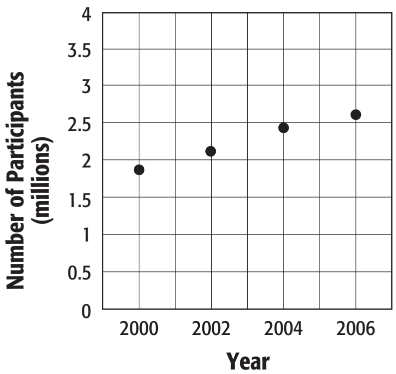
25. 
In each figure, the shading moves to the next area of the figure counter clockwise.

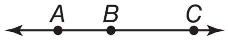
27. 
The shading of the lower triangle in the upper right quadrant of the first figure moves clockwise through each set of triangles from one figure to the next.

29. Sample answer: It is drier in the west and hotter in the south than other parts of the country, so less water would be readily available.

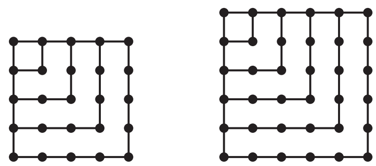
31 First, list examples: $1 \cdot 3 = 3$, $3 \cdot 5 = 15$, $7 \cdot 9 = 63$, $11 \cdot 11 = 121$. All the products are odd numbers. So, a conjecture about the product of two odd numbers is that the product is an odd number.
33. They are equal. **35.** The points equidistant from A and B form the perpendicular bisector of $\overline{AB}$. **37.** The area of the rectangle is two times the area of the square.

39a. **Hockey Participation by Year**

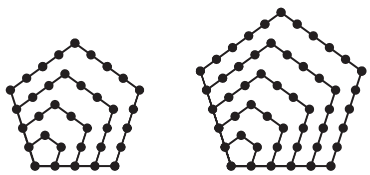


39b. Sample answer: More people over the age of 7 will play hockey in the future. The number of people playing hockey increases each year, so the graph suggests that even more people will play hockey in subsequent years. **41.** False; sample answer: Suppose $x = 2$, then $-x = -2$.
43. False; sample answer: 

45. False; sample answer: The length could be 4 m and the width could be 5 m. **47a.** 1, 4, 9, 16
47b. Sample answer: Start by adding 3 to 1 to get the second number, 4. Continue adding the next odd number to the previous number to get the next number in the sequence. **47c.** Sample answer: Each figure is the previous figure with an additional row and column of points added, which is 2 (position number) − 1. One is subtracted since 2 (position number) counts the corner point twice. 2 (position number) − 1 is always an odd number.
47d. 25, 36



49a. 1, 5, 12, 22 **49b.** Sample answer: Start by adding 4 to 1 to get the second number, 5. Increase the amount added to the previous number by 3 each time to get the next number in sequence. So, add 4 + 3 or 7 to 5 to get 12, and add 4 + 3 + 3 or 10 to 12 to get 22. **49c.** Sample answer: The second figure is the previous figure with 4 points added to make a pentagon. The third figure is the previous figure with 7 more points added, which is 3 more than the last number of points added. The fourth figure is the previous figure with 10 points added, which is 3 more than the last number of points added.
49d. 35, 51



51 **a.** For each even number from 10 to 20, write the number as the sum of two primes. Sample answer: 10 = 5 + 5, 12 = 5 + 7, 14 = 7 + 7, 16 = 5 + 11, 18 = 7 + 11, 20 = 7 + 13 **b.** The number 3 can be written as 0 + 3 and as 1 + 2. Since neither 0 nor 1 is a prime number, 3 cannot be written as the sum of two primes. So, the conjecture is false.

53. In the sequence of perimeters, each measure is twice the previous measure. Therefore, doubling the side length of a regular hexagon appears to also double its perimeter. In the sequence of areas, each measure is four times the previous measure. Therefore, doubling the side length of a regular hexagon appears to quadruple its area.

55. Jack; 2 is an even prime number.

57. Sample answer: False; if the two points create a straight angle that includes the third point, then the conjecture is true. If the two points do not create a straight angle with the third point, then the conjecture is false. **59.** B **61.** G **63.** 132 m^2; 60 m^3 **65.** 54 cm^2; 27 cm^3 **67.** 26.69 **69.** plane **71.** 18 **73.** 8

Lesson 2-2

1. A week has seven days, and there are 60 minutes in an hour. p and r is true, because p is true and r is true.

3 $q \vee r$: There are 20 hours in a day, or there are 60 minutes in an hour. A disjunction is true if at least one of the statements is true. So, $q \vee r$ is true because r is true. It does not matter that q is false.

5. A week has seven days, or there are 60 minutes in an hour. $p \vee r$ is true, because p is true and r is true.

7.

p	q	$\sim q$	$p \vee \sim q$
T	T	F	T
T	F	T	T
F	T	F	F
F	F	T	T

9.

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

11. $\overrightarrow{DB}$ is the angle bisector of $\angle ADC$, and $\overline{AD} \cong \overline{DC}$. p and r is true because p is true and r is true.
13. $\overline{AD} \cong \overline{DC}$ or $\overrightarrow{DB}$ is not the angle bisector of $\angle ADC$. r or $\sim p$ is true because r is true and $\sim p$ is false.
15. $\overrightarrow{DB}$ is not the angle bisector of $\angle ADC$, or $\overline{AD} \not\cong \overline{DC}$. $\sim p$ or $\sim r$ is false because $\sim p$ is false and $\sim r$ is false. **17.** Springfield is the capital of Illinois, and Illinois shares a border with Kentucky. $p \wedge r$ is true because p is true and r is true. **19.** Illinois does not share a border with Kentucky, or Illinois is to the west of Missouri. $\sim r \vee s$ is false because $\sim r$ is false and s is false. **21.** Springfield is not the capital of Illinois, and Illinois does not share a border with Kentucky. $\sim p \wedge \sim r$ is false because $\sim p$ is false and $\sim r$ is false.

23.

p	q	$\sim p$	$\sim p \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

25.

p	r	$p \wedge r$
T	T	T
T	F	F
F	T	F
F	F	F

27.

p	r	$p \vee r$
T	T	T
T	F	T
F	T	T
F	F	F

29.

p	r	$\sim p$	$\sim p \wedge r$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

31 **a.** The students who dive are represented by the intersection of the two sets and the nonintersecting portion of the Dive region. So, there are 3 + 4 or 7 students who dive. **b.** The students who participate in swimming or diving or both are represented by the union of the sets. There are 19 + 3 + 4 or 26 students who swim, dive, or do both. **c.** The students who swim and dive are represented by the intersection of the two sets. There are 3 students who both swim and dive.

33a. 50 **33b.** 40 **33c.** 110 **33d.** 20 **33e.** These teens do not use any of the listed electronics.

35 Make columns with the headings p, q, $\sim q$, r, $\sim q \vee r$, and $p \wedge (\sim q \vee r)$. List the possible combinations of truth values for p, q, and r. Use the truth values of q to find the truth values of $\sim q$. Use the truth values for each part of $\sim q \vee r$ to find the truth value of the compound statement. Then use the truth values for each part of $p \wedge (\sim q \vee r)$ to find the truth value of the compound statement.

p	q	$\sim q$	r	$\sim q \vee r$	$p \wedge (\sim q \vee r)$
T	T	F	T	T	T
T	F	T	T	T	T
T	T	F	F	F	F
T	F	T	F	T	T
F	T	F	T	T	F
F	F	T	T	T	F
F	T	F	F	F	F
F	F	T	F	T	F

If p and r are true, and q is true or false, then $p \wedge (\sim q \vee r)$ is true.

37.

p	q	$\sim q$	r	$\sim r$	$\sim q \wedge \sim r$	$p \vee (\sim q \wedge \sim r)$
T	T	F	T	F	F	T
T	F	T	T	F	F	T
T	T	F	F	T	F	T
T	F	T	F	T	T	T
F	T	F	T	F	F	F
F	F	T	T	F	F	F
F	T	F	F	T	F	F
F	F	T	F	T	T	T

true

39.

p	$\sim p$	q	r	$\sim r$	$(\sim p \vee q)$	$(\sim p \vee q) \vee \sim r$
T	F	T	T	F	T	T
T	F	F	T	F	F	F
T	F	T	F	T	T	T
T	F	F	F	T	F	T
F	T	T	T	F	T	T
F	T	F	T	F	T	T
F	T	T	F	T	T	T
F	T	F	F	T	T	T

If r is true or false, then $(\sim p \vee q) \vee \sim r$ is true.
41. Never; integers are rational numbers, not irrational. **43.** There exists at least one square that is not a rectangle. **45.** No students have classes in C-wing. **47.** Every segment has a midpoint.
49. Sample answer: A triangle has three sides, and a square has four sides. Both are true, so the compound statement is true. **51.** 22 in^2; The area of a triangle is $\frac{1}{2}bh$. The base of the triangle is 11 inches and the height of the triangle is 4 inches, so the area of the triangle is $\frac{1}{2}(11)(4)$ or 22 in^2. **53.** A **55.** triangular prism; bases: $\triangle MNO$, $\triangle PQR$; faces: $\triangle MNO$, $\triangle PQR$, $OMPR$, $ONQR$, $PQNM$; edges: $\overline{MN}$, $\overline{NO}$, $\overline{OM}$, $\overline{PQ}$, $\overline{QR}$, $\overline{PR}$, $\overline{NQ}$, $\overline{MP}$, $\overline{OR}$; vertices: M, N, O, P, Q, and R
57. triangular pyramid; base: $\triangle HJK$; faces: $\triangle HJK$, $\triangle HLK$, $\triangle KLJ$, $\triangle HLJ$; edges: $\overline{HK}$, $\overline{KJ}$, $\overline{HJ}$, $\overline{HL}$, $\overline{KL}$, $\overline{JL}$; vertices: H, K, J, and L **59.** −1 **61.** −7 **63.** 25
65. 14 **67.** 10

Lesson 2-3

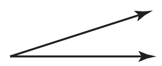
1. H: today is Friday; C: tomorrow is Saturday.
3. H: two angles are supplementary; C: the sum of the measures of the angles is 180.

5 **hypothesis:** You are sixteen years old.
conclusion: You are eligible to drive.
statement in if-then form: If you are sixteen years old, then you are eligible to drive.

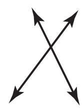
7. If the angle is acute, then its measure is between 0 and 90. **9a.** If moisture in the air condenses and falls, then it rains. **9b.** If a cumulonimbus cloud has supercooled moisture, then hail forms. **9c.** If the temperature is freezing in all or most of the atmosphere, then precipitation falls as snow.
11. False; Charlotte, Michigan; The hypothesis of the conditional is true, but the conclusion is false. The counterexample shows that the conditional statement is false. **13.** False; the animal could be a leopard. The hypothesis of the conditional is true, but the conclusion is false. This counterexample shows that the conditional statement is false. **15.** True; the hypothesis is false, since pigs cannot fly. A conditional with a false hypothesis is always true, so this conditional statement is true. **17.** If a number is a whole number, then it is an integer. Converse: If a number is an integer, then it is a whole number. False; sample answer: −3. Inverse: If a number is not a whole number, then it is not an integer. False: sample answer: −3. Contrapositive: If a number is not an integer, then it is not a whole number; true. **19.** H: you lead; C: I will follow. **21.** H: two angles are vertical; C: they are congruent. **23.** H: there is no struggle; C: there is no progress. **25.** H: a convex polygon has five sides; C: it is a pentagon. **27.** If you were at the party, then you received a gift. **29.** If a figure is a circle, then the area is πr^2. **31.** If an angle is right, then the angle measures 90 degrees. **33.** If the museum is the Andy Warhol Museum, then most of the collection is Andy Warhol's artwork.

35 To show that a conditional is false, you need only to find one counterexample. 9 is an odd number, but not divisible by 5. The hypothesis of the conditional is true, but the conclusion is false. So, this counterexample shows that the conditional statement is false.

37. False; the angle drawn is an acute angle whose measure is not 45. The hypothesis of the conditional is true, but the conclusion is false. This counterexample shows that the conditional statement is false.



39. True; when this hypothesis is true, the conclusion is also true, since an angle and its complement's sum is 90. So, the conditional statement is true. **41.** True; the hypothesis is false, since red and blue paint make purple paint. A conditional with a false hypothesis is always true, so this conditional statement is true. **43.** False; the animal could be a falcon. The hypothesis of the conditional is true, but the conclusion is false. This counterexample shows that the conditional statement is false.
45. False; these lines intersect, but do not form right angles. The hypothesis of the conditional is true, but the conclusion is false. This counterexample shows that the conditional statement is false.



47. Converse: If you live in Illinois, then you live in Chicago. False: You can live in Springfield. Inverse: If you do not live in Chicago, then you do not live in Illinois. False: You can live in Springfield. Contrapositive: If you do not live in Illinois, then you do not live in Chicago; true. **49.** Converse: If two angles are congruent, then they have the same measure; true. Inverse: If two angles do not have the same measure, then the angles are not congruent; true. Contrapositive: If two angles are not congruent, then they do not have the same measure; true. **51.** If segments are congruent, then they have the same length. Converse: If segments have the same length, then they are congruent; true. Inverse: If segments are not congruent, then they do not have the same length; true. Contrapositive: If segments do not have the same length, then they are not congruent; true. **53.** If an animal has stripes, then it is a zebra; false: a tiger has stripes.

55 The inverse is formed by negating both the hypothesis and the conclusion of the conditional. Inverse: If an animal does not have stripes, then it is not a zebra. This is a true statement.

57a. Sample answer: If a compound is an acid, it contains hydrogen. If a compound is a base, it contains hydroxide. If a compound is a hydrocarbon, it contains only hydrogen and carbon. **57b.** Sample answer: If a compound contains hydrogen, it is an acid. False; a hydrocarbon contains hydrogen. If a compound contains hydroxide, it is a base; true. If a compound contains only hydrogen and carbon, it is a hydrocarbon; true.

59 The blue area of the Venn diagram includes nonlinear functions but not quadratic functions. So, if a function is nonlinear, it may or may not be a quadratic function. Therefore, the conditional is false.

61. True; the deciduous area and the evergreen area have no common areas, so a deciduous tree cannot be evergreen. **63.** Sample answer: Kiri; when the hypothesis of a conditional is false, the conditional is always true. **65.** True; since the conclusion is false, the converse of the statement must be true. The converse and inverse are logically equivalent, so the inverse is also true. **67.** The hypothesis q of the inverse statement is *I received a detention*. The conclusion p of the inverse statement is *I did not arrive at school on time*. So the conditional A is $p \rightarrow q$: If I did not arrive at school on time, then I recieved a detention. So the converse of statement A is $\sim p \rightarrow \sim q$: If I did arrive at school on time, then I did not receive a detention. The contrapositve of Statement A is $\sim q \rightarrow \sim p$: If I did not receive a detention, then I arrived at school on time. **69.** A **71.** 0.00462

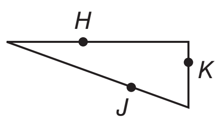
73.

p	q	p and q
T	T	T
T	F	F
F	T	F
F	F	F

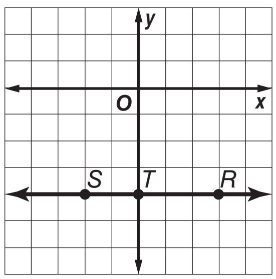
75.

p	q	$\sim p$	$\sim p \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

77. H, J, and K are noncollinear.

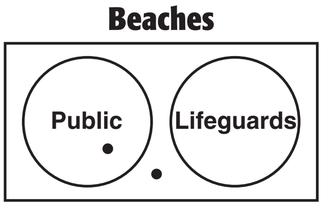


79. R, S, and T are collinear. **81.** $\overline{BC} \cong \overline{CD}$, $\overline{BE} \cong \overline{ED}$, $\overline{BA} \cong \overline{DA}$ **83.** about 9000 kg **85.** Divide each side by 8. **87.** Multiply each side by 3.



Lesson 2-4

1 Olivia is basing her conclusion on facts provided to her by her high school, not on a pattern of observations, so she is using deductive reasoning. **3.** valid; Law of Detachment **5.** Invalid: Bayview could be inside or outside the public beach's circle.

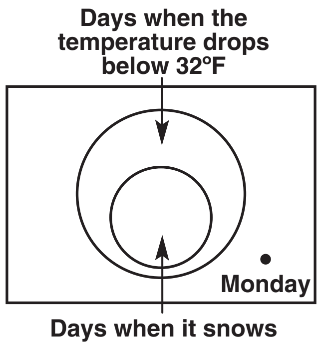


7. C **9.** No valid conclusion; ∠1 and ∠2 do not have to be vertical in order to be congruent. **11.** inductive reasoning **13.** deductive reasoning **15.** inductive reasoning

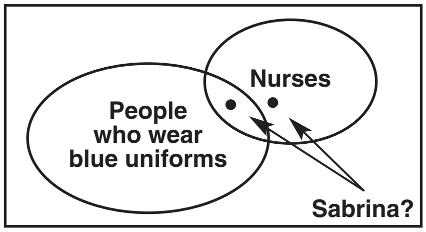
17 The given statement *Figure ABCD has four right angles* satisfies the conclusion of the true conditional. However, having a true conditional and a true conclusion does not make the hypothesis true. The figure could be a rectangle. So, the conclusion is invalid.

19. Invalid; your battery could be dead because it was old. **21.** valid; Law of Detachment

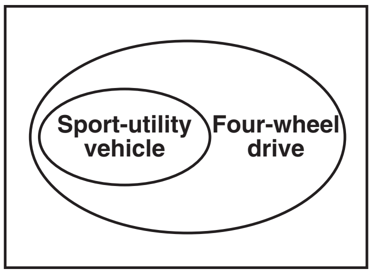
23. Valid; Monday is outside of the days when the temperature drops below 32°F, so it cannot be inside the days when it snows circle either, so the conclusion is valid.



25. Invalid; Sabrina could be inside just the nurses' circle or inside the intersection of the circles, so the conclusion is invalid.



27 The given statement *Ms. Rodriguez has just purchased a vehicle that has four-wheel drive* satisfies the conclusion of the true conditional. However, having a true conditional and a true conclusion does not make the hypothesis true. Ms. Rodriguez's car might be in the Four-wheel-drive section of the diagram that is not a sport-utility vehicle. So, the conclusion is invalid.

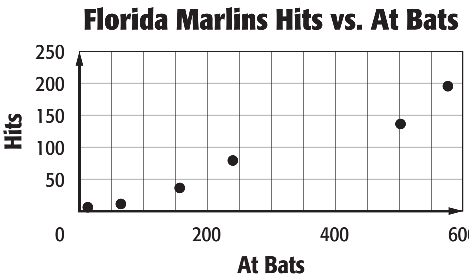


29. no valid conclusion **31.** no valid conclusion **33.** If two lines are not parallel, then they intersect in a point. **35.** Figure *ABCD* has all sides congruent; Law of Detachment.

37 You can reword the first given statement: If you are a ballet dancer, then you like classical music. Statement (1): If you are a ballet dancer, then you like classical music. Statement (2): If you like classical music, then you enjoy the opera. Since the conclusion of Statement (1) is the hypothesis of Statement (2), you can apply the Law of Syllogism. A valid conclusion: If you are a ballet dancer, then you enjoy the opera.

39. No valid conclusion; knowing a conclusion is true, does not imply the hypothesis will be true.

41a.



41b. Sample answer: About 91; inductive; a pattern was used to reach the conclusion. **41c.** Sample answer: The player with 240 at bats got more hits; deductive; the facts provided in the table were used to reach the conclusion. **43.** Law of Detachment: $[(p \to q) \land p] \to q$; Law of Syllogism: $[(p \to q) \land (q \to r)] \to (p \to r)$

45. Jonah's statement can be restated as, "Jonah is in group B and Janeka is in group B." In order for this compound statement to be true, both parts of the statement must be true. If Jonah was in group A, he would not be able to say that he is in group B, since students in group A must always tell the truth. Therefore the statement that Jonah is in group B is true. For the compound statement to be false, the statement that Janeka is in group B must be false. Therefore, Jonah is in group B and Janeka is in group A. **47.** D **49.** $\frac{26}{11}$

51a. If you live in Hawaii or Arizona then you do not observe Daylight Savings Time.

51b. If you do not observe Daylight Savings Time, then you live in Hawaii or Arizona; true.

53.

p	~*p*	*q*	~*q*	~*p* or ~*q*
T	F	T	F	F
T	F	F	T	T
F	T	T	F	T
F	T	F	T	T

55.

y	~*y*	*z*	~*y* or *z*
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	T

57. 18 **59.** Yes; the symbol denotes that $\angle DAB$ is a right angle. **61.** Yes; the sum of their measures is $m\angle ADC$, which is 90. **63.** No; we do not know $m\angle ABC$.

Lesson 2-5

1. The left side and front side have a common edge line *r*. Planes $\mathcal{P}$ and *Q* only intersect along line *r*. Postulate 2.7, which states if two planes intersect, then their intersection is a line. **3.** The front bottom edge of the figure is line *n* which contains points *D*, *C*, and *E*. Postulate 2.3, which states a line contains at least two points. **5.** Points *D* and *E*, which are on line *n*, lie in plane *Q*. Postulate 2.5, which states that if two points lie in a plane, then the entire line containing those points lies in that plane.

7 Postulate 2.7 states that if two planes intersect, then their intersection is a line. However, if three planes intersect, then their intersection may be a line or a point. So, the statement is sometimes true.

9. Always; Postulate 2.1 states through any two points, there is exactly one line. **11.** Postulate 2.3; a line contains at least two points. **13.** Postulate 2.4; a plane contains at least three noncollinear points. **15.** Since *C* is the midpoint of $\overline{AE}$ and $\overline{DB}$, $CA = CE = \frac{1}{2}AE$ and $CD = CB = \frac{1}{2}DB$ by the definition of midpoint. We are given $\overline{AE} \cong \overline{DB}$, so $AE = DB$ by the definition of congruent segments. By the multiplication property, $\frac{1}{2}DB = \frac{1}{2}AE$. So, by substitution, $AC = CB$. **17.** The edges of the sides of the bottom layer of the cake intersect. Plane $\mathcal{P}$ and *Q* of this cake intersect only once in line *m*. Postulate 2.7; if two planes intersect, then their intersection is a line. **19.** The top edge of the bottom layer of the cake is a straight line *n*. Points *C*, *D*, and *K* lie along this edge, so they lie along line *n*. Postulate 2.3; a line contains at least two points.

21 The bottom right part of the cake is a side. The side contains points *K*, *E*, *F*, and *G* and forms a plane. Postulate 2.2, which states that through any three noncollinear points, there is exactly one plane, shows that this is true.

23. The top edges of the bottom layer form intersecting lines. Lines *h* and *g* of this cake intersect only once at point *J*. Postulate 2.6; if two lines intersect, then their intersection is exactly one point. **25.** Never; Postulate 2.1 states through any two points, there is exactly one line. **27.** Always; Postulate 2.5 states if two points lie in a plane, then the entire line containing those points lies in that plane. **29.** Sometimes; the points must be noncollinear.

31. Given: *L* is the midpoint of $\overline{JK}$.
$\overline{JK}$ intersects $\overline{MK}$ at *K*. $\overline{MK} \cong \overline{JL}$

Prove: $\overline{LK} \cong \overline{MK}$

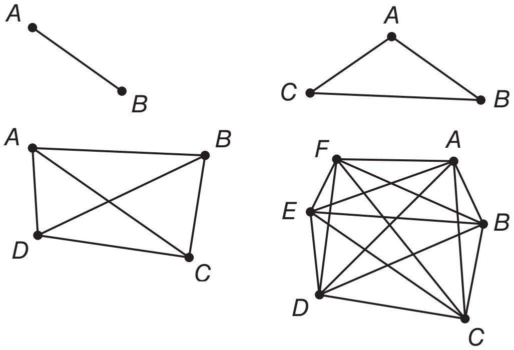
Proof: We are given that *L* is the midpoint of $\overline{JK}$ and $\overline{MK} \cong \overline{JL}$. By the Midpoint Theorem, $\overline{JL} \cong \overline{LK}$. By the Transitive Property of Equality, $\overline{LK} \cong \overline{MK}$.

33a. Southside Blvd.; sample answer: Since there is a line between any two points, and Southside Blvd. is the line between point *A* and point *B*, it is the shortest route between the two. **33b.** I-295

35. Points E, F, and G lie along the same line. Postulate 2.3 states that a line contains at least two points.

37. Postulate 2.1; through any two points, there is exactly one line. **39.** Postulate 2.4; a plane contains at least three noncollinear points. **41.** Postulate 2.7; if two planes intersect, then their intersection is a line.

43a.



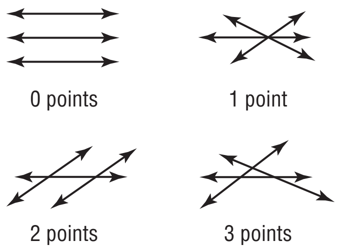
43b.

Number of Computers	Number of Connections
2	1
3	3
4	6
5	10
6	15

43c. $n - 1$

43d. $\frac{n(n-1)}{2}$

45. Lisa is correct. Sample answer: The proof should begin with the given, which is that $\overline{AB}$ is congruent to $\overline{BD}$ and A, B, and D are collinear. Therefore, Lisa began the proof correctly. **47a.** Plane Q is perpendicular to plane $\mathcal{P}$. **47b.** Line a is perpendicular to plane $\mathcal{P}$. **49.** Sometimes; three coplanar lines may have 0, 1, 2, or 3 points of intersection, as shown in the figures below.



Postulates 2.1–2.5 were used. Through points A and B there is exactly one line, n, satisfying Postulate 2.1. For the noncollinear points A, B, and C there is exactly one plane, $\mathcal{P}$, satisfying Postulate 2.2. Line n contains points A and B, satisfying Postulate 2.3. Plane $\mathcal{P}$ contains the noncollinear points A, B, and C, satisfying Postulate 2.4. Line n, containing points A and B, lies entirely in plane $\mathcal{P}$, satisfying Postulate 2.5. **51.** A **53.** H **55.** no conclusion **57.** If people are happy, then they rarely correct their faults. **59.** True; M is on $\overline{AB}$ and $AM + MB = AB$, so $p \wedge q$ is true. **61.** 19 m; 20 m of edging **63.** 5.5 **65.** 2, −2

Lesson 2-6

1. Trans. Prop. **3.** Sym. Prop.

5. Given: $\frac{y+2}{3} = 3$

Prove: $y = 7$

Proof:

Statements (Reasons)

a. $\frac{y+2}{3} = 3$ (Given)

b. $3\left(\frac{y+2}{3}\right) = 3(3)$ (Mult. Prop.)

c. $y + 2 = 9$ (Subst.)

d. $y = 7$ (Subt. Prop.)

7. Given: $\overline{AB} \cong \overline{CD}$

Prove: $x = 7$

Proof:

Statements (Reasons)

1. $\overline{AB} \cong \overline{CD}$ (Given)

2. $AB = CD$ (Def. of congruent segments)

3. $4x - 6 = 22$ (Subst. Prop.)

4. $4x = 28$ (Add. Prop.)

5. $x = 7$ (Div. Prop.)

9. Subt. Prop.

11.

$4x - 5 = x + 12$	Original equation
$4x - 5 + 5 = x + 12 + 5$	Addition Property of Equality
$4x = x + 17$	Simplify.

If $4x - 5 = x + 12$, then $4x = x + 17$ by the Addition Property of Equality.

13. Dist. Prop. **15.** Trans. Prop.

17. Given: $\frac{8-3x}{4} = 32$

Prove: $x = -40$

Proof:

Statements (Reasons)

a. $\frac{8-3x}{4} = 32$ (Given)

b. $4\left(\frac{8-3x}{4}\right) = 4(32)$ (Mult. Prop.)

c. $8 - 3x = 128$ (Subst.)

d. $-3x = 120$ (Subt. Prop.)

e. $x = -40$ (Div. Prop.)

19. Given: $-\frac{1}{3}n = 12$

Prove: $n = -36$

Proof:

Statements (Reasons)

1. $-\frac{1}{3}n = 12$ (Given)

2. $-3\left(-\frac{1}{3}n\right) = -3(12)$ (Mult. Prop.)

3. $n = -36$ (Subst.)

21. **a.** Use properties of equality to justify each step in solving the equation for a.

Given: $d = vt + \frac{1}{2}at^2$

Prove: $a = \frac{2d - 2vt}{t^2}$

Proof:

Statements (Reasons)

1. $d = vt + \frac{1}{2}at^2$ (Given)

2. $d - vt = vt - vt + \frac{1}{2}at^2$ (Subtraction Property)

3. $d - vt = \frac{1}{2}at^2$ (Substitution)

4. $2(d - vt) = 2\left(\frac{1}{2}at^2\right)$ (Multiplication Property)

5. $2(d - vt) = at^2$ (Substitution)
6. $2d - 2vt = at^2$ (Distributive Property)
7. $\frac{2d - 2vt}{t^2} = \frac{at^2}{t^2}$ (Division Property)
8. $\frac{2d - 2vt}{t^2} = a$ (Substitution)
9. $a = \frac{2d - 2vt}{t^2}$ (Symmetric Property)

b. $a = \frac{2d - 2vt}{t^2}$ Given
$= \frac{2(2850) - 2(50)(30)}{30^2}$ $d = 2850$, $t = 30$, $v = 50$
$= 3$ Simplify.

The acceleration of the object is 3 ft/s^2. The Substitution Property of Equality justifies this calculation.

23. Given: $\overline{DF} \cong \overline{EG}$
Prove: $x = 10$
Proof:
Statements (Reasons)
1. $\overline{DF} \cong \overline{EG}$ (Given)
2. $DF = EG$ (Def. of $\cong$ segs)
3. $11 = 2x - 9$ (Subst.)
4. $20 = 2x$ (Add. Prop.)
5. $10 = x$ (Div. Prop.)
6. $x = 10$ (Symm. Prop.)

25 Use properties of equality and the definition of congruent angles to justify each step in proving $x = 100$.
Given: $\angle Y \cong \angle Z$
Prove: $x = 100$
Proof:
Statements (Reasons)
1. $\angle Y \cong \angle Z$ (Given)
2. $m\angle Y = m\angle Z$ (Definition of $\cong$ ∠s)
3. $x + 10 = 2x - 90$ (Substitution)
4. $10 = x - 90$ (Subtraction Property)
5. $100 = x$ (Addition Property)
6. $x = 100$ (Symmetric Property)

27a. Given: $V = \frac{P}{I}$
Prove: $\frac{V}{2} = \frac{P}{2I}$
Proof:
Statements (Reasons)
1. $V = \frac{P}{I}$ (Given)
2. $\frac{1}{2} \cdot V = \frac{1}{2} \cdot \frac{P}{I}$ (Mult. Prop.)
3. $\frac{V}{2} = \frac{P}{2I}$ (Mult. Prop.)

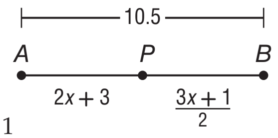
27b. Given: $V = \frac{P}{I}$
Prove: $2V = \frac{2P}{I}$
Proof:
Statements (Reasons)
1. $V = \frac{P}{I}$ (Given)
2. $2 \cdot V = 2 \cdot \frac{P}{I}$ (Mult. Prop)
3. $2V = \frac{2P}{I}$ (Mult. Prop.)

29. Given: $c^2 = a^2 + b^2$
Prove: $a = \sqrt{c^2 - b^2}$
Proof:
Statements (Reasons)
1. $a^2 + b^2 = c^2$ (Given)
2. $a^2 + b^2 - b^2 = c^2 - b^2$ (Subt. Prop.)
3. $a^2 = c^2 - b^2$ (Subs.)
4. $a = \pm\sqrt{c^2 - b^2}$ (Sq. Root Prop.)
5. $a = \sqrt{c^2 - b^2}$ (Length cannot be negative.)

31 The relation "is taller than" is not an equivalence relation because it fails the Reflexive and Symmetric properties. You cannot be taller than yourself (reflexive); if you are taller than your friend, then it does not imply that your friend is taller than you (symmetric).

33. The relation "$\neq$" is not an equivalence relation because it fails the Reflexive Property, since $a \neq a$ is not true. **35.** The relation "$\approx$" is not an equivalence relation because it fails the Reflexive Property, since $a \approx a$ is not true.

37. Given: $AP = 2x + 3$, $PB = \frac{3x + 1}{2}$, $AB = 10.5$
Prove: $\frac{AP}{AB} = \frac{2}{3}$



Proof:
Statements (Reasons)
1. $AP = 2x + 3$, $PB = \frac{3x + 1}{2}$, $AB = 10.5$ (Given)
2. $AP + PB = AB$ (Def. of a segment)
3. $2x + 3 + \frac{3x + 1}{2} = 10.5$ (Subt.)
4. $2 \cdot \left(2x + 3 + \frac{3x + 1}{2}\right) = 2 \cdot 10.5$ (Mult. Prop.)
5. $2 \cdot \left(2x + 3 + \frac{3x + 1}{2}\right) = 21$ (Subst.)
6. $2 \cdot 2x + 2 \cdot 3 + 2 \cdot \frac{3x + 1}{2} = 21$ (Dist. Prop.)
7. $4x + 6 + 3x + 1 = 21$ (Mult. Prop.)
8. $7x + 7 = 21$ (Add. Prop.)
9. $7x + 7 - 7 = 21 - 7$ (Subt. Prop.)
10. $7x = 14$ (Subs.)
11. $x = 2$ (Div. Prop.)
12. $AP = 2(2) + 3$ (Subst.)
13. $AP = 4 + 3$ (Mult. Prop.)
14. $AP = 7$ (Add. Prop.)
15. $\frac{AP}{AB} = \frac{7}{10.5}$ (Subst.)
16. $\frac{AP}{AB} = 0.\overline{6}$ (Div. Prop.)
17. $\frac{2}{3} = 0.\overline{6}$ (Div. Prop.)
18. $\frac{AP}{AB} = \frac{2}{3}$ (Trans. Prop.)

39. Sometimes; sample answer: If $a^2 = 1$ and $a = 1$, then $b = \sqrt{1}$ or 1. The statement is also true if $a = -1$, then $b = 1$. If $b = 1$, then $\sqrt{b} = 1$ since the square root of a number is nonnegative. Therefore, the statement is sometimes true. **41.** Sample answer: Depending on the purpose of the proof, one format may be preferable to another. For example, when writing an informal proof, you could use a paragraph proof to quickly convey your reasoning. When writing a more formal proof, a two-column proof may be preferable so that the justifications for each step are organized and easy to follow. **43.** 83° **45.** E **47.** Never; the sum of the measure of two supplementary angles is 180, so two obtuse angles can never be supplementary. **49.** Yes, by the Law of Detachment. **51.** (4, −3) **53.** (1, 2) **55.** (−1, −1) **57.** 2.4 cm

Lesson 2-7

1. Given: $\overline{LK} \cong \overline{NM}$, $\overline{KJ} \cong \overline{MJ}$
Prove: $\overline{LJ} \cong \overline{NJ}$
Proof:
Statements (Reasons)
a. $\overline{LK} \cong \overline{NM}$, $\overline{KJ} \cong \overline{MJ}$ (Given)
b. $LK = NM$, $KJ = MJ$ (Def. of ≅ segs.)
c. $LK + KJ = NM + MJ$ (Add. Prop.)
d. $LJ = LK + KJ$; $NJ = NM + MJ$ (Seg. Add. Post.)
e. $LJ = NJ$ (Subst.)
f. $\overline{LJ} \cong \overline{NJ}$ (Def. of ≅ segs.)

3 Use the definition of congruent segments and the Substitution Property of Equality.
Given: $\overline{AR} \cong \overline{CR}$; $\overline{DR} \cong \overline{BR}$
Prove: $AR + DR \cong CR + BR$
Proof:
Statements (Reasons)
1. $\overline{AR} \cong \overline{CR}$, $\overline{DR} \cong \overline{BR}$ (Given)
2. $AR = CR$, $DR = BR$ (Definition of ≅ segments)
3. $AR + DR = AR + BR$ (Addition Property)
4. $AR + DR = CR + BR$ (Substitution Property)

5. Given: $\overline{AB} \cong \overline{CD}$, $AB + CD = EF$
Prove: $2AB = EF$
Proof:
Statements (Reasons)
1. $\overline{AB} \cong \overline{CD}$, $AB + CD = EF$ (Given)
2. $AB = CD$ (Def. of ≅ segs.)
3. $AB + AB = EF$ (Subst.)
4. $2AB = EF$ (Subst.)

7 Use the Reflexive Property of Equality and the definition of congruent segments.
Given: $\overline{AB}$
Prove: $\overline{AB} \cong \overline{AB}$
Proof:
Statements (Reasons)
1. $\overline{AB}$ (Given)
2. $AB = AB$ (Reflexive Property)
3. $\overline{AB} \cong \overline{AB}$ (Definition of ≅ segments)

9. Given: $\overline{SC} \cong \overline{HR}$ and $\overline{HR} \cong \overline{AB}$
Prove: $\overline{SC} \cong \overline{AB}$
Proof:
Statements (Reasons)
1. $\overline{SC} \cong \overline{HR}$ and $\overline{HR} \cong \overline{AB}$ (Given)
2. $SC = HR$ and $HR = AB$ (Def. of ≅ segs.)
3. $SC = AB$ (Trans. Prop.)
4. $\overline{SC} \cong \overline{AB}$ (Def. of ≅ segs.)

11. Given: E is the midpoint of $\overline{DF}$ and $\overline{CD} \cong \overline{FG}$.
Prove: $\overline{CE} \cong \overline{EG}$
Proof:
Statements (Reasons)
1. E is the midpoint of $\overline{DF}$ and $\overline{CD} \cong \overline{FG}$. (Given)
2. $DE = EF$ (Def. of midpoint)
3. $CD = FG$ (Def. of ≅ segs.)
4. $CD + DE = EF + FG$ (Add. Prop.)
5. $CE = CD + DE$ and $EG = EF + FG$ (Seg. Add. Post.)
6. $CE = EG$ (Subst.)
7. $\overline{CE} \cong \overline{EG}$ (Def. of ≅ segs.)

13a. Given: $\overline{AC} \cong \overline{GI}$, $\overline{FE} \cong \overline{LK}$, $AC + CF + FE = GI + IL + LK$
Prove: $\overline{CF} \cong \overline{IL}$
Proof:
Statements (Reasons)
1. $\overline{AC} \cong \overline{GI}$, $\overline{FE} \cong \overline{LK}$, $AC + CF + FE = GI + IL + LK$ (Given)
2. $AC + CF + FE = AC + IL + LK$ (Subst.)
3. $AC - AC + CF + FE = AC - AC + IL + LK$ (Subt. Prop.)
4. $CF + FE = IL + LK$ (Subst.)
5. $CF + FE = IL + FE$ (Subs.)
6. $CF + FE - FE = IL + FE - FE$ (Subt. Prop.)
7. $CF = IL$ (Subst.)
8. $\overline{CF} \cong \overline{IL}$ (Def. of ≅ segs.)

13b. Sample answer: I measured $\overline{CF}$ and $\overline{IL}$, and both were 1.5 inches long, so the two segments are congruent.

15 **a.** Use the definition of midpoint and the segment addition postulate to prove.
Given: $\overline{SH} \cong \overline{TF}$; P is the midpoint of $\overline{SH}$ and $\overline{TF}$.
Prove: $\overline{SP} \cong \overline{TP}$
Proof:
Statements (Reasons)
1. $\overline{SH} \cong \overline{TF}$, P is the midpoint of $\overline{SH}$, P is the midpoint of $\overline{TF}$. (Given)
2. $SH = TF$ (Definition of ≅ segments)
3. $SP = PH$, $TP = PF$ (Definition of midpoint)
4. $SH = SP + PH$, $TF = TP + PF$ (Segment Addition Postulate)
5. $SP + PH = TP + PF$ (Substitution)
6. $SP + SP = TP + TP$ (Substitution)
7. $2SP = 2TP$ (Substitution)
8. $SP = TP$ (Division Property)
9. $\overline{SP} \cong \overline{TP}$ (Definition of ≅ segments)

b. $SP = \frac{1}{2}SH$ — Definition of midpoint
$= \frac{1}{2}(127.3)$ or 63.54 — Substitution

Since $\overline{TF} \cong \overline{SH}$, then $FP = SP = 63.54$.

Since $\triangle SPF$ is a right triangle, use the Pythagorean Theorem to find SF, the distance from first base to second base.

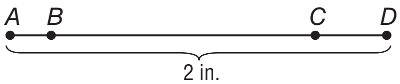
$c^2 = a^2 + b^2$ — Pythagorean Theorem
$SF^2 = SP^2 + FP^2$ — Substitution
$SF^2 = 63.54^2 + 63.54^2$ — Substitution

$SF^2 \approx 8074.6632$	Simplify.
$SF \approx 90$	Take the positive square root of each side.

The distance from first base to second base is about 90 feet.

17. Neither; since $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{BF}$, then $\overline{AB} \cong \overline{BF}$ by the Transitive Property of Congruence. **19.** No; congruence refers to segments. Segments cannot be added, only the measures of segments.

21.



23. D **25.** 18

27. Given: $AC = DF$, $AB = DE$
Prove: $BC = EF$
Proof:
Statements (Reasons)
1. $AC = DF$, $AB = DE$ (Given)
2. $AC = AB + BC$; $DF = DE + EF$ (Seg. Add. Post.)
3. $AB + BC = DE + EF$ (Subst.)
4. $BC = EF$ (Subt. Prop.)

29. 60, 30, 90, 60, 120, 60 **31.** $9\sqrt{2}$ **33.** $3y^4\sqrt{5x}$ **35.** 8

Lesson 2-8

1 $m\angle 1 = 90$ because $\angle 1$ is a right angle.

$m\angle 2 + m\angle 3 = 90$	Complement Theorem
$26 + m\angle 3 = 90$	$m\angle 2 = 26$
$26 + m\angle 3 - 26 = 90 - 26$	Subtraction Property
$m\angle 3 = 64$	Substitution

3. $m\angle 4 = 114$, $m\angle 5 = 66$; Suppl. Thm.

5. Given: $\angle 2 \cong \angle 6$
Prove: $\angle 4 \cong \angle 8$
Proof:
Statements (Reasons)
1. $\angle 2 \cong \angle 6$ (Given)
2. $m\angle 2 + m\angle 4 = 180$, $m\angle 6 + m\angle 8 = 180$ (Suppl. Thm.)
3. $m\angle 2 + m\angle 8 = 180$ (Subst.)
4. $m\angle 2 - m\angle 2 + m\angle 4 = 180 - m\angle 2$, $m\angle 2 - m\angle 2 + m\angle 8 = 180 - m\angle 2$ (Subt. Prop.)
5. $m\angle 4 = 180 - m\angle 2$, $m\angle 8 = 180 - m\angle 2$ (Subt. Prop.)
6. $m\angle 4 = m\angle 8$ (Subst.)
7. $\angle 4 \cong \angle 8$ (Def. ≅ ∠s)

7. Given: $\angle 4 \cong \angle 7$
Prove: $\angle 5 \cong \angle 6$
Proof:
Statements (Reasons)
1. $\angle 4 \cong \angle 7$ (Given)
2. $\angle 4 \cong \angle 5$ and $\angle 6 \cong \angle 7$ (Vert. ∠s Thm.)
3. $\angle 7 \cong \angle 5$ (Subst.)
4. $\angle 5 \cong \angle 6$ (Subst.)

9. $m\angle 3 = 62$, $m\angle 1 = m\angle 4 = 45$ (≅ Comp. and Suppl. Thm.) **11.** $m\angle 9 = 156$, $m\angle 10 = 24$ (≅ Suppl. Thm.)

13

$m\angle 6 + m\angle 7 = 180$	Supplement Thm.
$2x - 21 + 3x - 34 = 180$	Substitution
$5x - 55 = 180$	Substitution
$5x - 55 + 55 = 180 + 55$	Addition Property
$5x = 235$	Substitution
$\frac{5x}{5} = \frac{235}{5}$	Division Property
$x = 47$	Substitution

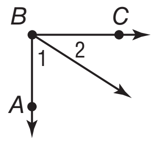
$m\angle 6 = 2x - 21$	Given
$m\angle 6 = 2(47) - 21$ or 73	Substitution

$m\angle 7 = 3x - 34$	Given
$m\angle 7 = 3(47) - 34$ or 107	Substitution

$\angle 8 \cong \angle 6$	Vertical Angles Theorem
$m\angle 8 = m\angle 6$	Definition of ≅ ∠s
$= 73$	Substitution

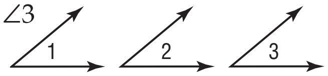
15. Given: $\angle 5 \cong \angle 6$
Prove: $\angle 4$ and $\angle 6$ are supplementary.
Proof:
Statements (Reasons)
1. $\angle 5 \cong \angle 6$ (Given)
2. $m\angle 5 = m\angle 6$ (Def. of ≅ ∠s)
3. $\angle 4$ and $\angle 5$ are supplementary. (Def. of linear pairs)
4. $m\angle 4 + m\angle 5 = 180$ (Def. of ∠s)
5. $m\angle 4 + m\angle 6 = 180$ (Subst.)
6. $\angle 4$ and $\angle 6$ are supplementary. (Def. of ∠s)

17. Given: $\angle ABC$ is a right angle.
Prove: $\angle 1$ and $\angle 2$ are complementary angles.



Proof:
Statements (Reasons)
1. $\angle ABC$ is a right angle. (Given)
2. $m\angle ABC = 90$ (Def. of rt. ∠s)
3. $m\angle ABC = m\angle 1 + m\angle 2$ (∠ Add. Post.)
4. $90 = m\angle 1 + m\angle 2$ (Subst.)
5. $\angle 1$ and $\angle 2$ are complementary angles. (Def. of comp. ∠s)

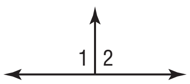
19. Given: $\angle 1 \cong \angle 2$, $\angle 2 \cong \angle 3$
Prove: $\angle 1 \cong \angle 3$



Proof:
Statements (Reasons)
1. $\angle 1 \cong \angle 2$, $\angle 2 \cong \angle 3$ (Given)
2. $m\angle 1 = m\angle 2$, $m\angle 2 = m\angle 3$ (Def. of ≅ ∠s)
3. $m\angle 1 = m\angle 3$ (Trans. Prop.)
4. $\angle 1 \cong \angle 3$ (Def. of ≅ ∠s)

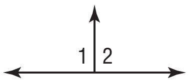
21. Given: $\angle 1 \cong \angle 4$
Prove: $\angle 2 \cong \angle 3$
Proof:
Statements (Reasons)
1. $\angle 1 \cong \angle 4$ (Given)
2. $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$ (Vert. ∠s are ≅.)
3. $\angle 1 \cong \angle 3$ (Trans. Prop.)
4. $\angle 2 \cong \angle 3$ (Subst.)

23. Given: $\angle 1$ and $\angle 2$ are rt. ∠s.
Prove: $\angle 1 \cong \angle 2$



Proof:
Statements (Reasons)
1. $\angle 1$ and $\angle 2$ are rt. ∠s. (Given)
2. $m\angle 1 = 90$, $m\angle 2 = 90$ (Def. of rt. ∠s)
3. $m\angle 1 = m\angle 2$ (Subst.)
4. $\angle 1 \cong \angle 2$ (Def. of ≅ ∠s)

25. Given: $\angle 1 \cong \angle 2$, $\angle 1$ and $\angle 2$ are supplementary.
Prove: $\angle 1$ and $\angle 2$ are rt. ∠s.
Proof:



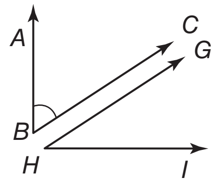
Statements (Reasons)
1. $\angle 1 \cong \angle 2$, $\angle 1$ and $\angle 2$ are supplementary. (Given)
2. $m\angle 1 + m\angle 2 = 180$ (Def. of supp. ∠s)
3. $m\angle 1 = m\angle 2$ (Def. of ≅ ∠s)
4. $m\angle 1 + m\angle 1 = 180$ (Subst.)
5. $2(m\angle 1) = 180$ (Subst.)
6. $m\angle 1 = 90$ (Div. Prop.)
7. $m\angle 2 = 90$ (Subst. (steps 3, 6))
8. $\angle 1$ and $\angle 2$ are rt. ∠s. (Def. of rt. ∠s)

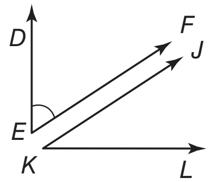
27. Since the path of the pendulum forms a right angle, $\angle ABC$ is a right angle, or measures 90. $\overrightarrow{BR}$ divides $\angle ABC$ into $\angle ABR$ and $\angle CBR$. By the Angle Addition Postulate, $m\angle ABR + m\angle CBR = m\angle ABC$, and, using substitution, $m\angle ABR + m\angle CBR = 90$. Substituting again, $m\angle 1 + m\angle 2 = 90$. We are given that $m\angle 1$ is 45°, so, substituting, $45 + m\angle 2 = 90$. Using the Subtraction Property, $45 - 45 + m\angle 2 = 90 - 45$, or $m\angle 2 = 45$. Since $m\angle 1$ and $m\angle 2$ are equal, $\overrightarrow{BR}$ is the bisector of $\angle ABC$ by the definition of angle bisector.

29 To prove lines ℓ and m are perpendicular, show that $\angle 1$, $\angle 3$, and $\angle 4$ are right ∠s.
Given: $\angle 2$ is a right angle.
Prove: $\ell \perp m$
Proof:
Statements (Reasons)
1. $\angle 2$ is a right angle. (Given)
2. $m\angle 2 = 90$ (Definition of a rt. ∠)
3. $\angle 2 \cong \angle 3$ (Vert. ∠s are ≅.)
4. $m\angle 3 = 90$ (Substitution)
5. $m\angle 1 + m\angle 2 = 180$ (Supplement Theorem)
6. $m\angle 1 + 90 = 180$ (Substitution)
7. $m\angle 1 + 90 - 90 = 180 - 90$ (Subtraction Property)
8. $m\angle 1 = 90$ (Substitution)
9. $\angle 1 \cong \angle 4$ (Vertical ∠s are ≅.)
10. $\angle 4 \cong \angle 1$ (Symmetric Property)
11. $m\angle 4 = m\angle 1$ (Definition of ≅ ∠s)
12. $m\angle 4 = 90$ (Substitution)
13. $\ell \perp m$ (⊥ lines intersect to form four rt. ∠s.)

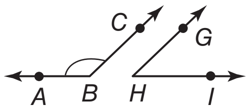
31. Given: $\overrightarrow{XZ}$ bisects $\angle WXY$, and $m\angle WXZ = 45$.
Prove: $\angle WXY$ is a right angle.
Proof:
Statements (Reasons)
1. $\overrightarrow{XZ}$ bisects $\angle WXY$ and $m\angle WXZ = 45$. (Given)
2. $\angle WXZ \cong \angle ZXY$ (Def. of ∠ bisector)
3. $m\angle WXZ = m\angle ZXY$ (Def. of ≅ ∠s)
4. $m\angle ZXY = 45$ (Subst.)
5. $m\angle WXY = m\angle WXZ + m\angle ZXY$ (∠ Add. Post.)
6. $m\angle WXY = 45 + 45$ (Subst.)
7. $m\angle WXY = 90$ (Subst.)
8. $\angle WXY$ is a right angle. (Def. of rt. ∠s)

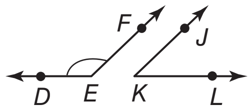
33. Each of these theorems uses the words "or to congruent angles" indicating that this case of the theorem must also be proven true. The other proofs only addressed the "to the same angle" case of the theorem.





Given: $\angle ABC \cong \angle DEF$, $\angle GHI$ is complementary to $\angle ABC$, $\angle JKL$ is complementary to $\angle DEF$.
Prove: $\angle GHI \cong \angle JKL$
Proof:
Statements (Reasons)
1. $\angle ABC \cong \angle DEF$, $\angle GHI$ is complementary to $\angle ABC$, $\angle JKL$ is complementary to $\angle DEF$. (Given)
2. $m\angle ABC + m\angle GHI = 90$, $\angle DEF + \angle JKL = 90$ (Def. of compl. ∠s)
3. $m\angle ABC + m\angle JKL = 90$ (Subst.)
4. $90 = m\angle ABC + m\angle JKL$ (Symm. Prop.)
5. $m\angle ABC + m\angle GHI = m\angle ABC + m\angle JKL$ (Trans. Prop.)
6. $m\angle ABC - m\angle ABC + m\angle GHI = m\angle ABC - m\angle ABC + m\angle JKL$ (Subt.)
7. $m\angle GHI = m\angle JKL$ (Subst.)
8. $\angle GHI \cong \angle JKL$ (Def. of ≅ ∠s)





Given: $\angle ABC \cong \angle DEF$, $\angle GHI$ is supplementary to $\angle ABC$, $\angle JKL$ is supplementary to $\angle DEF$.
Prove: $\angle GHI \cong \angle JKL$
Proof:
Statements (Reasons)
1. $\angle ABC \cong \angle DEF$, $\angle GHI$ is supplementary to $\angle ABC$, $\angle JKL$ is supplementary to $\angle DEF$. (Given)
2. $m\angle ABC + m\angle GHI = 180$, $m\angle DEF + m\angle JKL = 180$ (Def. of suppl. ∠s)
3. $m\angle ABC + m\angle JKL = 180$ (Subst.)
4. $180 = m\angle ABC + m\angle JKL$ (Symm. Property)
5. $m\angle ABC + m\angle GHI = m\angle ABC + m\angle JKL$ (Trans. Prop.)
6. $m\angle ABC - m\angle ABC + m\angle GHI = m\angle ABC - m\angle ABC + m\angle JKL$ (Subt.)
7. $m\angle GHI = m\angle JKL$ (Subst.)
8. $\angle GHI \cong \angle JKL$ (Def. of ≅ ∠s)

35. Sample answer: Since protractors have the scale for both acute and obtuse angles along the top, the supplement is the measure of the given angle on the other scale. **37.** A **39.** B **41.** Subtratcion Prop. **43.** Substitution **45.** true **47.** true **49.** line n **51.** point W **53.** Yes; it intersects both m and n when all three lines are extended.

Chapter 2 Study Guide and Review

1. false; theorem **3.** true **5.** true **7.** true **9.** false; negation **11.** false; two nonadjacent supplementary angles **13.** Sample answer: Dogs or other pets may threaten or chase wildlife that might not be present in his local park. **15.** A plane contains at least three

noncollinear points and the sum of the measures of two complementary angles is not 180; true.
17a. 18 **17b.** 14 **17c.** 22 **19.** true **21.** *PQRS* is a parallelogram; Law of Detachment. **23.** valid; Law of Detachment **25.** Sometimes; if the three points are collinear, they will be contained in multiple planes, but if they are noncollinear, they will be contained in only one plane. **27.** Sometimes; if the angles are adjacent, they will form a right angle, but if they are not adjacent, they will not. **29.** Symmetric
31. Distributive Property **33.** Transitive Property
35. Statements (Reasons)
1. $PQ = RS$, $PQ = 5x + 9$, $RS = x - 31$ (Given)
2. $5x + 9 = x - 31$ (Subst.)
3. $4x = 9 = -31$ (Subt.)
4. $4x = -40$ (Subt. Prop.)
5. $x = -10$ (Div. Prop.)

37. Statements (Reasons)
1. X is the midpoint of $\overline{WX}$ and $\overline{VZ}$. (Given)
2. $\overline{WX} \cong \overline{YX}$, $\overline{VX} \cong \overline{ZX}$ (Def. of midpoint)
3. $WX = YX$, $VX = ZX$ (Def. of $\cong$)
4. $VX = VW = WX$, $ZX = ZY + YX$ (Seg. Add. Post.)
5. $VW + WX = ZY + YX$ (Subs.)
6. $VW = ZY$ (Subt. Prop.)

39. Segment Addition Postulate **41.** 127
43. Statements (Reasons)
1. $\angle 1 \cong \angle 4$, $\angle 2 \cong \angle 3$ (Given)
2. $m\angle 1 = m\angle 4$, $m\angle 2 = m\angle 3$ (Def. of $\cong$)
3. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$ (Add. Prop.)
4. $m\angle 1 + m\angle 2 = m\angle AFC$, $m\angle 3 + m\angle 4 = m\angle EFC$ ($\angle$ Add. Post.)
5. $m\angle AFC = m\angle EFC$ (Subst.)
6. $\angle AFC = \angle EFC$ (Def. of $\cong$)

CHAPTER 3
Parallel and Perpendicular LInes

Chapter 3 Get Ready

1. 4 **3.** Yes, points *C* and *D* lie in plane *CBD*. **5.** 142 **7.** 90 **9.** −1 **11.** $\frac{1}{3}$

Lesson 3-1

1. *TUV* **3.** $\overline{YX}$, $\overline{TU}$, $\overline{ZW}$

5 Angle $\angle 1$ and $\angle 8$ are nonadjacent exterior angles that lie on opposite sides of the transversal. So, they are alternate exterior angles.

7. alternate interior **9.** line *n*; corresponding
11. line *m*; consecutive interior **13.** $\overline{CL}$, $\overline{EN}$, $\overline{BK}$, $\overline{AJ}$

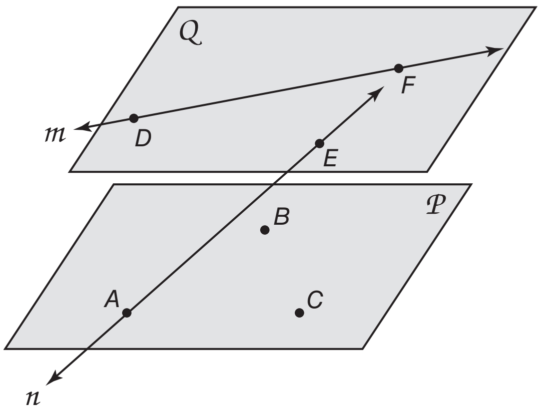
15 The segments that do not intersect $\overline{BC}$ and are not in the same plane as $\overline{BC}$ are skew to $\overline{BC}$. Any of the following are skew to $\overline{BC}$: $\overline{EN}$, $\overline{AJ}$, $\overline{DM}$, $\overline{NM}$, $\overline{NJ}$, $\overline{JK}$, or $\overline{ML}$.

17. $\overline{KL}$, $\overline{CL}$, $\overline{BK}$, $\overline{ML}$, $\overline{DM}$, $\overline{NM}$, $\overline{KJ}$ **19.** $\overline{JK}$ **21.** line *s*; corresponding **23.** line *t*; alternate interior **25.** line *t*; alternate exterior **27.** line *t*; consecutive interior

29. line *s*; alternate exterior **31.** line *a*; vertical
33. line *c*; alternate interior **35.** line *f*; corresponding
37a. Sample answer: Since the lines are coplanar and they cannot touch, they are parallel. **37b.** Line *q* is a transversal of lines *p* and *m*. **39.** skew **41.** parallel
43. intersecting

45 **a.** The treads, or the upper horizontal parts of the stairs, are parallel. **b.** The treads of the two steps at the top of the incline lie in the same plane. So, they are coplanar. **c.** The treads of the steps on the incline of the escalator do not intersect and are not parallel to the treads of the steps on the bottom of the conveyor. So, they are skew.

47a.



47b. parallel **47c.** skew **49.** Sometimes; $\overleftrightarrow{AB}$ intersects $\overleftrightarrow{EF}$ depending on where the planes intersect. **51.** B **53.** (0, 4), (−6, 0) **55.** $m\angle 9 = 86$, $m\angle 10 = 94$ **57.** $m\angle 19 = 140$, $m\angle 20 = 40$
59. 3.75 **61.** 90 **63.** 45

Lesson 3-2

1. 94; Corresponding Angle Postulate **3.** 86; Corresponding Angle Postulate and Supplement Angle Theorem **5.** 79; Vertical Angle Theorem, Consecutive Interior Angles Theorem **7.** $m\angle 2 = 93$, $m\angle 3 = 87$, $m\angle 4 = 87$ **9.** $x = 114$ by the Alternate Exterior Angles Theorem **11.** 62; Corresponding Angles Postulate **13.** 118; Def. of Supplementary Angles **15.** 38; Corresponding Angles Postulate **17.** 142; Supplement Angles Theorem **19.** 38; Alternate Exterior Angles Postulate

21 If the radiation rays form parallel lines, then $\angle 1$ and $\angle 3$ are corresponding angles. So, according to the Corresponding Angles Postulate, $\angle 1$ and $\angle 3$ are congruent.

23. Supplementary; since $\angle 3$ and $\angle 5$ are a linear pair, they are supplementary. $\angle 4$ and $\angle 5$ are congruent because they are alternate exterior angles, so $\angle 3$ is supplementary to $\angle 4$.

25
$3x - 15 = 105$ Corresponding Angles Postulate
$3x = 120$ Add 15 to each side.
$x = 40$ Divide each side by 3.

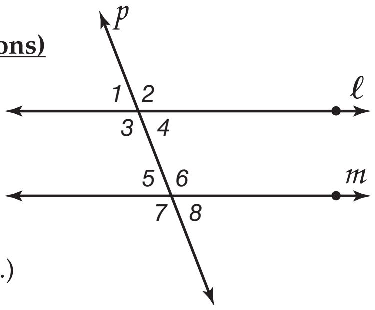
$(3x - 15) + (y + 25) = 180$ Supplement Theorem
$105 + y + 25 = 180$ Substitution

$y + 130 = 180$ Simplify.
$y = 50$ Subtract 130 from each side.

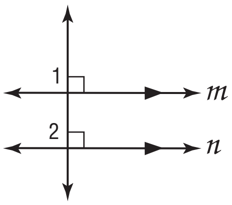
So, $x = 40$ by the Corresponding Angles Postulate; $y = 50$ by the Supplement Theorem.

27. $x = 42$ by the Consecutive Interior Angles Theorem; $y = 14$ by the Consecutive Interior Angles Theorem **29.** $x = 60$ by the Consecutive Interior Angles Theorem; $y = 10$ by the Supplement Theorem **31.** Congruent; Alternate Interior Angles **33.** Congruent; vertical angles are congruent.

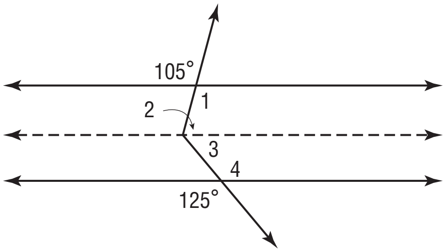
35. Given: $\ell \parallel m$
Prove: $\angle 1 \cong \angle 8$
$\angle 2 \cong \angle 7$
Proof:
Statements (Reasons)
1. $\ell \parallel m$ (Given)
2. $\angle 1 \cong \angle 5$, $\angle 2 \cong \angle 6$ (Corr. ∠s Post.)
3. $\angle 5 \cong \angle 8$, $\angle 6 \cong \angle 7$ (Vertical ∠s Thm.)
4. $\angle 1 \cong \angle 8$, $\angle 2 \cong \angle 7$ (Trans. Prop.)



37. Given: $m \parallel n$, $t \perp m$
Prove: $t \perp n$
Proof:
Statements (Reasons)
1. $m \parallel n$, $t \perp m$ (Given)
2. $\angle 1$ is a right angle. (Def. of ⊥)
3. $m\angle 1 = 90$ (Def. of rt. ∠s)
4. $\angle 1 \cong \angle 2$ (Corr. ∠s Post.)
5. $m\angle 1 = m\angle 2$ (Def. of ≅ ∠s)
6. $m\angle 2 = 90$ (Subst.)
7. $\angle 2$ is a right angle. (Def. of rt. ∠s)
8. $t \perp n$ (Def. of ⊥ lines)



39 Draw a line parallel to the two given lines and label angles 1, 2, 3, and 4. So, $x = m\angle 2 + m\angle 3$.



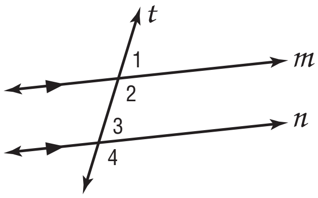
$m\angle 1 = 105$ because vertical angles are congruent and their measures are equal. $\angle 1$ and $\angle 2$ are supplementary by the Consecutive Interior Angles Theorem.

$m\angle 1 + m\angle 2 = 180$ Definition of supplementary angles
$105 + m\angle 2 = 180$ Substitution
$m\angle 2 = 75$ Subtract 105 from each side.

$m\angle 4 = 125$ because vertical angles are congruent and their measures are equal. $\angle 3$ and $\angle 4$ are supplementary by the Consecutive Interior Angles Theorem.

$m\angle 3 + m\angle 4 = 180$ Definition of supplementary angles
$m\angle 3 + 125 = 180$ Substitution
$m\angle 3 = 55$ Subtract 125 from each side.
$m\angle 2 + m\angle 3 = x$ Angle Addition Postulate
$75 + 55 = x$ Substitution
$130 = x$ Simplify.

41a. Sample answer for m and n:

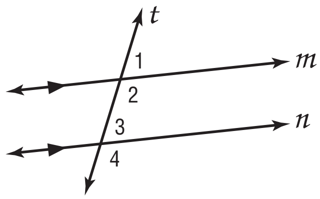


41b. Sample answer:

$m\angle 1$	$m\angle 2$	$m\angle 3$	$m\angle 4$
60	120	60	120
45	135	45	135
70	110	70	110
90	90	90	90
25	155	25	155

41c. Sample answer: Angles on the exterior of a pair of parallel lines located on the same side of the transversal are supplementary. **41d.** Inductive; a pattern was used to make a conjecture.

41e. Given: parallel lines m and n cut by transversal t
Prove: $\angle 1$ and $\angle 4$ are supplementary.



Proof:
Statements (Reasons)
1. Lines m and n are parallel and cut by transversal t. (Given)
2. $m\angle 1 + m\angle 2 = 180$ (Suppl. Thm.)
3. $\angle 2 \cong \angle 4$ (Corr. ∠s are ≅.)
4. $m\angle 2 = m\angle 4$ (Def. of congruence)
5. $m\angle 1 + m\angle 4 = 180$ (Subst.)
6. $\angle 1$ and $\angle 4$ are supplementary. (Definition of supplementary angles)

43. In both theorems, a pair of angles is formed when two parallel lines are cut by a transversal. However, in the Alternate Interior Angles Theorem, each pair of alternate interior angles that is formed are congruent, whereas in the Consecutive Interior Angles Theorem, each pair of angles formed is supplementary.
45. $x = 171$ or $x = 155$; $y = 3$ or $y = 5$ **47.** C **49.** I and II **51.** Skew lines; the planes are flying in different directions and at different altitudes.
53. $m\angle 6 = 43$, $m\angle 7 = 90$ **55.** 15 **57.** $\frac{7}{3}$ **59.** 1
61. $-\frac{3}{5}$

Lesson 3-3

1. -1 **3.** $\frac{6}{5}$

5 slope of $\overleftrightarrow{WX} = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$= \frac{5-4}{4-2}$ $(x_1, y_1) = (2, 4), (x_2, y_2) = (4, 5)$

$= \frac{1}{2}$ Simplify.

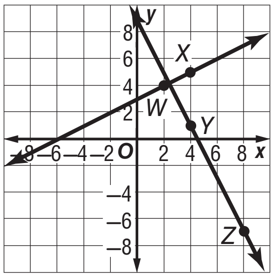
slope of $\overleftrightarrow{YZ} = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$= \frac{-7-1}{8-4}$ $(x_1, y_1) = (4, 1)$, $(x_2, y_2) = (8, -7)$

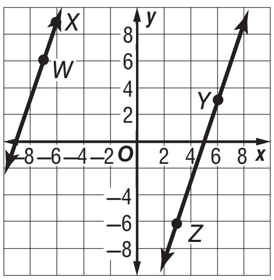
$= \frac{-8}{4}$ or -2 Simplify.

$\frac{1}{2}(-2) = -1$ Product of slopes

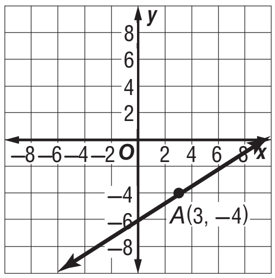
Since the product of the slopes is -1, $\overleftrightarrow{WX}$ is perpendicular to $\overleftrightarrow{YZ}$.



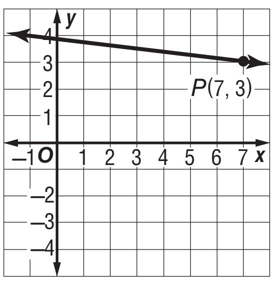
7. parallel



9.



11.



13 Substitute $(-3, 1)$ for (x_1, y_1) and $(4, -2)$ for (x_2, y_2).

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

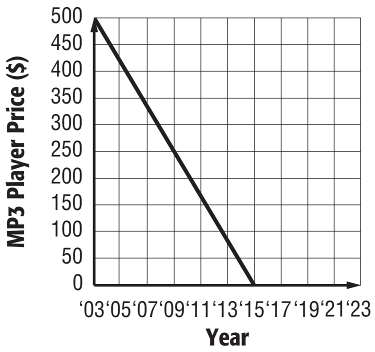
$= \frac{-2-1}{4-(-3)}$ Substitution

$= -\frac{3}{7}$ Simplify.

15. 8 **17.** undefined **19.** 1 **21.** 0 **23.** undefined

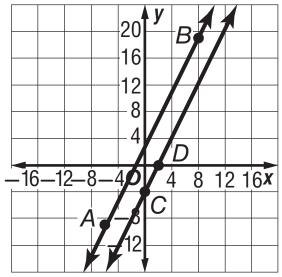
25. $-\frac{1}{6}$

27a.

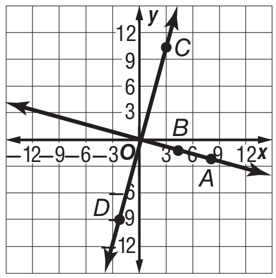


27b. $41.50
27c. $84

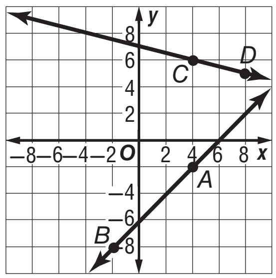
29. parallel



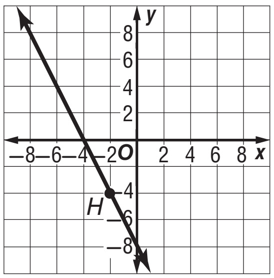
31. perpendicular



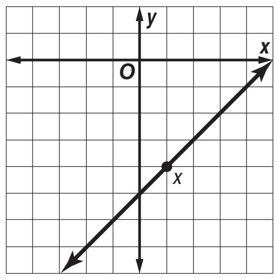
33. neither



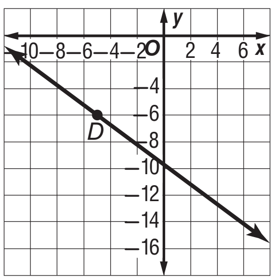
35.



37.



39.



41. Line 2

43 slope of Line 1 $= \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$= \frac{0-(-4)}{7-(-9)}$ $(x_1, y_1) = (-9, -4)$, $(x_2, y_2) = (7, 0)$

$= \frac{4}{16}$ Subtract.

$= \frac{1}{4}$ Simplify.

slope of Line 2 $= \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

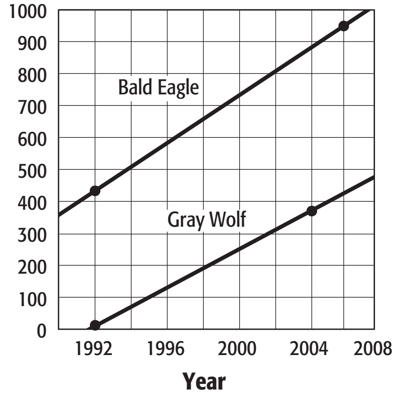
$= \frac{4-1}{7-0}$ $(x_1, y_1) = (0, 1)$, $(x_2, y_2) = (7, 4)$

$= \frac{3}{7}$ Subtract.

Since $\frac{1}{4} < \frac{3}{7}$, the slope of Line 2 is steeper than the slope of Line 1.

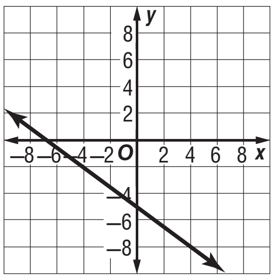
45a. the bald eagle

45b.

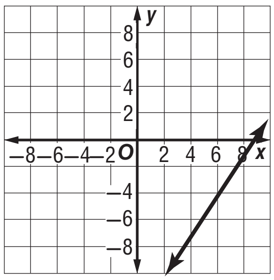


45c. 1189 bald eagles; 494 gray wolves

47. $y = -8$



49. $y = 0$

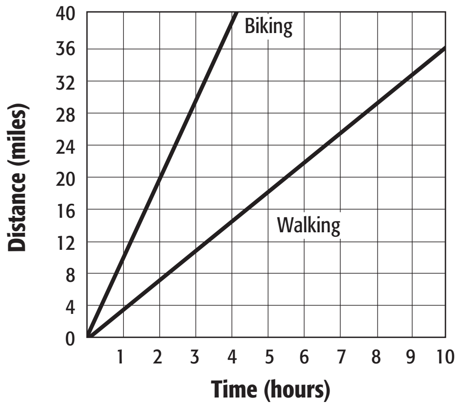


51 a.

Time (hours)	Distance Walking (miles)	Distance Riding Bikes (miles)
0	0	0
1	3.5	10
2	7	20
3	10.5	30
4	14	40

Multiply the time spent walking by 3.5 and the time spent riding by 10 to find the distances travelled.

b. Use the table from part **a** to create a graph with time on the horizontal axis and distance on the vertical axis.

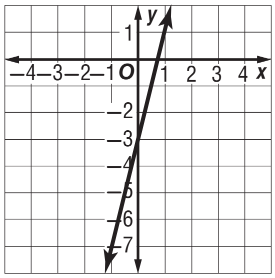


c. their speed **d.** Sample answer: Yes, they can make it if they ride their bikes. If they walk, it takes over two hours to go eight miles, so they wouldn't be home in time and they wouldn't get to spend any time in the store. If they ride their bikes, they can travel there in 24 minutes. If they spend 30 minutes in the store and spend 24 minutes riding home, the total amount of time they will use is $24 + 30 + 24 = 78$ minutes, which is 1 hour and 18 minutes.

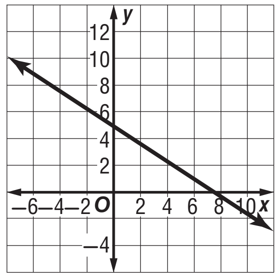
53. Terrell; Hale subtracted the x-coordinates in the wrong order. **55.** The Sears Tower has a vertical or undefined and the Leaning Tower of Pisa has a positive slope. **57.** Sample answer: $(4, -3)$ and $(5, -5)$ lie along the same line as point X and Y. The slope between all of the points is -2. To find additional points, you can take any point on the line and subtract 2 from the y-coordinate and add 1 to the x-coordinate. **59.** 2 : 5 **61.** C **63.** 123 **65.** 57 **67.** ABC, ABQ, PQR, CDS, APU, DET **69.** valid **71.** 6 **73.** $y = -2x + 3$

Lesson 3-4

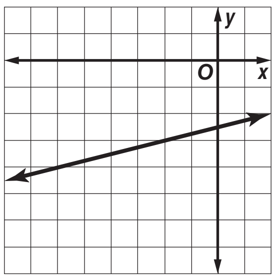
1. $y = 4x - 3$



3. $y = -\frac{2}{3}x + 5$



5. $y + 3 = \frac{1}{4}(x + 2)$

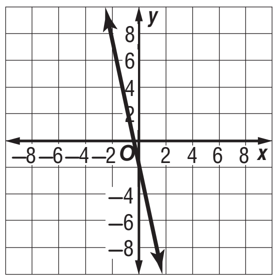


7. $y = \frac{5}{4}x - 1$

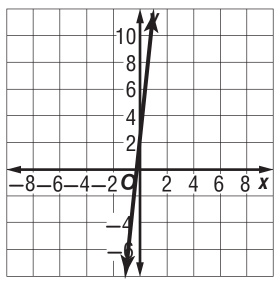
9. $y = \frac{9}{7}x - \frac{19}{7}$

11. $y = 4x + 9$

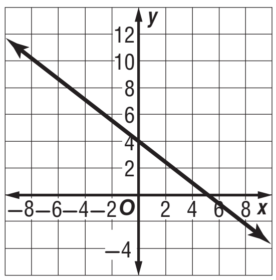
13. $y = -5x - 2$



15. $y = 9x + 2$

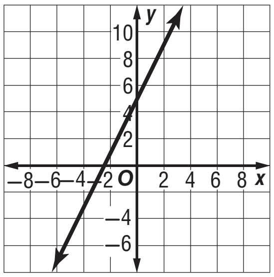


17. $y = -\frac{3}{4}x + 4$

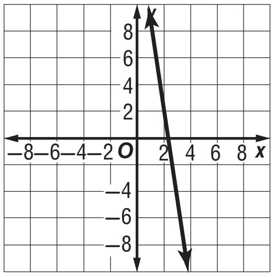


19 $y - y_1 = m(x - x_1)$ Point-slope form
$y - 11 = 2(x - 3)$ $m = 2, (x_1, y_1) = (3, 11)$
$y = 2x + 5$ Simplify.

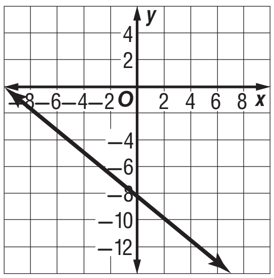
Graph the given point (3, 11). Use the slope 2 or $\frac{2}{1}$ to find another point 2 units up and 1 unit to the right.



21. $y - 9 = -7(x - 1)$

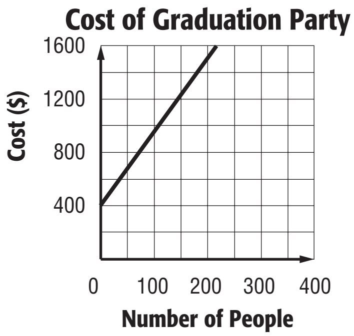


23. $y + 6 = -\frac{4}{5}(x + 3)$



25. $y = -4$ **27.** $x = -3$ **29.** $y = -\frac{3}{4}x + 3$
31. $y = -\frac{10}{3}x + \frac{38}{3}$ **33.** $y = \frac{3}{2}x - \frac{1}{2}$ **35.** $y = \frac{2}{3}x - 2$
37. $y = -2x - 18$ **39.** $y = -\frac{2}{3}x + 6$

41 **a.** For each person, the cost increases $5.50. So, the rate of change, or slope, is 5.5. The y-intercept represents the cost when there are 0 people, or $400. Let y represent the total cost and x represent the number of people who attend the party.
$y = mx + b$ Slope-intercept form
$y = 5.5x + 400$ $m = 5.5, b = 400$
b. Let the x-axis represent the number of people and the y-axis represent the total cost.



c. If $\frac{2}{3}$ of the class attends, then $\frac{2}{3} \cdot 285$ or 190 people attend.
$y = 5.5x + 400$ Write the equation.
$= 5.5(190) + 400$ $x = 190$
$= 1045 + 400$ Multiply.
$= 1445$ Simplify.
The party will cost $1445.
d. $y = 5.5x + 400$ Write the equation.
$2000 = 5.5x + 400$ $y = 2000$
$1600 = 5.5x$ Subtract.
$y = 290.9$ Simplify.
If they raise $2000, 290 people can attend the party.

43. p **45.** n, p, or r **47.** perpendicular **49.** neither

51 First, find the slope of the line through (3, 2) and (−7, 2)
$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula
$= \frac{2 - 2}{-7 - 3}$ $(x_1, y_1) = (3, 2), (x_2, y_2) = (-7, 2)$
$= \frac{0}{-10}$ Subtract.
$= 0$ Simplify.
Since the slope of the line is 0, it is a horizontal line. A line is perpendicular to a horizontal line if it is vertical, so find the vertical line through the point (−8, 12). The line through the point (−8, 12) perpendicular to the line through (3, 2) and (−7, 2) is $x = -8$.

53. $C = 15(x - 1) + 40$ or $C = 15x + 25$ **55.** 14
57. Sample answer: $y = 2x - 1$, $y = -\frac{1}{2}x - \frac{17}{2}$ **59.** Sample answer: When given the slope and y-intercept, the slope-intercept form is easier to use. When given two points, the point-slope form is easier to use. When given the slope and a point, the point-slope form is easier to use.
61. H **63.** E **65.** 2 **67.** $x = 3$, $y \approx 26.33$
69. Gas-O-Rama is also a quarter mile from Lacy's home; the two gas stations are half a mile apart.
71. consecutive interior **73.** alternate exterior

Lesson 3-5

1. $j \parallel k$; Converse of Corresponding Angles Postulate

3 Angle 3 and ∠10 are alternate exterior angles of lines ℓ and m. Since $\angle 3 \cong \angle 10$, $\ell \parallel m$ by the Alternate Exterior Angles Converse Theorem.

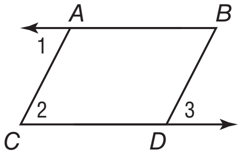
5. 20 **7.** Yes; Sample answer: Since the alternate exterior angles are congruent, the backrest and footrest are parallel. **9.** $u \parallel v$; Alternate Exterior ∠s Converse
11. $r \parallel s$; Consecutive Interior ∠s Converse
13. $u \parallel v$; Alternate Interior ∠s Converse **15.** $r \parallel s$; Corresponding ∠s Converse **17.** 22; Conv. Corr. ∠s Post.

19 The angles are consecutive interior angles. For lines m and n to be parallel, consecutive interior angles must be supplementary, according to the Consecutive Interior Angles Converse Theorem.
$(7x - 2) + (10 - 3x) = 180$ Definition of supp. ∠s
$4x + 8 = 180$ Simplify.
$4x = 172$ Subtract 8 from each side.
$x = 43$ Divide each side by 4.

21. 36; Alt. Ext. ∠s Conv.
23a. ∠1 and ∠2 are supplementary. **23b.** Def. of linear pair **23c.** ∠2 and ∠3 are supplementary; Suppl. Thm.

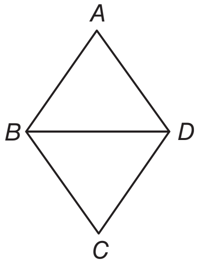
23d. $\cong$ Suppl. Thm. **23e.** Converse of Corr. $\angle$s Post.

25. Given: $\angle 1 \cong \angle 3$, $\overline{AC} \parallel \overline{BD}$
Prove: $\overline{AB} \parallel \overline{CD}$



Proof:
Statements (Reasons)
1. $\angle 1 \cong \angle 3$, $\overline{AC} \parallel \overline{BD}$ (Given)
2. $\angle 2 \cong \angle 3$ (Corr. $\angle$s Post.)
3. $\angle 1 \cong \angle 2$ (Trans. Prop.)
4. $\overline{AB} \parallel \overline{CD}$ (If alternate $\angle$s are $\cong$, then lines are $\parallel$.)

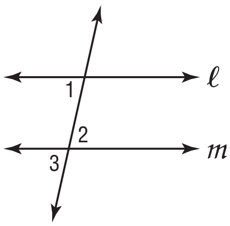
27. Given: $\angle ABC \cong \angle ADC$, $m\angle A + m\angle ABC = 180$
Prove: $\overline{AB} \parallel \overline{CD}$



Proof:
Statements (Reasons)
1. $\angle ABC \cong \angle ADC$, $m\angle A + m\angle ABC = 180$ (Given)
2. $m\angle ABC = m\angle ADC$ (Def. of $\cong$ $\angle$s)
3. $m\angle A + m\angle ADC = 180$ (Substitution)
4. $\angle A$ and $\angle ADC$ are supplementary. (Def. of supplementary $\angle$s)
5. $\overline{AB} \parallel \overline{CD}$ (If consec. int. $\angle$s are supplementary, then lines are $\parallel$.)

29 The Converse of the Perpendicular Transversal Theorem states that two lines perpendicular to the same line are parallel. Since the slots, or the bottom of each rectangular opening, are perpendicular to each of the sides, the slots are parallel. Since any pair of slots is perpendicular to the sides, they are also parallel.

31. Given: $\angle 1 \cong \angle 2$
Prove: $\ell \parallel m$



Proof:
Statements (Reasons)
1. $\angle 1 \cong \angle 2$ (Given)
2. $\angle 2 \cong \angle 3$ (Vertical $\angle$s are $\cong$)
3. $\angle 1 \cong \angle 3$ (Transitive Prop.)
4. $\ell \parallel m$ (If corr $\angle$s are $\cong$, then lines are $\parallel$.)

33. $r \parallel s$; Sample answer: The corresponding angles are congruent. Since the angles are equal, the lines are parallel. **35.** $r \parallel s$; Sample answer: The alternate exterior angles are congruent. Since the angles are equal, the lines are parallel. **37.** Daniela; $\angle 1$ and $\angle 2$ are alternate interior angles for $\overline{WX}$ and $\overline{YZ}$, so if alternate interior angles are congruent, then the lines are parallel.

39. Sample answer:
Given: $a \parallel b$ and $b \parallel c$
Prove: $a \parallel c$
Statements (Reasons)
1. $a \parallel b$ and $b \parallel c$ (Given)
2. $\angle 1 \cong \angle 3$ (Alt. Int. $\angle$s Thm.)
3. $\angle 3 \cong \angle 2$ (Vert. $\angle$s are $\cong$)
4. $\angle 2 \cong \angle 4$ (Alt. Int. $\angle$s Thm.)
5. $\angle 1 \cong \angle 4$ (Trans. Prop.)
6. $a \parallel b$ (Alt. Int. $\angle$s Conv. Thm.)

41a. We know that $m\angle 1 + m\angle 2 = 180$. Since $\angle 2$ and $\angle 3$ are linear pairs, $m\angle 2 + m\angle 3 = 180$. By substitution, $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$. By subtracting $m\angle 2$ from both sides we get $m\angle 1 = m\angle 3$. $\angle 1 \cong \angle 3$, by the definition of congruent angles. Therefore, $a \parallel c$ since the corresponding angles are congruent. **41b.** We know that $a \parallel c$ and $m\angle 1 + m\angle 3 = 180$. Since $\angle 1$ and $\angle 3$ are corresponding angles, they are congruent and their measures are equal. By substitution, $m\angle 3 + m\angle 3 = 180$ or $2m\angle 3 = 180$. By dividing both sides by 2, we get $m\angle 3 = 90$. Therefore, $t \perp c$ since they form a right angle. **43.** Yes; sample answer: A pair of angles can be both supplementary and congruent if the measure of both angles is 90, since the sum of the angle measures would be 180.

45. G

47. J

49. $y = \frac{4}{5}x - 9$

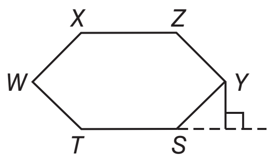
51. 6 hours

53.

55. 8.6 m; 3.5 m^2 **57.** 10, 8.3

Lesson 3-6

1.



3. The formation should be that of two parallel lines that are also parallel to the 50-yard line; the band members have formed two lines that are equidistant from the 50-yard line, so by Theorem 3.9, the two lines formed are parallel. **5.** $\sqrt{10}$ units

7 Let ℓ represent $y = -2x + 4$ and let m represent $y = -2x + 14$. The slope of the lines is -2. Write an equation for line p. The slope of p is the opposite reciprocal of -2, or $\frac{1}{2}$. Use the y-intercept of line ℓ, (0, 4), as one of the endpoints of the perpendicular segment.

$(y - y_1) = m(x - x_1)$	Point-slope form
$(y - 4) = \frac{1}{2}(x - 0)$	$(x_1, y_1) = (0, 4)$, $m = \frac{1}{2}$
$y - 4 = \frac{1}{2}x$	Simplify.
$y = \frac{1}{2}x + 4$	Add 4 to each side.

Use a system of equations to find the point of intersection of lines m and p.
Equation for m: $y = -2x + 14$
Equation for p: $y = \frac{1}{2}x + 4$

$y = -2x + 14$	Equation for m
$\frac{1}{2}x + 4 = -2x + 14$	Use Equation p to substitute $\frac{1}{2}x + 4$ for y.
$\frac{5}{2}x + 4 = 14$	Add $2x$ to each side.

$\frac{5}{2}x = 10$ Subtract 4 from each side.
$x = 4$ Multiply each side by $\frac{2}{5}$.

$y = \frac{1}{2}x + 4$ Equation for p

$y = \frac{1}{2}(4) + 4$ Substitute 4 for x.

$y = 6$ Simplify.

The point of intersection is (4, 6). Use the Distance Formula to find the distance between (0, 4) and (4, 6).

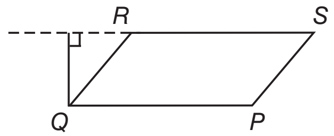
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

$= \sqrt{(4 - 0)^2 + (6 - 4)^2}$ $x_1 = 0, y_1 = 4,$ $x_2 = 4, y_2 = 6$

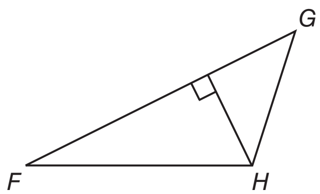
$= \sqrt{20}$ or $2\sqrt{5}$ Simplify.

The distance between the lines is $2\sqrt{5}$ units.

9.



11.



13. No; a driveway perpendicular to the road would be the shortest. The angle the driveway makes with the road is less than 90°, so it is not the shortest possible driveway.

(15) Find the slope and y-intercept of line ℓ and write the equation for the line.

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-3)}{7 - 0}$ or 1

Since ℓ contains (0, −3), the y-intercept is −3. So, the equation for line ℓ is $y = 1x + (-3)$ or $y = x - 3$. The slope of a line perpendicular to ℓ, line w, is −1. Write the equation of line w through (4, 3) with slope −1.

$y = mx + b$ Slope-intercept form
$3 = -1(4) + b$ $m = -1, (x, y) = (4, 3)$
$3 = -4 + b$ Simplify.
$7 = b$ Add 4 to each side.

So, the equation for line w is $y = -x + 7$. Solve the system of equations to determine the point of intersection.

line ℓ: $y = x - 3$
line w: $(+)\ y = -x + 7$
$2y = 4$ Add the two equations.
$y = 2$ Divide each side by 2.

Solve for x.

$y = x - 3$ Equation for line ℓ
$2 = x - 3$ $y = 2$
$5 = x$ Add 3 to each side.

The point of intersection is (5, 2). Let this be point Q. Use the Distance Formula to determine the distance between $P(4, 3)$ and $Q(5, 2)$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

$= \sqrt{(5 - 4)^2 + (2 - 3)^2}$ $x_1 = 4, y_1 = 3,$ $x_2 = 5, y_2 = 2$

$= \sqrt{2}$ Simplify.

The distance between the lines is $\sqrt{2}$ units.

17. 6 units **19.** $\sqrt{10}$ units **21.** 6 units **23.** $\sqrt{26}$ units **25.** 21 units **27.** $4\sqrt{17}$ units **29.** $\sqrt{14.76}$ units **31.** 5 units **33.** 6 units

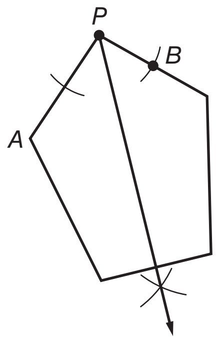
(35) He can conclude that the right and left sides of the bulletin board are not parallel, since the perpendicular distance between one line and any point on the other line must be equal anywhere on the lines for the two lines to be parallel. In this case, the length of the top of the bulletin board is not equal to the length of the bottom of the bulletin board.

39b. Place point C any place on line m. The area of the triangle is $\frac{1}{2}$ the height of the triangle times the length of the base of the triangle. The numbers stay constant regardless of the location of C on line m. **39c.** 16.5 in^2

(41) To find out if the lines will intersect, determine if they are parallel. If they are parallel the perpendicular distance between the two lines at any point will be equal. Use a ruler to measure the distance between points A and C and points B and D. $AC = 1.2$ centimeters and $BD =$ 1.35 centimeters. The lines are not parallel, so they will intersect. Shenequa is correct.

43. $a = \pm 1$; $y = \frac{1}{2}x + 6$ and $y = \frac{1}{2}x + \frac{7}{2}$ or $y = -\frac{1}{2}x + 6$ and $y = -\frac{1}{2}x + \frac{7}{2}$

45a. Sample answer:



45b. Sample answer: Using a protractor, the measurement of the constructed angle is equal to 90. So, the line constructed from vertex P is perpendicular to the nonadjacent side chosen.

45c. Sample answer: The same compass setting was used to construct points A and B. Then the same compass setting was used to construct the perpendicular line to the side chosen. Since the compass setting was equidistant in both steps, a perpendicular line was constructed.

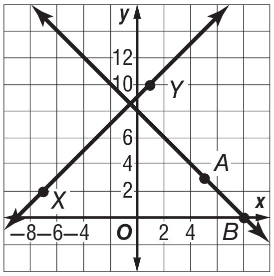
47. Sample answer: First, a point on one of the parallel lines is found. Then the line perpendicular to the pair of parallel lines is found. Then the point of intersection is found between the perpendicular line and the other line not used in the first step. Last, the Distance Formula is used to determine the distance between the pair of intersection points. This value is the distance between the pair of parallel lines. **49.** C **51.** B **53.** $y + 1 = \frac{1}{4}(x - 3)$

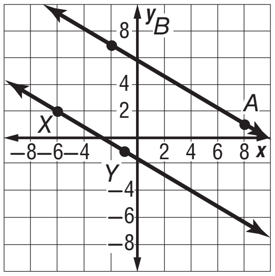
55. $y - 3 = -(x + 2)$
57. Given: $AB = BC$
Prove: $AC = 2BC$
Proof:
Statements (Reasons)
1. $AB = BC$ (Given)
2. $AC = AB + BC$ (Seg. Add. Post.)
3. $AC = BC + BC$ (Substitution)
4. $AC = 2BC$ (Substitution)

59. Sample answer: Robin ⊥ Cardinal; Bluebird divides two of the angles formed by Robin and Cardinal into pairs of complementary angles.
61. 5 **63.** 13 **65.** 5

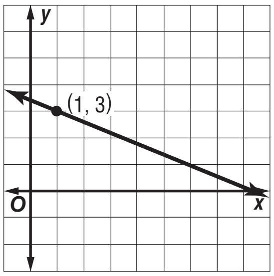
Chapter 3 Study Guide and Review

1. false; parallel **3.** true **5.** true **7.** false; congruent **9.** corresponding **11.** alternate exterior **13.** skew lines **15.** 57; $\angle 5 \cong \angle 13$ by Corr. ∠s Post. and 13 and 14 form a linear pair **17.** 123; $\angle 11 \cong \angle 5$ by Alt. Int. ∠s Thm. and $\angle 5 \cong \angle 1$ by Alt. Ext. ∠s Thm. **19.** 57; $\angle 1 \cong \angle 3$ by Corr. ∠s Post. and ∠3 and ∠6 form a linear pair.
21. perpendicular **23.** parallel



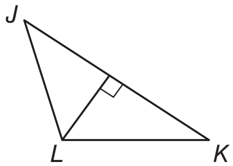


25.



27. $y + 9 = 2(x - 4)$
29. $y = 5x - 3$
31. $y = -\frac{2}{3}x + 10$
33. $C = 20h + 50$
35. none
37. $v \parallel z$; Alternate Exterior Angles Converse Thm.

39. 135 **41.**



CHAPTER 4

Congruent Triangles

Get Ready

1. right **3.** obtuse **5.** 84°; Alternate Exterior Angles **7.** ≈10.8 **9.** ≈18.0 **11.** 144.2 mi

Lesson 4-1

1. right **3.** equiangular **5.** obtuse; $\angle BDC > 90°$ **7.** isosceles

9 By the definition of midpoint, $FK = KH$.

$FK + KH = FH$	Segment Addition Postulate
$KH + KH = FH$	Substitution
$2KH = FH$	Simplify.
$2(2.5) = FH$	Substitution
$5 = FH$	Simplify.

Since $\overline{GH} \cong \overline{FG}$, $GH = FG$ or 5. Since $GH = FG = FH = 5$, the triangle has three sides with the same measure. Therefore, the triangle has three congruent sides, so it is equilateral.

11. scalene **13.** $x = 5$, $QR = RS = QS = 25$ **15.** obtuse **17.** right **19.** acute **21.** obtuse **23.** acute **25.** right **27.** equilateral **29.** scalene **31.** scalene **33.** scalene **35.** equilateral

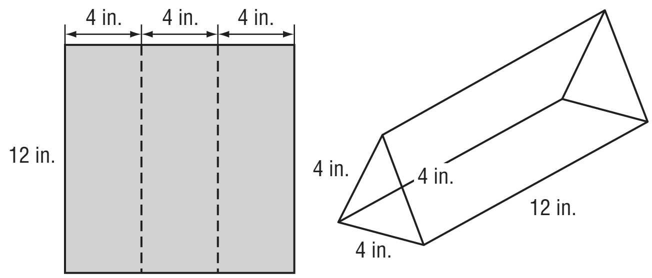
37 Since △FGH is equilateral, $FG = GH$.

$FG = GH$	Given
$3x + 10 = 9x - 8$	Substitution
$10 = 6x - 8$	Subtract $3x$ from each side.
$18 = 6x$	Add 8 to each side.
$3 = x$	Divide each side by 3.

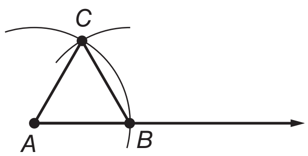
$FG = 3x + 10$	Given
$= 3(3) + 10$ or 19	$x = 3$

$FG = GH = HF = 19$

39 Because the base of the prism formed is an equilateral triangle, the mirror tile must be cut into three strips of equal width. Since the original tile is a 12-inch square, each strip will be 12 inches long by 12 ÷ 3 or 4 inches wide.

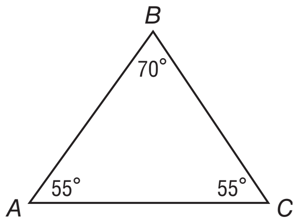


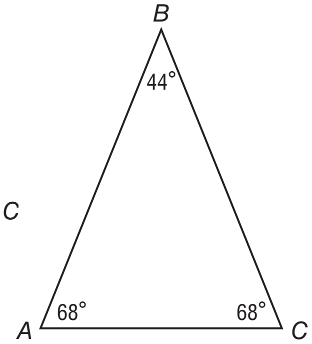
41. isosceles obtuse **43.** scalene; $XZ = 3\sqrt{5}$, $XY = \sqrt{113}$, $YZ = 2\sqrt{26}$ **45.** isosceles; $XZ = 2$, $XY = 2\sqrt{2}$, $YZ = 2$
47. Given: $m\angle ADC = 120$
Prove: △DBC is acute.
Proof: ∠ADC and ∠BDC form a linear pair. ∠ADC and ∠BDC are supplementary because if two angles form a linear pair, then they are supplementary. So, $m\angle ADC + m\angle BDC = 180$. We know $m\angle ADC = 120$, so by substitution, $120 + m\angle BDC = 180$. Subtract to find that $m\angle BDC = 60$. We already know that ∠B is acute because △ABC is acute. ∠BCD must also be acute because ∠C is acute and $m\angle C = m\angle ACD + m\angle BCD$. △$DBC$ is acute by definition.
49. $x = 15$; $FG = 35$, $GH = 35$, $HF = 35$ **51.** $x = 3$; $MN = 13$, $NP = 13$, $PM = 11$
53.



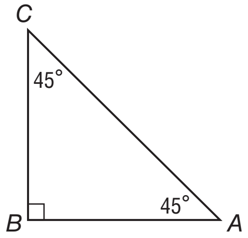
Sample answer: In △ABC, $AB = BC = AC = 1.3$ cm. Since all sides have the same length, they are all congruent. Therefore the triangle is equilateral. $\triangle ABC$ was constructed using AB as the length of each side. Since the arc for each segment is the same, the triangle is equilateral.

55 **a.** Sample answer:
acute isosceles

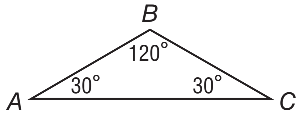




right isosceles



obtuse isosceles



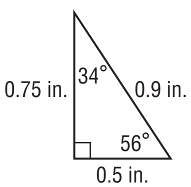
b.

$m\angle A$	$m\angle C$	$m\angle B$	Sum of Angle Measures
55	55	70	180
68	68	44	180
45	45	90	180
30	30	120	180

c. Sample answer: In an isosceles triangle, the angles opposite the congruent sides have the same measure. The sum of the angle measures of an isosceles triangle is 180. **d.** If the measures of the angles opposite the congruent sides of an isosceles triangle have the same measure, then if one angle measures x, then the other angle also measures x. If the sum of the measures of the angles of an isosceles triangle is 180, the measure of the third angle is $180 - (x + x)$ or $180 - 2x$. So, the other two angles measure x and $180 - 2x$.

57. Never; all equiangular triangles have three 60° angles, so they do not have a 90° angle. Therefore they cannot be right triangles. **59.** Never; all equilateral triangles are also equiangular, which means all of the angles are 60°. A right triangle has one 90° angle.

61. Sample answer:



63. Not possible; all equilateral triangles have three acute angles. **65.** A **67.** 13.5 **69.** 7 **71.** $2\sqrt{5}$ **73.** Two lines in a plane that are perpendicular to the same line are parallel. **75.** H: you are a teenager; C: you are at least 13 years old.

77. H: you have a driver's license; C: you are at least 16 years old **79.** Plane AEB intersects with plane $\mathcal{N}$ in $\overleftrightarrow{AB}$. **81.** Points D, C, and B lie in plane $\mathcal{N}$, but point E does not lie in plane $\mathcal{N}$. Thus, they are not coplanar. **83.** cons. int. **85.** alt. ext.

Lesson 4-2

1. 58 **3.** 80 **5.** 49 **7.** 78 **9.** 61 **11.** 151 **13.** 30

15

$m\angle L + m\angle M + m\angle 2 = 180$	Triangle Angle-Sum Theorem
$31 + 90 + m\angle 2 = 180$	Substitution
$121 + m\angle 2 = 180$	Simplify.
$m\angle 2 = 59$	Subtract 121 from each side.

$\angle 1$ and $\angle 2$ are congruent vertical angles. So, $m\angle 1 = 59$.

$m\angle 1 + m\angle 3 + m\angle P = 180$	Triangle Angle-Sum Theorem
$59 + m\angle 3 + 22 = 180$	Substitution
$81 + m\angle 3 = 180$	Simplify.
$m\angle 3 = 99$	Subtract 118 from each side.

$m\angle 1 = 59$, $m\angle 2 = 59$, $m\angle 3 = 99$

17. 79 **19.** 21

21

$m\angle A + m\angle B = 148$	Exterior Angle Theorem
$(2x - 15) + (x - 5) = 148$	Substitution
$3x - 20 = 148$	Simplify.
$3x = 168$	Add 20 to each side.
$x = 56$	Divide each side by 2.

So, $m\angle ABC = 56 - 5$ or 51.

23. 78 **25.** 39 **27.** 55 **29.** 35 **31.** $x = 30$; 30, 60

33 In $\triangle ABC$, $\angle B$ and $\angle C$ are congruent, so $m\angle B = m\angle C$.

$m\angle A = 3(m\angle B)$	$m\angle A$ is to be 3 times $m\angle B$.
$m\angle A + m\angle B + m\angle C = 180$	Triangle Angle-Sum Theorem
$3(m\angle B) + m\angle B + m\angle B = 180$	Substitution
$5(m\angle B) = 180$	Simplify.
$m\angle B = 36$	Divide each side by 5.

$m\angle C = m\angle B = 36$ $\quad$ $m\angle A = 3m\angle B = 3(36)$ or 108

35. **Given:** $\triangle MNO$; $\angle M$ is a right angle.
Prove: There can be at most one right angle in a triangle.
Proof: In $\triangle MNO$, M is a right angle. $m\angle M + m\angle N + m\angle O = 180$. $m\angle M = 90$, so $m\angle N + m\angle O = 90$. If N were a right angle, then $m\angle O = 0$. But that is impossible, so there cannot be two right angles in a triangle.
Given: $\triangle PQR$; $\angle P$ is obtuse.
Prove: There can be at most one obtuse angle in a triangle.
Proof: In $\triangle PQR$, $\angle P$ is obtuse. So $m\angle P > 90$. $m\angle P + m\angle Q + m\angle R = 180$. It must be that $m\angle Q + m\angle R < 90$. So, $\angle Q$ and $\angle R$ must be acute.

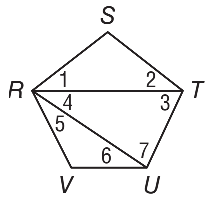
37. $m\angle 1 = 65$, $m\angle 2 = 20$, $m\angle 3 = 95$, $m\angle 4 = 40$, $m\angle 5 = 110$, $m\angle 6 = 45$, $m\angle 7 = 70$, $m\angle 8 = 65$ **39.** 67°, 23° **41.** $z < 23$; Sample answer: Since the sum of the measures of the angles of a triangle is 180 and $m\angle X = 157$, $157 + m\angle Y + m\angle Z = 180$, so $m\angle Y + m\angle Z = 23$. If $m\angle Y$ was 0, then $m\angle Z$ would equal 23. But since an angle must have a measure greater than 0, $m\angle Z$ must be less than 23, so $z < 23$.

43 Use the Triangle Angle-Sum Theorem and the Addition Property to prove the statement is true.

Given: *RSTUV* is a pentagon.

Prove: $m\angle S + m\angle STU + m\angle TUV + m\angle V + m\angle VRS = 540$

Proof:



Statements (Reasons)

1. *RSTUV* is a pentagon. (Given)

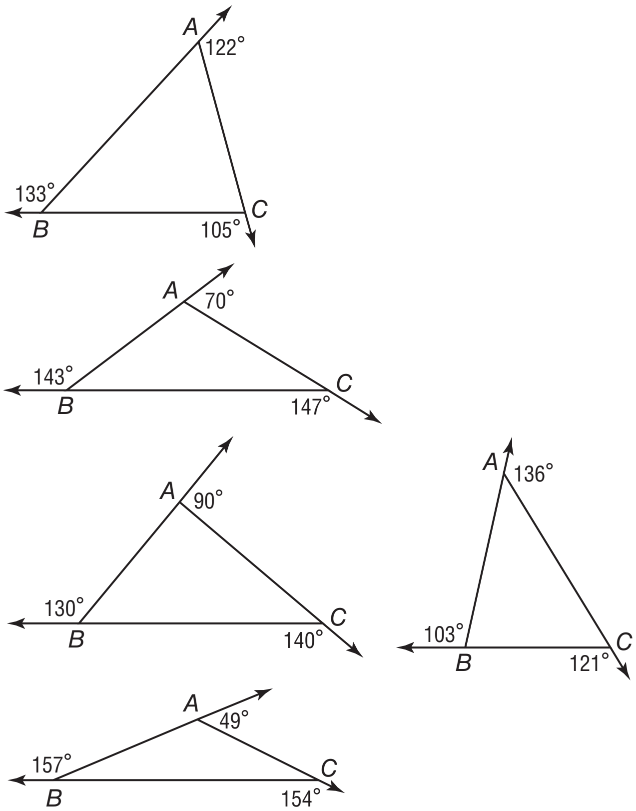
2. $m\angle S + m\angle 1 + m\angle 2 = 180$; $m\angle 3 + m\angle 4 + m\angle 7 = 180$; $m\angle 6 + m\angle V + m\angle 5 = 180$ (△ Sum. Thm.)

3. $m\angle S + m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 7 + m\angle 6 + m\angle V + m\angle 5 = 540$ (Addition Property)

4. $m\angle VRS = m\angle 1 + m\angle 4 + m\angle 5$; $m\angle TUV = m\angle 7 + m\angle 6$; $m\angle STU = m\angle 2 + m\angle 3$ (∠ Addition)

5. $m\angle S + m\angle STU + m\angle TUV + m\angle V + m\angle VRS = 540$ (Substitution)

45a. Sample answer:

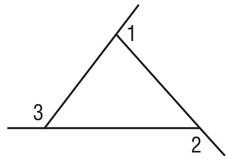


45b. Sample answer:

$m\angle 1$	$m\angle 2$	$m\angle 3$	Sum of Angle Measures
122	105	133	360
70	147	143	360
90	140	130	360
136	121	103	360
49	154	157	360

45c. Sample answer: The sum of the measures of the exterior angles of a triangle is 360.

45d. $m\angle 1 + m\angle 2 + m\angle 3 = 360$



45e. The Exterior Angle Theorem tells us that $m\angle 3 = m\angle BAC + m\angle BCA$, $m\angle 2 = m\angle BAC + m\angle CBA$, $m\angle 1 = m\angle CBA + m\angle BCA$. Through substitution, $m\angle 1 + m\angle 2 + m\angle 3 = m\angle CBA + m\angle BCA + m\angle BAC + m\angle CBA + m\angle BAC + m\angle BCA$. This can be simplified to $m\angle 1 + m\angle 2 + m\angle 3 = 2m\angle CBA + 2m\angle BCA + 2m\angle BAC$. The Distributive Property can be applied and gives $m\angle 1 + m\angle 2 + m\angle 3 = 2(m\angle CBA + m\angle BCA + m\angle BAC)$. The Triangle Angle-Sum Theorem tells us that $m\angle CBA + m\angle BCA + m\angle BAC = 180$. Through substitution we have $m\angle 1 + m\angle 2 + m\angle 3 = 2(180) = 360$.

47. The measure of $\angle a$ is the supplement of the exterior angle with measure 110, so $m\angle a = 180 - 110$ or 70. Because the angles with measures b and c are congruent, $b = c$. Using the Exterior Angle Theorem, $b + c = 110$. By substitution, $b + b = 110$, so $2b = 110$ and $b = 55$. Because $b = c$, $c = 55$. **49.** $y = 13$, $z = 14$

51. Sample answer: Since an exterior angle is acute, the adjacent angle must be obtuse. Since another exterior angle is right, the adjacent angle must be right. A triangle cannot contain both a right and an obtuse angle because it would be more than 180 degrees. Therefore, a triangle cannot have an obtuse, acute, and a right exterior angle. **53.** 100°, 115°, 145°

55. E **57.** obtuse **59.** $\sqrt{26}$ units

61. Each set of figures has one more triangle than the previous set and the direction of the triangles alternate between pointing up and and pointing to the right.

63. Multiplication Property **65.** Addition Property

67. Substitution Property

Lesson 4-3

1. $\angle Y \cong \angle S$, $\angle X \cong \angle R$, $\angle XZY \cong \angle RZS$, $\overline{YX} \cong \overline{SR}$, $\overline{YZ} \cong \overline{SZ}$, $\overline{XZ} \cong \overline{RZ}$; $\triangle YXZ \cong \triangle SRZ$ **3.** $\frac{1}{2}$ in.; Sample answer: The nut is congruent to the opening for the $\frac{1}{2}$-in. socket.

5

$\angle M \cong \angle R$	CPCTC
$m\angle M = m\angle R$	Definition of congruence
$y + 10 = 2y - 40$	Substitution
$10 = y - 40$	Subtract y from each side.
$50 = y$	Add 40 to each side.

7. 16; $\angle N$ corresponds to $\angle X$. By the Third Angles

Theorem, $m\angle N = 64$, so $4x = 64$. **9.** $\angle X \cong \angle A$, $\angle Y \cong \angle B$, $\angle Z \cong \angle C$, $\overline{XY} \cong \overline{AB}$, $\overline{XZ} \cong \overline{AC}$, $\overline{YZ} \cong \overline{BC}$; $\triangle XYZ \cong \triangle ABC$ **11.** $\angle R \cong \angle J$, $\angle T \cong \angle K$, $\angle S \cong \angle L$, $\overline{RT} \cong \overline{JK}$, $\overline{TS} \cong \overline{KL}$, $\overline{RS} \cong \overline{JL}$; $\triangle RTS \cong \triangle JKL$ **13.** 20

15

$\overline{ED} \cong \overline{UT}$	CPCTC
$ED = UT$	Definition of congruence
$3z + 10 = z + 16$	Substitution
$2z + 10 = 16$	Subtract z from each side.
$2z = 6$	Subtract 10 from each side.
$z = 3$	Divide each side by 3.

17a. $\triangle ABC \cong \triangle MNO$; $\triangle DEF \cong \triangle PQR$
17b. $\overline{AB} \cong \overline{MN}$, $\overline{BC} \cong \overline{NO}$, $\overline{AC} \cong \overline{MO}$, $\overline{DE} \cong \overline{PQ}$, $\overline{EF} \cong \overline{QR}$, $\overline{DF} \cong \overline{PR}$ **17c.** $\angle A \cong \angle M$, $\angle B \cong \angle N$, $\angle C \cong \angle O$, $\angle D \cong \angle P$, $\angle E \cong \angle Q$, $\angle F \cong \angle R$

19

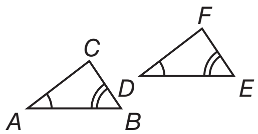
$148 + 18 + a = 180$	Triangle Angle-Sum Theorem
$166 + a = 180$	Simplify.
$a = 14$	Subtract 166 from each side.

If two angles of one triangle are congruent to two angles of another triangle, then the third angles of the triangles are congruent. So, $3x + y = 14$ and $5x - y = 18$. Solve the system of equations.

$$\begin{aligned} 3x + y &= 14 \\ (+)\ 5x - y &= 18 \\ \hline 8x \quad &= 32 \end{aligned}$$ Add the equations.
$x = 4$ Divide each side by 8.

$3x + y = 14$	Original equation
$3(4) + y = 14$	$x = 4$
$12 + y = 14$	Simplify.
$y = 2$	Subtract 12 from each side.

21. Given: $\angle A \cong \angle D$
$\angle B \cong \angle E$
Prove: $\angle C \cong \angle F$
Proof:

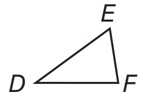


Statements (Reasons)
1. $\angle A \cong \angle D$, $\angle B \cong \angle E$ (Given)
2. $m\angle A = m\angle D$, $m\angle B = m\angle E$ (Def. of $\cong$ ∠s)
3. $m\angle A + m\angle B + m\angle C = 180$, $m\angle D + m\angle E + m\angle F = 180$ ($\angle$ Sum Theorem)
4. $m\angle A + m\angle B + m\angle C = m\angle D + m\angle E + m\angle F$ (Trans. Prop.)
5. $m\angle D + m\angle E + m\angle C = m\angle D + m\angle E + m\angle F$ (Subst.)
6. $m\angle C = m\angle F$ (Subt. Prop.)
7. $\angle C \cong \angle F$ (Def. of $\cong$ ∠s)

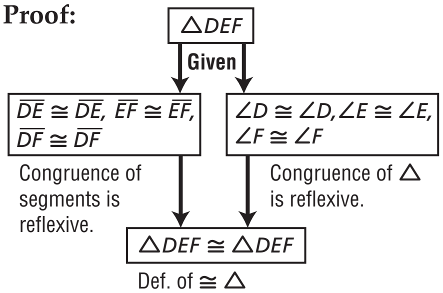
23. Given: $\overline{BD}$ bisects $\angle B$.
$\overline{BD} \perp \overline{AC}$
Prove: $\angle A \cong \angle C$
Proof:
Statements (Reasons)
1. $\overline{BD}$ bisects $\angle B$, $\overline{BD} \perp \overline{AC}$. (Given)
2. $\angle ABD \cong \angle DBC$ (Def. of angle bisector)
3. $\angle ADB$ and $\angle BDC$ are right angles. ($\perp$ lines form rt. ∠s.)
4. $\angle ADB \cong \angle BDC$ (All rt. ∠s are $\cong$.)
5. $\angle A \cong \angle C$ (Third $\angle$ Thm.)

25. Sample answer: Both of the punched flowers are congruent to the flower on the stamp, because it was used to create the images. According to the Transitive Property of Polygon Congruence, the two stamped images are congruent to each other because they are both congruent to the flowers on the punch.

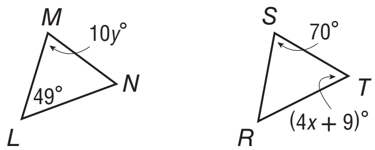
27. Given: $\triangle DEF$
Prove: $\triangle DEF \cong \triangle DEF$



Proof:



29.

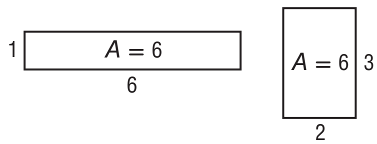


$x = 13$; $y = 7$

31 **a.** All the longer sides of the triangles are congruent and all the shorter sides are congruent. Sample answer: $\overline{AB} \cong \overline{CB}$, $\overline{AB} \cong \overline{DE}$, $\overline{AB} \cong \overline{FE}$, $\overline{CB} \cong \overline{DE}$, $\overline{CB} \cong \overline{FE}$, $\overline{DE} \cong \overline{FE}$, $\overline{AC} \cong \overline{DF}$ **b.** If the area is a square, then each of the four sides measures $\sqrt{100}$ or 10 feet. So, the perimeter of the square is 4(10) or 40 ft. The pennant string will need to be 40 ft long. **c.** Each pennant and the distance to the next pennant is 6 in. or 0.5 ft. So, the number of pennants is $40 \div 0.5$ or 80.

33a. If two triangles are congruent, then their areas are equal. **33b.** If the areas of a pair of triangles are equal, then the triangles are congruent; false; If one triangle has a base of 2 and a height of 6 and a second triangle has a base of 3 and a height of 4, then their areas are equal, but they are not congruent. **33c.** No; sample answer: Any pair of equilateral triangles that have the same base also have the same height, so it is not possible to draw a pair of equilateral triangles with the same area that are not congruent.

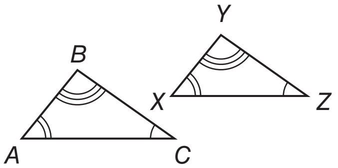
33d. yes;
sample answer:



33e. No; any pair of squares that have the same area have the same side length, which is the square root of the area. If their areas are equal, they are congruent.
33f. Regular n-gons; If two regular n-gons are congruent, then they have the same area. All regular n-gons have the same shape, but may have different sizes. If two regular n-gons have the same area, then they not only have the same shape but also the same

size. Therefore, they are congruent. **35.** diameter, radius, or circumerence; Sample answer: Two circles are the same size if they have the same diameter, radius, or circumference, so she can determine if the hoops are congruent if she measures any of them. **37.** Both; Sample answer: $\angle A$ corresponds with $\angle Y$, $\angle B$ corresponds with $\angle X$, and $\angle C$ corresponds with $\angle Z$. $\triangle CAB$ is the same triangle as $\triangle ABC$ and $\triangle ZXY$ is the same triangle as $\triangle XYZ$. **39.** $x = 16$, $y = 8$

41. False; $\angle A \cong \angle X$, $\angle B \cong \angle Y$, $\angle C \cong \angle Z$, but corresponding sides are not congruent.



43. Sometimes; Equilateral triangles will be congruent if one pair of corresponding sides are congruent. **45.** 5 **47.** C **49.** 59 **51.** $JK = 2\sqrt{146}$, $KL = \sqrt{290}$, $JL = \sqrt{146}$; scalene **53.** $JK = 5$, $KL = 5\sqrt{2}$, $JL = 5$; isosceles **55.** always **57.** complementary angles

Lesson 4-4

1a. two

1b. Given: $ABCD$ is a square

Prove: $\triangle ABC \cong \triangle CDA$

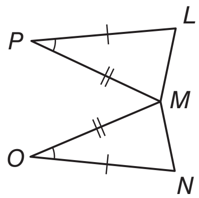
Proof:

__Statements (Reasons)__

1. $ABCD$ is a square (Given)
2. $\overline{AB} \cong \overline{CD}$, $\overline{BC} \cong \overline{DA}$ (Def. of $\cong$ a square)
3. $\overline{AC} \cong \overline{CA}$ (Reflex. Prop. $\cong$)
4. $\triangle ABC \cong \triangle CDA$ (SSS)

1c. Sample answer: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$; $\overleftrightarrow{AC}$ is a transversal to $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$, so $\angle CAB$ and $\angle ACD$ are alternate interior angles. Since $\triangle ABC \cong \triangle CDA$, $\angle CAB$ and $\angle ACD$ are congruent corresponding angles. Therefore, the lines are parallel.

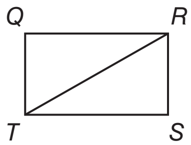
3 Sample answer: We are given that $\overline{LP} \cong \overline{NO}$ and $\angle LPM \cong \angle NOM$. Since $\triangle MOP$ is equilateral, $\overline{MO} \cong \overline{MP}$ by the definition of an equilateral triangle. So, two sides and the included angle of $\triangle LMP$ are congruent to two sides and the included angle of $\triangle NMO$. Therefore, $\triangle LMP$ is congruent to $\triangle NMO$ by the Side-Angle-Side Congruence Postulate.



5. Given: $\overline{QR} \cong \overline{SR}$ and $\overline{ST} \cong \overline{QT}$

Prove: $\triangle QRT \cong \triangle SRT$

Proof: We know that $\overline{QR} \cong \overline{SR}$ and $\overline{ST} \cong \overline{QT}$. $\overline{RT} \cong \overline{RT}$ by the Reflexive Property. Since $\overline{QR} \cong \overline{SR}$, $\overline{ST} \cong \overline{QT}$, and $\overline{RT} \cong \overline{RT}$, $\triangle QRT \cong \triangle SRT$ by SSS.



7. Given: $\overline{AB} \cong \overline{ED}$, $\angle ABC$ and $\angle EDC$ are right angles, and C is the midpoint of $\overline{BD}$.

Prove: $\triangle ABC \cong \triangle EDC$

Proof:

__Statements (Reasons)__

1. $\overline{AB} \cong \overline{ED}$, $\angle ABC$ and $\angle EDC$ are right angles, and C is the midpoint of $\overline{BD}$. (Given)
2. $\angle ABC \cong \angle EDC$ (All rt. $\angle$s $\cong$)
3. $\overline{BC} \cong \overline{DC}$ (Midpoint Thm.)
4. $\triangle ABC \cong \triangle EDC$ (SAS)

9 Use $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to find the lengths of the sides of $\triangle MNO$.

$MN = \sqrt{(-1-0)^2 + [-4-(-1)]^2}$ $(x_1, y_1) = (0, -1)$, $(x_2, y_2) = (-1, -4)$

$= \sqrt{1+9}$ or $\sqrt{10}$ Simplify.

$NO = \sqrt{[-4-(-1)]^2 + [-3-(-4)]^2}$ $(x_1, y_1) = (-1, -4)$, $(x_2, y_2) = (-4, -3)$

$= \sqrt{9+1}$ or $\sqrt{10}$ Simplify.

$MO = \sqrt{(-4-0)^2 + [-3-(-1)]^2}$ $(x_1, y_1) = (0, -1)$, $(x_2, y_2) = (-4, -3)$

$= \sqrt{16+4}$ or $\sqrt{20}$ Simplify.

Find the lengths of the sides of $\triangle QRS$.

$QR = \sqrt{(4-3)^2 + [-4-(-3)]^2}$ $(x_1, y_1) = (3, -3)$, $(x_2, y_2) = (4, -4)$

$= \sqrt{1+1}$ or $\sqrt{2}$ Simplify.

$RS = \sqrt{(3-4)^2 + [3-(-4)]^2}$ $(x_1, y_1) = (4, -4)$, $(x_2, y_2) = (3, 3)$

$= \sqrt{1+49}$ or $\sqrt{50}$ Simplify.

$QS = \sqrt{(3-3)^2 + [3-(-3)]^2}$ $(x_1, y_1) = (3, -3)$, $(x_2, y_2) = (3, 3)$

$= \sqrt{0+36}$ or 6 Simplify.

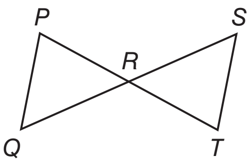
$MN = \sqrt{10}$, $NO = \sqrt{10}$, $MO = \sqrt{20}$, $QR = \sqrt{2}$, $RS = \sqrt{50}$, and $QS = 6$. The corresponding sides are not congruent, so the triangles are not congruent.

11. $MN = \sqrt{10}$, $NO = \sqrt{10}$, $MO = \sqrt{20}$, $QR = \sqrt{10}$, $RS = \sqrt{10}$, and $QS = \sqrt{20}$. Each pair of corresponding sides has the same measure, so they are congruent. $\triangle MNO \cong \triangle QRS$ by SSS.

13. Given: R is the midpoint of $\overline{QS}$ and $\overline{PT}$.

Prove: $\triangle PRQ \cong \triangle TRS$

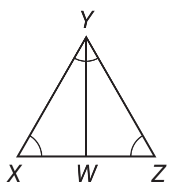
Proof: Since R is the midpoint of $\overline{QS}$ and $\overline{PT}$, $\overline{PR} \cong \overline{RT}$ and $\overline{RQ} \cong \overline{RS}$ by definition of a midpoint. $\angle PRQ \cong \angle TRS$ by the Vertical Angles Theorem. So, $\triangle PRQ \cong \triangle TRS$ by SAS.



15. Given: $\triangle XYZ$ is equilateral. $\overline{WY}$ bisects $\angle Y$.

Prove: $\overline{XW} \cong \overline{ZW}$

Proof: We know that $\overline{WY}$ bisects $\angle Y$, so $\angle XYW \cong \angle ZYW$. Also, $\overline{YW} \cong \overline{YW}$ by the Reflexive

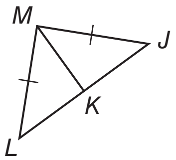


Property. Since $\triangle XYZ$ is equilateral it is a special type of isosceles triangle, so $\overline{XY} \cong \overline{ZY}$. By the Side-Angle-Side Congruence Postulate, $\triangle XYW \cong \triangle ZYW$. By CPCTC, $\overline{XW} \cong \overline{ZW}$.

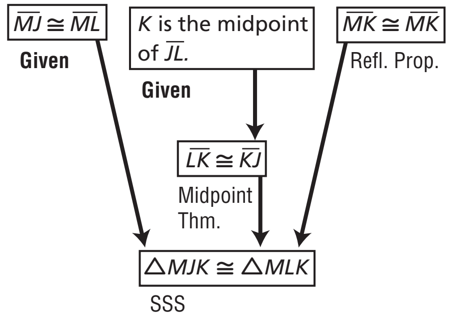
17 The triangles have two pairs of congruent sides. Since triangles cannot be proven congruent using only two sides, it is not possible to prove congruence.

19. SAS

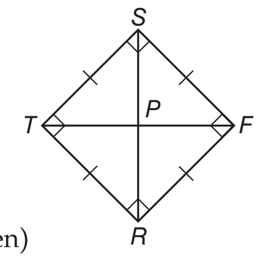
21. Given: $\overline{MJ} \cong \overline{ML}$;
K is the midpoint of $\overline{JL}$.
Prove: $\triangle MJK \cong \triangle MLK$



Proof:



23a. Given: $\overline{TS} \cong \overline{SF} \cong \overline{FR} \cong \overline{RT}$; $\angle TSF$, $\angle SFR$, $\angle FRT$, and $\angle RTS$ are right angles.
Prove: $\overline{RS} \cong \overline{TF}$

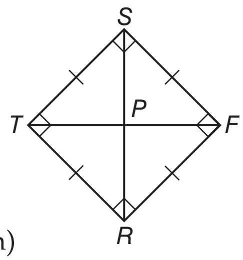


Proof:

Statements (Reasons)

1. $\overline{TS} \cong \overline{SF} \cong \overline{FR} \cong \overline{RT}$ (Given)
2. $\angle TSF$, $\angle SFR$, $\angle FRT$, and $\angle RTS$ are right angles. (Given)
3. $\angle STR \cong \angle TRF$ (All rt. ∡ are ≅.)
4. $\triangle STR \cong \triangle TRF$ (SAS)
5. $\overline{RS} \cong \overline{TF}$ (CPCTC)

23b. Given: $\overline{TS} \cong \overline{SF} \cong \overline{FH} \cong \overline{HT}$; $\angle TSF$, $\angle SFH$, $\angle FHT$, and$\angle HTS$ are right angles.
Prove: $\angle SRT \cong \angle SRF$

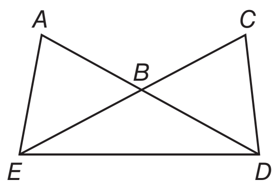


Proof:

Statements (Reasons)

1. $\overline{TS} \cong \overline{SF} \cong \overline{FR} \cong \overline{RT}$ (Given)
2. $\angle TSF$, $\angle SFR$, $\angle FRT$, and $\angle RTS$ are right angles. (Given)
3. $\angle STR \cong \angle SFR$ (All rt. ∡ are ≅.)
4. $\triangle STR \cong \triangle SFR$ (SAS)
5. $\angle SRT \cong \angle SRF$ (CPCTC)

25. Given: $\triangle EAB \cong \triangle DCB$
Prove: $\triangle EAD \cong \triangle DCE$



Proof:

Statements (Reasons)

1. $\triangle EAB \cong \triangle DCB$ (Given)
2. $\overline{EA} \cong \overline{DC}$ (CPCTC)
3. $\overline{ED} \cong \overline{DE}$ (Reflex. Prop.)
4. $\overline{AB} \cong \overline{CB}$ (CPCTC)
5. $\overline{DB} \cong \overline{EB}$ (CPCTC)
6. $AB = CB$, $DB = EB$ (Def. ≅ segments)
7. $AB + DB = CB + EB$ (Add. Prop. =)
8. $AD = AB + DB$, $CE = CB + EB$ (Seg. addition)
9. $AD = CE$ (Subst. Prop. =)
10. $\overline{AD} \cong \overline{CE}$ (Def. ≅ segments)
11. $\triangle EAD \cong \triangle DCE$ (SSS)

27 Given that $\triangle WXY \cong \triangle WXZ$, by CPCTC, $\angle WXZ \cong \angle WXY$ and $\overline{XY} \cong \overline{XZ}$.

$\angle WXZ \cong \angle WXY$	CPCTC
$m\angle WXZ = m\angle WXY$	Definition of congruence
$90 = 20y + 10$	Substitution
$80 = 20y$	Subtract 10 from each side.
$y = 4$	Divide each side by 20.

$\overline{XY} \cong \overline{XZ}$	CPCTC
$XY = XZ$	Definition of congruence
$3y + 7 = 19$	Substitution
$3y = 12$	Subtract 7 from each side.
$y = 4$	Divide each side by 3.

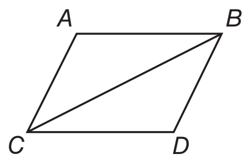
29a. Sample answer: Method 1: You could use the Distance Formula to find the length of each of the sides, and then use the Side-Side-Side Congruence Postulate to prove the triangles congruent. Method 2: You could find the slopes of $\overline{ZX}$ and $\overline{WY}$ to prove that they are perpendicular and that $\angle WYZ$ and $\angle WYX$ are both right angles. You can use the Distance Formula to prove that $\overline{XY}$ is congruent to $\overline{ZY}$. Since the triangles share the leg $\overline{WY}$, you can use the Side-Angle-Side Congruence Postulate; Sample answer: I think that method 2 is more efficient, because you only have two steps instead of three.

29b. Sample answer: Yes; the slope of $\overline{WY}$ is -1 and the slope of $\overline{ZX}$ is 1, and -1 and 1 are opposite reciprocals, so $\overline{WY}$ is perpendicular to $\overline{ZX}$. Since they are perpendicular, $\angle WYZ$ and $\angle WYX$ are both 90°. Using the Distance Formula, the length of $\overline{ZY}$ is $\sqrt{(4-1)^2 + (5-2)^2}$ or $3\sqrt{2}$, and the length of $\overline{XY}$ is $\sqrt{(7-4)^2 + (8-5)^2}$ or $3\sqrt{2}$. Since $\overline{WY}$ is congruent to $\overline{WY}$, $\triangle WYZ$ is congruent to $\triangle WYX$ by the Side-Angle-Side Congruence Postulate. **31.** Shada; for SAS the angle must be the included angle and here it is not included. **33.** Case 1: You know the hypotenuses are congruent and two corresponding legs are congruent. Then the Pythagorean Theorem says that the other legs are congruent so the triangles are congruent by SSS. Case 2: You know the legs are congruent and the right angles are congruent, then the triangles are congruent by SAS. **35.** F **37.** D **39.** 18

41. $y = -\frac{1}{5}x - 4$ **43.** $y = x + 3$ **45.** False; a 16-year-old could be a freshman, sophomore, junior, or senior. The hypothesis of the conditional is true, but the conclusion is false. This counterexample shows that the conditional statement is false. **47.** Trans. Prop. **49.** Substitution

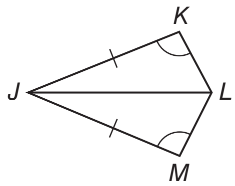
Lesson 4-5

1. Given: $\overline{CB}$ bisects $\angle ABD$ and $\angle ACD$.
Prove: $\triangle ABC \cong \triangle DBC$



Proof:
Statements (Reasons)
1. $\overline{CB}$ bisects $\angle ABD$ and $\angle ACD$. (Given)
2. $\angle ABC \cong \angle DBC$ (Def. of $\angle$ bisector)
3. $\overline{BC} \cong \overline{BC}$ (Refl. prop.)
4. $\angle ACB \cong \angle DCB$ (Def. of $\angle$ bisector)
5. $\triangle ABC \cong \triangle DBC$ (ASA)

3. Given: $\angle K \cong \angle M$, $\overline{JK} \cong \overline{JM}$, $\overline{JL}$ bisects $\angle KLM$.
Prove: $\triangle JKL \cong \triangle JML$



Proof: We are given $\angle K \cong \angle M$, $\overline{JK} \cong \overline{JM}$, and $\overline{JL}$ bisects $\angle KLM$. Since $\overline{JL}$ bisects $\angle KLM$, we know $\angle KLJ \cong \angle MLJ$. So, $\triangle JKL \cong \triangle JML$ is congruent by the AAS Congruence Theorem.

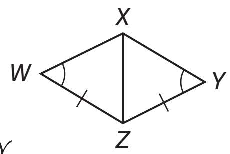
5 **a.** We know $\angle BAE$ and $\angle DCE$ are congruent because they are both right angles. $\overline{AE}$ is congruent to $\overline{EC}$ by the Midpoint Theorem. From the Vertical Angles Theorem, $\angle DEC \cong \angle BEA$. So, two angles and the included side of $\triangle DCE$ are congruent to two angles and the included side of $\triangle BAE$. By ASA, the surveyor knows that $\triangle DCE \cong \triangle BAE$. By CPCTC, $\overline{DC} \cong \overline{AB}$, so the surveyor can measure $\overline{DC}$ and know the distance between A and B.

b.

$\triangle DCE \cong \triangle BAE$	SAS
$\overline{DC} \cong \overline{AB}$	CPCTC
$DC = AB$	Definition of congruence
$550 = AB$	Substitution

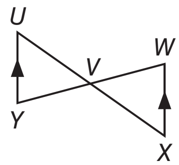
So, by the definition of congruence, $AB = 550$ m.

7. Given: $\angle W \cong \angle Y$, $\overline{WZ} \cong \overline{YZ}$, $\overline{XZ}$ bisects $\angle WZY$.
Prove: $\triangle XWZ \cong \triangle XYZ$



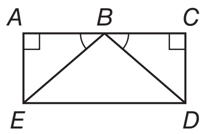
Proof: It is given that $\angle W \cong \angle Y$, $\overline{WZ} \cong \overline{YZ}$, and $\overline{XZ}$ bisects $\angle WZY$. By the definition of angle bisector, $\angle WZX \cong \angle YZX$. The Angle-Side-Angle Congruence Postulate tells us that $\triangle XWZ \cong \triangle XYZ$.

9 Use the Alternate Interior Angle Theorem and AAS to prove the triangles congruent.
Given: V is the midpoint of $\overline{YW}$; $\overline{UY} \parallel \overline{XW}$.
Prove: $\triangle UVY \cong \triangle XVW$

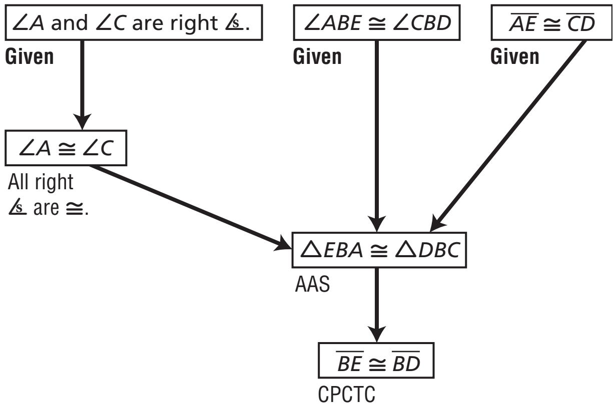


Proof:
Statements (Reasons)
1. V is the midpoint of $\overline{YW}$; $\overline{UY} \parallel \overline{XW}$. (Given)
2. $\overline{YV} \cong \overline{VW}$ (Midpoint Theorem)
3. $\angle VWX \cong \angle VYU$ (Alt. Int. $\angle$s Thm.)
4. $\angle VUY \cong \angle VXW$ (Alt. Int. $\angle$s Thm.)
5. $\triangle UVY \cong \triangle XVW$ (AAS)

11. Given: $\angle A$ and $\angle C$ are right angles. $\angle ABE \cong \angle CBD$, $\overline{AE} \cong \overline{CD}$
Prove: $\overline{BE} \cong \overline{BD}$



Proof:

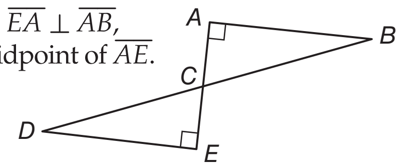


13a. $\angle HJK \cong \angle GFK$ since all right angles are congruent. We are given that $\overline{JK} \cong \overline{KF}$. $\angle HKJ$ and $\angle FKG$ are vertical angles, so $\angle HKJ \cong \angle FKG$ by the Vertical Angles Theorem. By ASA, $\triangle HJK \cong \triangle GFK$, so $\overline{FG} \cong \overline{HJ}$ by CPCTC. **13b.** No; $HJ = 1350$ m, so $FG = 1350$ m. If the regatta is to be 1500 m, the lake is not long enough, since $1350 < 1500$.

15 If the triangles are congruent, $HJ = QJ$. Solve for y.

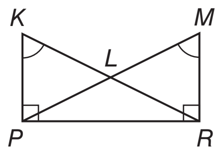
$HJ = QJ$	CPCTC
$9 = 2y - 1$	$HJ = 9$, $QJ = 2y - 1$
$y = 5$	Simplify.

17. Given: $\overline{AE} \perp \overline{DE}$, $\overline{EA} \perp \overline{AB}$, C is the midpoint of $\overline{AE}$.
Prove: $\overline{CD} \cong \overline{CB}$



Proof: We are given that $\overline{AE}$ is perpendicular to $\overline{DE}$, $\overline{EA}$ is perpendicular to $\overline{AB}$, and C is the midpoint of $\overline{AE}$. Since $\overline{AE}$ is perpendicular to $\overline{DE}$, $m\angle CED = 90$. Since $\overline{EA}$ is perpendicular to $\overline{AB}$, $m\angle BAC = 90$. $\angle CED \cong \angle BAC$ because all right angles are congruent. $\overline{AC} \cong \overline{CE}$ from the Midpt. Thm. $\angle ECD \cong \angle ACB$ because they are vertical angles. Angle-Side-Angle gives us that $\triangle CED \cong \triangle CAB$. $\overline{CD} \cong \overline{CB}$ because corresponding parts of congruent triangles are congruent.

19. Given: $\angle K \cong \angle M$, $\overline{KP} \perp \overline{PR}$, $\overline{MR} \perp \overline{PR}$
Prove: $\angle KPL \cong \angle MNL$

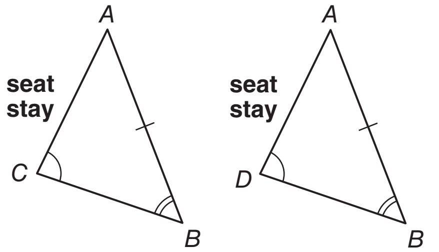


Proof:
Statements (Reasons)
1. $\angle K \cong \angle M$, $\overline{KP} \perp \overline{PR}$, $\overline{MR} \perp \overline{PR}$ (Given)
2. $\angle KPR$ and $\angle MRP$ are both right angles. (Def. of $\perp$)
3. $\angle KPR \cong \angle MRP$ (All rt. $\angle$s are congruent.)
4. $\overline{PR} \cong \overline{PR}$ (Refl. Prop.)
5. $\triangle KPR \cong \triangle MRP$ (AAS)
6. $\overline{KP} \cong \overline{MR}$ (CPCTC)
7. $\angle KLP \cong \angle MLR$ (Vertical angles are $\cong$.)
8. $\triangle KLP \cong \triangle MLR$ (AAS)

Selected Answers and Solutions

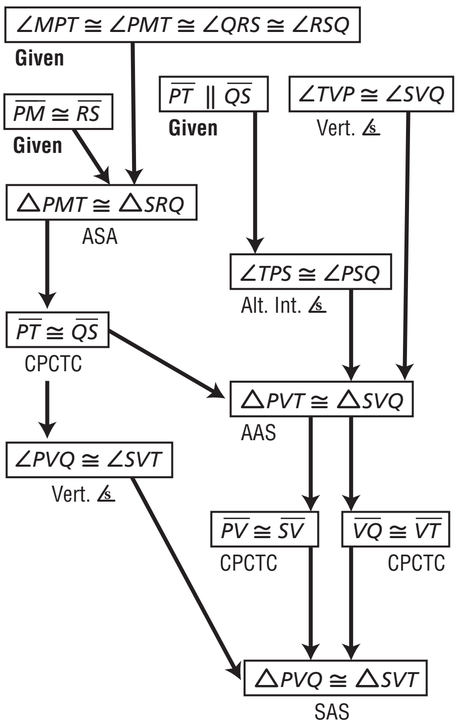
9. $\angle KPL \cong \angle MRL$ (CPCTC)

21 Since $m\angle ACB = m\angle ADB = 44$, and $m\angle CBA = m\angle DBA = 68$, then $\angle ACB \cong \angle ADB$ and $\angle CBA \cong \angle DBA$ by the definition of congruence. $\overline{AB} \cong \overline{AB}$ by the Reflexive Property, so $\triangle ACB \cong \triangle ADB$ by AAS. Then $\overline{AC} \cong \overline{AD}$ by CPCTC. Since $AC = AD$ by the definition of congruence, the two seat stays are the same length.



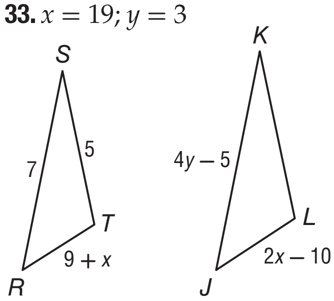
23. Tyrone; Lorenzo showed that all three corresponding angles were congruent, but AAA is not a proof of triangle congruence.

25.



27. B **29.** J **31.** $AB = \sqrt{125}$, $BC = \sqrt{221}$, $AC = \sqrt{226}$, $XY = \sqrt{125}$, $YZ = \sqrt{221}$, $XZ = \sqrt{226}$. The corresponding sides have the same measure and are congruent. $\triangle ABC \cong \triangle XYZ$ by SSS.

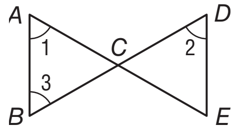
33. $x = 19$; $y = 3$



35.

p	q	$\sim p$	$\sim p \vee q$
F	T	T	T
T	T	F	T
F	F	T	T
T	F	F	F

37. Given: $\angle 2 \cong \angle 1$
$\angle 1 \cong \angle 3$
Prove: $\overline{AB} \parallel \overline{DE}$



<u>**Statements (Reasons)**</u>
1. $\angle 2 \cong \angle 1$, $\angle 1 \cong \angle 3$ (Given)
2. $\angle 2 \cong \angle 3$ (Trans. Prop.)
3. $\overline{AB} \parallel \overline{DE}$ (If alt. int. ∠s are ≅, lines are ∥.)

Lesson 4-6

1. $\angle BAC$ and $\angle BCA$ **3.** 12 **5.** 12

7. Given: $\triangle ABC$ is isosceles; $\overline{EB}$ bisects $\angle ABC$.
Prove: $\triangle ABE \cong \triangle CBE$
Proof:
<u>**Statements (Reasons)**</u>
1. $\triangle ABC$ is isosceles; $\overline{EB}$ bisects $\angle ABC$. (Given)
2. $\overline{AB} \cong \overline{BC}$ (Def. of isosceles)
3. $\angle ABE \cong \angle CBE$ (Def. of ∠ bisector)
4. $\overline{BE} \cong \overline{BE}$ (Refl. Prop.)
5. $\triangle ABE \cong \triangle CBE$ (SAS)

9 $\angle ABE$ is opposite $\overline{AE}$ and $\angle AEB$ is opposite $\overline{AB}$. Since $\overline{AE} \cong \overline{AB}$, $\angle ABE \cong \angle AEB$.

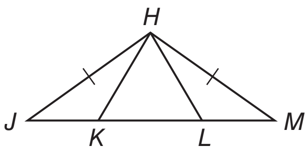
11. $\angle ACD$ and $\angle ADC$ **13.** $\overline{BF}$ and $\overline{BC}$ **15.** 60 **17.** 4

19 The triangle is equiangular, so it is also equilateral. All the sides are congruent.

$2x + 11 = 6x - 9$	Definition of congruence
$11 = 4x - 9$	Subtract $2x$ from each side.
$20 = 4x$	Add 9 to each side.
$5 = x$	Divide each side by 4.

21. $x = 11$, $y = 11$

23. Given: $\triangle HJM$ is an isosceles triangle and $\triangle HKL$ is an equilateral triangle. $\angle JKH$, $\angle HKL$ and $\angle HLK$, $\angle MLH$ are supplementary.



Prove: $\angle JHK \cong \angle MHL$
Proof: We are given that $\triangle HJM$ is an isosceles triangle and $\triangle HKL$ is an equilateral triangle, $\angle JKH$ and $\angle HKL$ are supplementary and $\angle HLK$ and $\angle MLH$ are supplementary. From the Isosceles Triangle Theorem, we know that $\angle HJK \cong \angle HML$. Since $\triangle HKL$ is an equilateral triangle, we know $\angle HLK \cong \angle LKH \cong \angle KHL$ and $\overline{HL} \cong \overline{KL} \cong \overline{HK}$. $\angle JKH$, $\angle HKL$ and $\angle HLK$, $\angle MLH$ are supplementary, and $\angle HKL \cong \angle HLK$, we know $\angle JKH \cong \angle MLH$ by the Congruent Supplements Theorem. By AAS, $\triangle JHK \cong \triangle MLH$. By CPCTC, $\angle JHK \cong \angle MHL$.

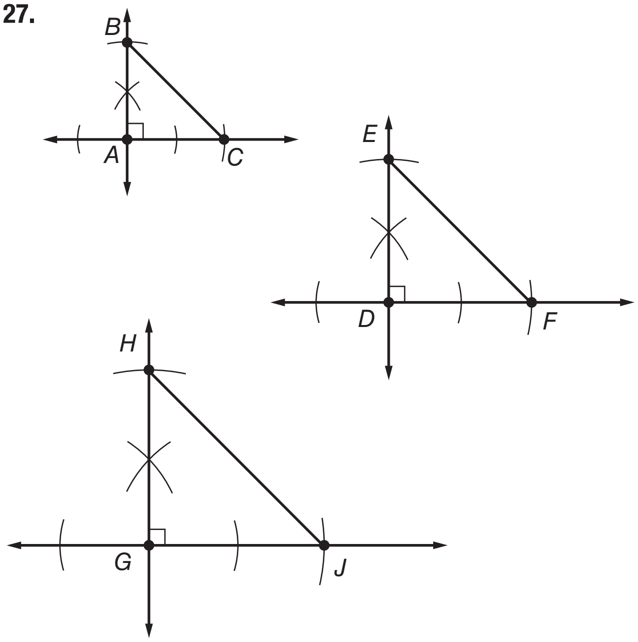
25a. 65°; Since $\triangle ABC$ is isosceles, $\angle ABC \cong \angle ACB$, so $180 - 50 = 130$ and $\frac{130}{2}$ or 65.

25b. Given: $\overline{BE} \cong \overline{CD}$
Prove: $\triangle AED$ is isosceles.
Proof:
<u>**Statements (Reasons)**</u>
1. $\overline{AB} \cong \overline{AC}$, $\overline{BE} \cong \overline{CD}$ (Given)
2. $AB = AC$, $BE = CD$ (Def. of congruence)
3. $AB + BE = AE$, $AC + CD = AD$ (Seg. Add. Post.)
4. $AB + BE = AC + CD$ (Add. Prop. of Eq.)
5. $AE = AD$ (Subst.)
6. $\overline{AE} \cong \overline{AD}$ (Def. of congruence)
7. $\triangle AED$ is isosceles. (Def. of isosceles)

25c. Given: $\overline{BC} \parallel \overline{ED}$ and $\overline{ED} \cong \overline{AD}$
Prove: $\triangle ADE$ is equilateral.
Proof:
<u>Statements (Reasons)</u>
1. $\overline{AB} \cong \overline{AC}$, $\overline{BC} \parallel \overline{ED}$ and $\overline{ED} \cong \overline{AD}$ (Given)
2. $\angle ABC \cong \angle ACB$ (Isos. $\triangle$ Thm.)
3. $m\angle ABC = m\angle ACB$ (Def. of $\cong$)
4. $\angle ABC \cong \angle AED$, $\angle ACB \cong \angle ADE$ (Corr. ∡ Thm.)
5. $m\angle ABC = m\angle AED$, $m\angle ACB = m\angle ADE$ (Def. of $\cong$)
6. $m\angle AED = m\angle ACB$ (Subst.)
7. $m\angle AED = m\angle ADE$ (Subst.)
8. $\angle AED \cong \angle ADE$ (Def. of $\cong$)
9. $\overline{AD} \cong \overline{AE}$ (Conv. of Isos. $\triangle$ Thm.)
10. $\triangle ADE$ is equilateral. (Def. of equilateral $\triangle$)

25d. One pair of congruent corresponding sides and one pair of congruent corresponding angles; since you know that the triangle is isosceles, if one leg is congruent to a leg of $\triangle ABC$, then you know that both pairs of legs are congruent. Because the base angles of an isosceles triangle are congruent, if you know that $\angle K \cong \angle B$ you know that $\angle K \cong \angle L$, $\angle B \cong \angle C$, and $\angle C \cong \angle L$. Therefore, with one pair of congruent corresponding sides and one pair of congruent corresponding angles, the triangles can be proved congruent using either ASA or SAS.

27.



Sample answer: I constructed a pair of perpendicular segments and then used the same compass setting to mark points equidistant from their intersection. I measured both legs for each triangle. Since $AB = AC = 1.3$ cm, $DE = DF = 1.9$ cm, and $GH = GJ = 2.3$ cm, the triangles are isosceles. I used a protractor to confirm that $\angle A$, $\angle D$, and $\angle G$ are all right angles.

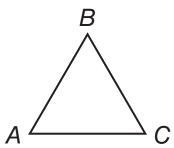
29 Since $\overline{AD} \cong \overline{CD}$, base angles CAD and ACD are congruent by the Isosceles Triangle Theorem. So, $m\angle CAD = m\angle ACD$.

$m\angle CAD + m\angle ACD + m\angle D = 180$ — Triangle Angle-Sum Theorem
$m\angle CAD + m\angle CAD + 92 = 180$ — Substitution
$2m\angle CAD + 92 = 180$ — Simplify.
$2m\angle CAD = 88$ — Subtract 92 from each side.
$m\angle CAD = 44$ — Simplify.

31. 136

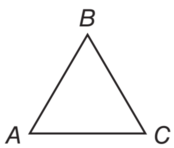
33. Given: Each triangle is isosceles, $\overline{BG} \cong \overline{HC}$, $\overline{HD} \cong \overline{JF}$, $\angle G \cong \angle H$, and $\angle H \cong \angle J$.
Prove: The distance from B to F is three times the distance from D to F.
Proof:
<u>Statements (Reasons)</u>
1. Each triangle is isosceles, $\overline{BG} \cong \overline{HC}$, $\overline{HD} \cong \overline{JF}$, $\angle G \cong \angle H$, and $\angle H \cong \angle J$. (Given)
2. $\angle G \cong \angle J$ (Trans. Prop.)
3. $\overline{BG} \cong \overline{CG}$, $\overline{HC} \cong \overline{HD}$, $\overline{JD} \cong \overline{JF}$ (Def. of Isosceles)
4. $\overline{BG} \cong \overline{JD}$ (Trans. Prop.)
5. $\overline{HC} \cong \overline{JD}$ (Trans. Prop.)
6. $\overline{CG} \cong \overline{JF}$ (Trans. Prop.)
7. $\triangle BCG \cong \triangle CDH \cong \triangle DFJ$ (SAS)
8. $\overline{BC} \cong \overline{CD} \cong \overline{DF}$ (CPCTC)
9. $BC = CD = DF$ (Def. of congruence)
10. $BC + CD + DF = BF$ (Seg. Add. Post.)
11. $DF + DF + DF = BF$ (Subst.)
12. $3DF = BF$ (Addition)

35. Case I
Given: $\triangle ABC$ is an equilateral triangle.
Prove: $\triangle ABC$ is an equiangular triangle.



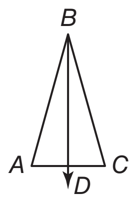
Proof:
<u>Statements (Reasons)</u>
1. $\triangle ABC$ is an equilateral triangle. (Given)
2. $\overline{AB} \cong \overline{AC} \cong \overline{BC}$ (Def. of equilateral $\triangle$)
3. $\angle A \cong \angle B \cong \angle C$ (Isosceles $\triangle$ Th.)
4. $\triangle ABC$ is an equiangular triangle. (Def. of equiangular)

Case II
Given: $\triangle ABC$ is an equiangular triangle.
Prove: $\triangle ABC$ is an equilateral triangle.



Proof:
<u>Statements (Reasons)</u>
1. $\triangle ABC$ is an equiangular triangle. (Given)
2. $\angle A \cong \angle B \cong \angle C$ (Def. of equiangular $\triangle$)
3. $\overline{AB} \cong \overline{AC} \cong \overline{BC}$ (If 2 ∡ of a $\triangle$ are $\cong$ then the sides opp. those ∡ are $\cong$.)
4. $\triangle ABC$ is an equilateral triangle. (Def. of equilateral)

37. Given: $\triangle ABC$, $\angle A \cong \angle C$
Prove: $\overline{AB} \cong \overline{CB}$



Proof:
<u>Statements (Reasons)</u>
1. Let $\overrightarrow{BD}$ bisect $\angle ABC$. (Protractor Post.)
2. $\angle ABD \cong \angle CBD$ (Def. of $\angle$ bisector)
3. $\angle A \cong \angle C$ (Given)

4. $\overline{BD} \cong \overline{BD}$ (Refl. Prop.)
5. $\triangle ABD \cong \triangle CBD$ (AAS)
6. $\overline{AB} \cong \overline{CB}$ (CPCTC)

39. 14

41

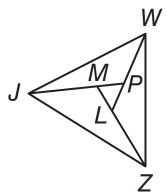
$m\angle LPM + m\angle LPQ = 180$	Supplement Theorem
$(3x - 55) + (2x + 10) = 180$	Substitution
$5x - 45 = 180$	Simplify.
$5x = 225$	Add 45 to each side.
$x = 45$	Divide each side by 45.

$m\angle LPM = 3x - 55$	Given
$= 3(45) - 55$	Substitution
$= 135 - 55$ or 80	Simplify.

Because $\overline{LM} \cong \overline{LP}$, base angles LMP and LPM are congruent by the Isosceles Triangle Theorem. So, $m\angle LMP = m\angle LPM = 80$.

43. 80

45. Given: $\triangle WJZ$ is equilateral, and $\angle ZWP \cong \angle WJM \cong \angle JZL$.
Prove: $\overline{WP} \cong \overline{ZL} \cong \overline{JM}$



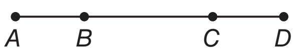
Proof: We know that $\triangle WJZ$ is equilateral, since an equilateral $\triangle$ is equiangular, $\angle ZWJ \cong \angle WJZ \cong \angle JZW$. So, $m\angle ZWJ = m\angle WJZ = m\angle JZW$, by the definition of congruence. Since $\angle ZWP \cong \angle WJM \cong \angle JZL$, $m\angle ZWP = m\angle WJM = m\angle JZL$, by the definition of congruence. By the Angle Addition Postulate, $m\angle ZWJ = m\angle ZWP + m\angle PWJ$, $m\angle WJZ = m\angle WJM + m\angle MJZ$, $m\angle JZW = m\angle JZL + m\angle LZW$. By substitution, $m\angle ZWP + m\angle PWJ = m\angle WJM + m\angle MJZ = m\angle JZL + m\angle LZW$. Again by substitution, $m\angle ZWP + m\angle PWJ = m\angle ZWP + m\angle PJZ = m\angle ZWP + m\angle LZW$. By the Subtraction Property, $m\angle PWJ = m\angle PJZ = m\angle LZW$. By the definition of congruence, $\angle PWJ \cong \angle PJZ \cong \angle LZW$. So, by ASA, $\triangle WZL \cong \triangle ZJM \cong \triangle JWP$. By CPCTC, $\overline{WP} \cong \overline{ZL} \cong \overline{JM}$.

47. Never; the measure of the vertex angle will be 180 − 2(measure of the base angle) so if the base angles are integers, then 2(measure of the base angle) will be even and 180 − 2(measure of the base angle) will be even. **49.** It is not possible because a triangle cannot have more than one obtuse angle.

51. Sample answer: If a triangle is already classified, you can use the previously proven properties of that type of triangle in the proof. Doing this can save you steps when writing the proof. **53.** 185 **55.** E

57. $SU = \sqrt{17}$, $TU = \sqrt{2}$, $ST = 5$, $XZ = \sqrt{29}$, $YZ = 2$, $XY = 5$; the corresponding sides are not congruent; the triangles are not congruent.

59. Given: $AC = BD$
Prove: $AB = CD$



1. $AC = BD$ (Given)
2. $AC = AB + BC$, $BD = BC + CD$ (Seg. Add. Post.)
3. $AB + BC = BC + CD$ (Subst.)
4. $\overline{BC} \cong \overline{BC}$ (Reflexive)
5. $BC = BC$ (Def. of ≅ Segs.)
6. $AB = CD$ (Subt. Prop.)

61. Add. Prop. **63.** Trans. Prop. **65.** *A, K, B* or *B, J, C*

67. Given: $\angle ACB \cong \angle ABC$
Prove: $\angle XCA \cong \angle YBA$

Statements (Reasons)

1. $\angle ACB \cong \angle ABC$ (Given)
2. $\angle XCA$ and $\angle ACB$ are a linear pair. $\angle ABC$ and $\angle ABY$ are a linear pair. (Def. of linear pair)
3. $\angle XCA$, $\angle ACB$ and $\angle ABC$, $\angle ABY$ are supplementary. (Suppl. Thm.)
4. $\angle XCA \cong \angle YBA$ (∡ suppl. to ≅ ∡ are ≅.)

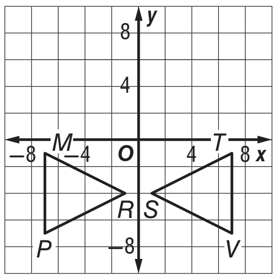
Lesson 4-7

1. translation **3.** reflection **5.** $\triangle LKJ$ is a reflection of $\triangle XYZ$. $XY = 7$, $YZ = 8$, $XZ = \sqrt{113}$, $KJ = 8$, $LJ = \sqrt{113}$, $LK = 7$. $\triangle XYZ \cong \triangle LKJ$ by SSS. **7.** reflection

9 The image in green can be found by translating the blue figure to the right and up, by reflecting the figure in the line $y = -x$, or by rotating the figure 180° in either direction.

11. rotation **13.** translation **15.** rotation

17



Use $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to find the lengths of the sides of $\triangle MPR$.

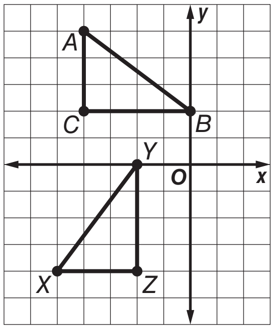
$MP = \sqrt{[-7-(-7)]^2 + [-7-(-1)]^2}$	$(x_1, y_1) = (-7, -1)$, $(x_2, y_2) = (-7, -7)$
$= \sqrt{0 + 36}$ or 6	Simplify.
$PR = \sqrt{[-1-(-7)]^2 + [-4-(-7)]^2}$	$(x_1, y_1) = (-7, -7)$, $(x_2, y_2) = (-1, -4)$
$= \sqrt{36 + 9}$ or $\sqrt{45}$	Simplify.
$MR = \sqrt{[-1-(-7)]^2 + [-4-(-1)]^2}$	$(x_1, y_1) = (-7, -1)$, $(x_2, y_2) = (-1, -4)$
$= \sqrt{36 + 9}$ or $\sqrt{45}$	Simplify.

Find the lengths of the sides of $\triangle TVS$.

$ST = \sqrt{(7-1)^2 + [-1-(-4)]^2}$	$(x_1, y_1) = (1, -4)$, $(x_2, y_2) = (7, -1)$
$= \sqrt{36 + 9}$ or $\sqrt{45}$	Simplify.
$TV = \sqrt{(7-7)^2 + [-7-(-1)]^2}$	$(x_1, y_1) = (7, -1)$, $(x_2, y_2) = (7, -7)$
$= \sqrt{0 + 36}$ or 6	Simplify.
$SV = \sqrt{(7-1)^2 + [-7-(-4)]^2}$	$(x_1, y_1) = (1, -4)$, $(x_2, y_2) = (7, -7)$
$= \sqrt{36 + 9}$ or $\sqrt{45}$	Simplify.

In $\triangle MPR$, $MP = 6$, $PR = \sqrt{45}$, and $MR = \sqrt{45}$. In

$\triangle TVS$, $ST = \sqrt{45}$, $TV = 6$, $SV = \sqrt{45}$. $\triangle MPR \cong \triangle TVS$ by SSS. $\triangle TVS$ is a reflection of $\triangle MPR$.

19. 

$\triangle XYZ$ is a rotation of $\triangle ABC$. $AB = 5$, $BC = 4$, $AC = 3$, $XY = 5$, $YZ = 4$, $XZ = 3$. Since $AB = XY$, $BC = YZ$, and $AC = XZ$, $\overline{AB} \cong \overline{XY}$, $\overline{BC} \cong \overline{YZ}$, and $\overline{AC} \cong \overline{XZ}$, $\triangle ABC \cong \triangle XYZ$ by SSS.

21. rotation

23. reflection

25. rotation

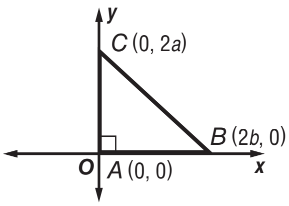
27. Rotation; the knob is the center of rotation.

29 **a.** Tionne used the stencil on one side, then flipped it and used the other side, then flipped it again to create the third flower in the design. She could have also used the stencil, then turned it to create the second flower, and turned it again to create the third flower. So, she could have used reflections or rotations. **b.** Tionne used the stamp, then turned it to create the second flower, and turned it again to create the third flower. So, she used rotations.

31a. translation, reflection **31b.** Sample answer: The triangles must be either isosceles or equilateral. When triangles are isosceles or equilateral, they have a line of symmetry, so reflections result in the same figure. **33.** Sample answer: A person looking in a mirror sees a reflection of himself or herself.

35. Sample answer: A faucet handle rotates when you turn the water on. **37.** no; 75% **39.** J **41.** 4 **43.** 10 **45.** yes; Law of Detachment **47.** (8, 9.5) **49.** (−7.5, 3.5) **51.** (1, 9.5)

Lesson 4-8

1. 

3. $T(2a, 0)$

5. $DC = \sqrt{[-a-(-a)]^2 + (b-0)^2}$ or b

$GH = \sqrt{(a-a)^2 + (b-0)^2}$ or b

Since $DC = GH$, $\overline{DC} \cong \overline{GH}$.

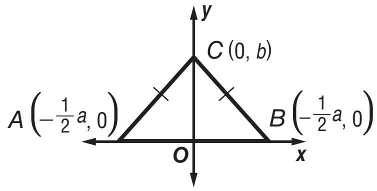
$DF = \sqrt{(0-a)^2 + \left(\frac{b}{2} - b\right)^2}$ or $\sqrt{a^2 + \frac{b^2}{4}}$

$GF = \sqrt{(a-0)^2 + \left(b - \frac{b}{2}\right)^2}$ or $\sqrt{a^2 + \frac{b^2}{4}}$

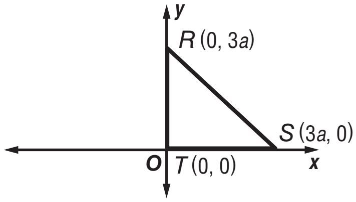
$CF = \sqrt{(0-a)^2 + \left(\frac{b}{2} - 0\right)^2}$ or $\sqrt{a^2 + \frac{b^2}{4}}$

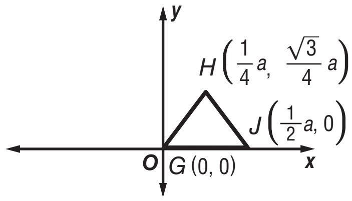
$HF = \sqrt{(a-0)^2 + \left(0 - \frac{b}{2}\right)^2}$ or $\sqrt{a^2 + \frac{b^2}{4}}$

Since $DF = GF = CF = HF$, $\overline{DF} \cong \overline{GF} \cong \overline{CF} \cong \overline{HF}$. $\triangle FGH \cong \triangle FDC$ by SSS.

7. 

9 Since this is a right triangle, each of the legs can be located on an axis. Placing the right angle of the triangle, $\angle T$, at the origin will allow the two legs to be along the x- and y-axes. Position the triangle in the first quadrant. Since R is on the y-axis, its x-coordinate is 0. Its y-coordinate is $3a$ because the leg is $3a$ units long. Since S is on the x-axis, its y-coordinate is 0. Its x-coordinate is $3a$ because the leg is $3a$ units long.



11. 

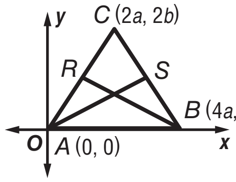
13. $C(a, a)$, $Y(a, 0)$

15 Vertex N is positioned at the origin. So, its coordinates are (0, 0). Vertex L is on the x-axis, so its y-coordinate is 0. The coordinates of vertex L are $(3a, 0)$. $\triangle NJL$ is isosceles, so the x-coordinate of J is located halfway between 0 and $3a$, or $1.5a$. The coordinates of vertex J are $(1.5a, b)$. So, the vertices are $N(0, 0)$, $J(1.5a, b)$, $L(3a, 0)$.

17. $H(2b, 2b\sqrt{3})$, $N(0, 0)$, $D(4b, 0)$

19. Given: Isosceles $\triangle ABC$ with $\overline{AC} \cong \overline{BC}$; R and S are midpoints of legs $\overline{AC}$ and $\overline{BC}$.

Prove: $\overline{AS} \cong \overline{BR}$



Proof:

The coordinates of S are $\left(\frac{2a+4a}{2}, \frac{2b+0}{2}\right)$ or $(3a, b)$.

The coordinates of R are $\left(\frac{2a+0}{2}, \frac{2b+0}{2}\right)$ or (a, b).

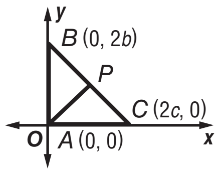
$AS = \sqrt{(3a-0)^2 + (b-0)^2}$ or $\sqrt{9a^2 + b^2}$

$BR = \sqrt{(4a-a)^2 + (0-b)^2}$ or $\sqrt{9a^2 + b^2}$

Since $AS = BR$, $\overline{AS} \cong \overline{BR}$.

21. Given: Right $\triangle ABC$ with right $\angle BAC$; P is the midpoint of $\overline{BC}$.

Prove: $AP = \frac{1}{2}BC$



Proof:
Midpoint P is $\left(\frac{0+2c}{2}, \frac{2b+0}{2}\right)$ or (c, b).

$AP = \sqrt{(c-0)^2 + (b-0)^2}$ or $\sqrt{c^2 + b^2}$

$BC = \sqrt{(2c - 0)^2 + (0 - 2b)^2} = \sqrt{4c^2 + 4b^2}$ or $2\sqrt{c^2 + b^2}$

$\frac{1}{2}BC = \sqrt{c^2 + b^2}$

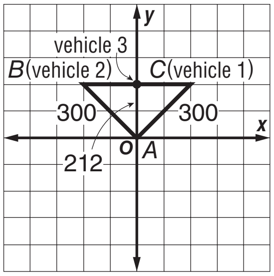
So, $AP = \frac{1}{2}BC$.

23. The distance between Raleigh and Durham is about 0.32 units, between Raleigh and Chapel Hill is about 0.41 units, and between Durham and Chapel Hill is about 0.15 units. Since none of these distances are the same, the Research Triangle is scalene.

25. slope of $\overline{XY} = 1$, slope of $\overline{YZ} = -1$, slope of $\overline{ZX} = 0$; since $1(-1) = -1$, $\overline{XY} \perp \overline{YZ}$. Therefore, $\triangle XYZ$ is a right triangle.

27. The slope between the tents is $-\frac{4}{3}$. The slope between the ranger's station and the tent located at (12, 9) is $\frac{3}{4}$. Since $-\frac{4}{3} \cdot \frac{3}{4} = -1$, the triangle formed by the tents and ranger's station is a right triangle.

29 **a.**



The equation of the line along which the first vehicle lies is $y = x$. The slope is 1 because the vehicle travels the same number of units north as it does east of the origin and the y-intercept is 0. The equation of the line along which the second vehicle lies is $y = -x$. The slope is -1 because the vehicle travels the same number of units north as it does west of the origin and the y-intercept is 0.

b. The paths taken by both the first and second vehicles are 300 yards long. Therefore, the paths are congruent. If two sides of a triangle are congruent, then the triangle is isosceles. You can also write a coordinate proof to prove the triangle formed is isosceles.

Given: $\triangle ABC$

Prove: $\triangle ABC$ is an isosceles right triangle.

Proof: By the Distance Formula,

$AB = \sqrt{(-a - 0)^2 + (a - 0)^2}$ or $\sqrt{2a^2}$ and

$AC = \sqrt{(a - 0)^2 + (a - 0)^2}$ or $\sqrt{2a^2}$. So, $AB = AC$ and $\overline{AB} \cong \overline{AC}$. The triangle is isosceles. By the Slope Formula, the slope of $\overline{AB}$ is $\frac{a}{-a}$ or -1 and the slope of $\overline{AC}$ is $\frac{a}{a}$ or 1. Since the slopes are negative reciprocals, the sides of the triangle are perpendicular and therefore form a right angle. So, $\triangle ABC$ is an isosceles right triangle.

c. The paths taken by the first two vehicles form the hypotenuse of isosceles right triangles.

$a^2 + b^2 = c^2$ Pythagorean Theorem

$a^2 + a^2 = 300^2$ $b = a$

$2a^2 = 90{,}000$ Simplify.

$a^2 = 45{,}000$ Divide each side by 2.

$a = 150\sqrt{2}$ Take the positive square root of each side.

First vehicle: $(a, a) = (150\sqrt{2}, 150\sqrt{2})$;
second vehicle: $(-a, a) = (-150\sqrt{2}, 150\sqrt{2})$
The third vehicle travels due north and therefore, remains on the y-axis; third vehicle: (0, 212)

d. The y-coordinates of the first two vehicles are $150\sqrt{2} \approx 212.13$, while the y-coordinate of the third vehicle is 212. Since all three vehicles have approximately the same y-coordinate, they are approximately collinear. The midpoint between the first and second vehicles is

$\left(\frac{150\sqrt{2} + (-150\sqrt{2})}{2}, \frac{150\sqrt{2} + 150\sqrt{2}}{2}\right)$

or approximately (0, 212.13). This is the approximate location of the third vehicle.

31. Sample answer: $(a, 0)$ **33.** Sample answer: $(4a, 0)$

35. Given: $\triangle ABC$ with coordinates $A(0, 0)$, $B(a, b)$, and $C(c, d)$ and $\triangle DEF$ with coordinates $D(0 + n, 0 + m)$, $E(a + n, b + m)$, and $F(c + n, d + m)$

Prove: $\triangle DEF \cong \triangle ABC$

Proof:

$AB = \sqrt{(a - 0)^2 + (b - 0)^2}$ or $\sqrt{a^2 + b^2}$

$DE = \sqrt{[a + n - (0 + n)]^2 + [b + m - (0 + m)]^2}$ or $\sqrt{a^2 + b^2}$

Since $AB = DE$, $\overline{AB} \cong \overline{DE}$.

$BC = \sqrt{(c - a)^2 + (d - b)^2}$ or $\sqrt{c^2 - 2ac + a^2 + d^2 - 2bd + b^2}$

$EF = \sqrt{[c + n - (a + n)]^2 + [d + m - (b + m)]^2}$ or $\sqrt{c^2 - 2ac + a^2 + d^2 - 2bd + b^2}$

Since $BC = EF$, $\overline{BC} \cong \overline{EF}$.

$CA = \sqrt{(c - 0)^2 + (d - 0)^2}$ or $\sqrt{c^2 + d^2}$

$FD = \sqrt{[0 + n - (c + n)]^2 + [0 + m - (d + m)]^2}$ or $\sqrt{c^2 + d^2}$

Since $CA = FD$, $\overline{CA} \cong \overline{FD}$.

Therefore, $\triangle DEF \cong \triangle ABC$ by the SSS Postulate.

37a. Using the origin as a vertex of the triangle makes calculations easier because the coordinates are (0, 0).

37b. Placing at least one side of the triangle on the x- or y-axis makes it easier to calculate the length of the side since one of the coordinates will be 0. **37c.** Keeping a triangle within the first quadrant makes all of the coordinates positive, and makes the calculations easier. **39.** D **41.** C **43.** reflection, translation, or rotation **45.** Sample answer: $\angle TSR \cong \angle TRS$

47. Sample answer: $\triangle RQV \cong \triangle SQV$ **49.** 4.2 **51.** 3.6

Chapter 4 Study Guide & Review

1. true **3.** true **5.** false; base **7.** true **9.** false;

coordinate proof **11.** obtuse **13.** right **15.** $x = 6$, $JK = KL = JL = 24$ **17.** 70 **19.** 82 **21.** $\angle D \cong \angle J$, $\angle A \cong \angle F$, $\angle C \cong \angle H$, $\angle B \cong \angle G$, $\overline{AB} \cong \overline{FG}$, $\overline{BC} \cong \overline{HG}$, $\overline{DC} \cong \overline{JH}$, $\overline{DA} \cong \overline{JF}$; polygon $ABCD \cong$ polygon $FGHJ$ **23.** $\triangle BFG \cong \triangle CGH \cong \triangle DHE \cong \triangle AEF$, $\triangle EFG \cong \triangle FGH \cong \triangle GHE \cong \triangle HEF$ **25.** No, the corresp. sides of the 2 △s are not ≅. **27.** not possible

29. Given: $\overline{AB} \parallel \overline{DC}$, $\overline{AB} \cong \overline{DC}$

Prove: $\triangle ABE \cong \triangle CDE$

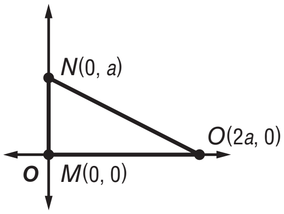
Proof:

Statements (Reasons)

1. $\overline{AB} \parallel \overline{DC}$ (Given)
2. $\angle A \cong \angle DCE$ (Alt. Int. ∠s Thm.)
3. $\overline{AB} \parallel \overline{DC}$ (Given)
4. $\angle ABE \cong \angle D$ (Alt. Int. ∠s Thm.)
5. $\triangle ABE \cong \triangle CDE$ (ASA)

31. 3
33. 77.5
35. reflection
37. rotation

39.



41. Given: $\triangle DSH$ with vertices $D(8, 28)$, $S(0, 0)$, and $H(19, 7)$

Prove: $\triangle DSH$ is scalene.

Proof:

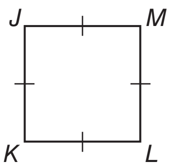
Statements (Reasons)

1. $D(8, 28)$, $S(0, 0)$, and $H(19, 7)$ (Given)
2. $DS = \sqrt{(8-0)^2 + (28-0)^2}$ or $\sqrt{848}$ (Distance Formula)
3. $SH = \sqrt{(19-0)^2 + (7-0)^2}$ or $\sqrt{410}$ (Distance Formula)
4. $DH = \sqrt{(8-19)^2 + (28-7)^2}$ or $\sqrt{562}$ (Distance Formula)
5. $\overline{DS} \ncong \overline{SH} \ncong \overline{DH}$
6. $\triangle DSH$ is a scalene triangle. (Definition of scalene)

CHAPTER 5
Relationships in Triangles

Chapter 5 Get Ready

1. 9 **3.** 10 ft
5. $JK = KL = LM = MJ$



7. Sometimes; the conjecture is true when E is between D and F, otherwise it is false. **9.** $-6 > x$
11. $x < 41$

Lesson 5-1

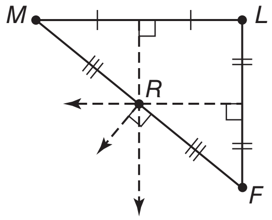
1. 12 **3.** 15 **5.** 8 **7.** 12

9 $\overrightarrow{MP}$ is the perpendicular bisector of $\overline{LN}$.

$LP = NP$	Perpendicular Bisector Theorem
$2x - 4 = x + 5$	Substitution
$x - 4 = 5$	Subtract x from each side.
$x = 9$	Add 4 to each side.

$NP = 9 + 5$ or 14

11. 6 **13.** 4 **15.**



17. $\overline{CD}$, $\overline{BD}$
19. $\overline{BH}$ **21.** 11

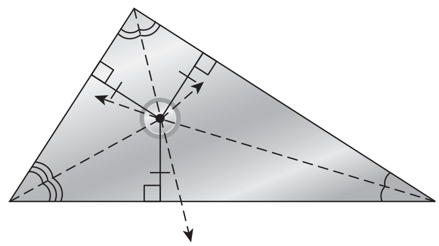
23 Since $\overline{QM} \perp \overrightarrow{NM}$, $\overline{QP} \perp \overrightarrow{NP}$, and $QM = QP$, Q is equidistant from the sides of $\angle PNM$. By the Converse of the Angle Bisector Theorem, $\overrightarrow{NQ}$ bisects $\angle PNM$.

$\angle PNQ \cong \angle QNM$	Definition of angle bisector
$m\angle PNQ = m\angle QNM$	Definition of congruent angles
$4x - 8 = 3x + 5$	Substitution
$x - 8 = 5$	Subtract $3x$ from each side.
$x = 13$	Add 8 to each side.

$m\angle PNM = m\angle PNQ + m\angle QNM$	Angle Addition Postulate
$= (4x - 8) + (3x + 5)$	Substitution
$= 7x - 3$	Simplify.
$= 7(13) - 3$ or 88	$x = 13$

25. 42 **27.** 7.1 **29.** 33

31 Sketch the table and draw the three angle bisectors of the triangle.



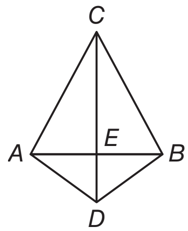
Find the point of concurrency of the angle bisectors of the triangle, the incenter. This point is equidistant from each side of the triangle. So, the centerpiece should be placed at the incenter.

33. No; we need to know whether the perpendicular segments are congruent to each other. **35.** No; we need to know whether the hypotenuses of the triangles are congruent.

37. Given: $\overline{CA} \cong \overline{CB}$, $\overline{AD} \cong \overline{BD}$

Prove: C and D are on the perpendicular bisector of $\overline{AB}$.

Proof:

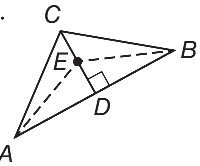


Statements (Reasons)

1. $\overline{CA} \cong \overline{CB}$, $\overline{AD} \cong \overline{BD}$ (Given)
2. $\overline{CD} \cong \overline{CD}$ (Congruence of segments is reflexive.)
3. $\triangle ACD \cong \triangle BCD$ (SSS)
4. $\angle ACD \cong \angle BCD$ (CPCTC)
5. $\overline{CE} \cong \overline{CE}$ (Congruence of segments is reflexive.)
6. $\triangle CEA \cong \triangle CEB$ (SAS)
7. $\overline{AE} \cong \overline{BE}$ (CPCTC)
8. E is the midpoint of $\overline{AB}$. (Def. of midpoint)
9. $\angle CEA \cong \angle CEB$ (CPCTC)
10. $\angle CEA$ and $\angle CEB$ form a linear pair. (Def. of linear pair)
11. $\angle CEA$ and $\angle CEB$ are supplementary.

(Supplement Theorem)
12. $m\angle CEA + m\angle CEB = 180$ (Def. of supplementary)
13. $m\angle CEA + m\angle CEA = 180$ (Substitution Prop.)
14. $2m\angle CEA = 180$ (Substitution Prop.)
15. $m\angle CEA = 90$ (Division Prop.)
16. $\angle CEA$ and $\angle CEB$ are rt. $\angle$. (Def. of rt. $\angle$)
17. $\overline{CD} \perp \overline{AB}$ (Def. of $\perp$)
18. $\overline{CD}$ is the perpendicular bisector of $\overline{AB}$. (Def. of $\perp$ bisector)
19. C and D are on the perpendicular bisector of $\overline{AB}$. (Def. of point on a line)

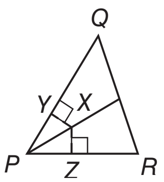
39. Given: $\overline{CD}$ is the $\perp$ bisector of $\overline{AB}$. E is a point on $\overline{CD}$.
Prove: $EA = EB$



Proof: $\overline{CD}$ is the $\perp$ bisector of $\overline{AB}$. By definition of $\perp$ bisector, D is the midpoint of $\overline{AB}$. Thus, $\overline{AD} \cong \overline{BD}$ by the Midpoint Theorem. $\angle CDA$ and $\angle CDB$ are right angles by the definition of perpendicular. Since all right angles are congruent, $\angle CDA \cong \angle CDB$. Since E is a point on $\overline{CD}$, $\angle EDA$ and $\angle EDB$ are right angles and are congruent. By the Reflexive Property, $\overline{ED} \cong \overline{ED}$. Thus, $\triangle EDA \cong \triangle EDB$ by SAS. $\overline{EA} \cong \overline{EB}$ because CPCTC, and by definition of congruence, $EA = EB$.

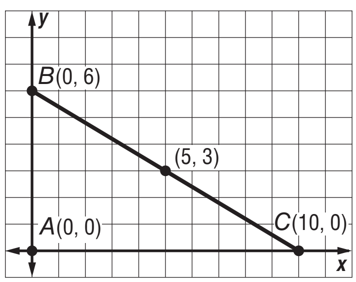
41. $y = -\frac{7}{2}x + \frac{15}{4}$; The perpendicular bisector bisects the segment at the midpoint of the segment. The midpoint is $\left(\frac{1}{2}, 2\right)$. The slope of the given segment is $\frac{2}{7}$, so the slope of the perpendicular bisector is $-\frac{7}{2}$.

43. Given: $\overline{PX}$ bisects $\angle QPR$. $\overline{XY} \perp \overline{PQ}$ and $\overline{XZ} \perp \overline{PR}$
Prove: $\overline{XY} \cong \overline{XZ}$



Proof:
Statements (Reasons)
1. $\overline{PX}$ bisects $\angle QPR$, $\overline{XY} \perp \overline{PQ}$, and $\overline{XZ} \perp \overline{PR}$. (Given)
2. $\angle YPX \cong \angle ZPX$ (Definition of angle bisector)
3. $\angle PYX$ and $\angle PZX$ are right angles. (Definition of perpendicular)
4. $\angle PYX \cong \angle PZX$ (Right angles are congruent.)
5. $\overline{PX} \cong \overline{PX}$ (Reflexive Property)
6. $\triangle PYX \cong \triangle PZX$ (AAS)
7. $\overline{XY} \cong \overline{XZ}$ (CPCTC)

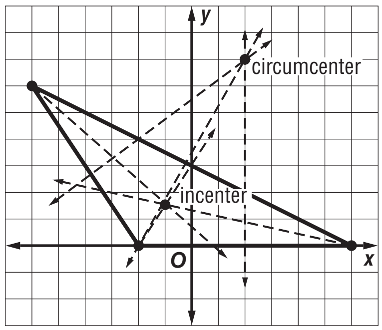
45 The circumcenter is the point where the perpendicular bisectors of a triangle intersect. You can find the circumcenter by locating the point of intersection of two of the perpendicular bisectors.



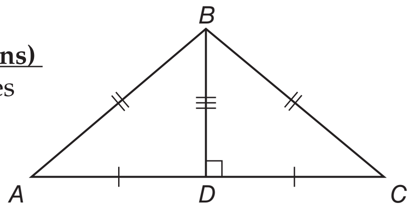
The equation of the perpendicular bisector of $\overline{AB}$ is $y = 3$. The equation of the perpendicular bisector of $\overline{AC}$ is $x = 5$. These lines intersect at (5, 3). The circumcenter is located at (5, 3).

47. a plane perpendicular to the plane in which $\overline{CD}$ lies and bisecting $\overline{CD}$

49. Sample answer:



51. always
Given: $\triangle ABC$ is isosceles with legs $\overline{AB}$ and $\overline{BC}$; $\overline{BD}$ is the $\perp$ bisector of $\overline{AC}$.
Prove: $\overline{BD}$ is the angle bisector of $\angle ABC$.

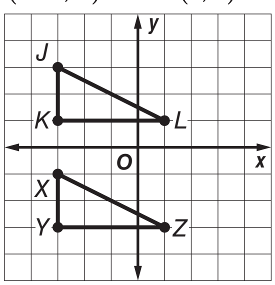


Proof:
Statements (Reasons)
1. $\triangle ABC$ is isosceles with legs $\overline{AB}$ and $\overline{BC}$. (Given)
2. $\overline{AB} \cong \overline{BC}$ (Def. of isosceles $\triangle$)
3. $\overline{BD}$ is the $\perp$ bisector of $\overline{AC}$. (Given)
4. D is the midpoint of $\overline{AC}$. (Def. of segment bisector)
5. $\overline{AD} \cong \overline{DC}$ (Def. of midpoint)
6. $\overline{BD} \cong \overline{BD}$ (Reflexive Property)
7. $\triangle ABD \cong \triangle CBD$ (SSS)
8. $\angle ABD \cong \angle CBD$ (CPCTC)
9. $\overline{BD}$ is the angle bisector of $\angle ABC$. (Def. of $\angle$ bisector)

53. Given: Plane $\mathcal{Z}$ is an angle bisector of $\angle KJH$. $\overline{KJ} \cong \overline{HJ}$
Prove: $\overline{MH} \cong \overline{MK}$
Proof:
Statements (Reasons)
1. Plane $\mathcal{Z}$ is an angle bisector of $\angle KJH$; $\overline{KJ} \cong \overline{HJ}$ (Given)
2. $\angle KJM \cong \angle HJM$ (Definition of angle bisector)
3. $\overline{JM} \cong \overline{JM}$ (Reflexive Property)
4. $\triangle KJM \cong \triangle HJM$ (SAS)
5. $\overline{MH} \cong \overline{MK}$ (CPCTC)

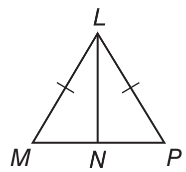
55. A **57.** D **59.** $L(a, b)$ **61.** $S(-2b, 0)$ and $R(0, c)$

63. $\triangle JKL$ is a translation of $\triangle XYZ$; $JK = 2$, $KL = 4$, $JL = \sqrt{20}$, $XY = 2$, $YZ = 4$, $XZ = \sqrt{20}$. $\triangle JKL \cong \triangle XYZ$ by SSS.



65. $\sqrt{5}$
67. $m = 42t + 450$; \$1164

69. Given: $\triangle MLP$ is isosceles. N is the midpoint of $\overline{MP}$.
Prove: $\overline{LN} \perp \overline{MP}$



Proof:
Statements (Reasons)
1. $\triangle MLP$ is isosceles. (Given)
2. $\overline{ML} \cong \overline{PL}$ (Definition of isosceles $\triangle$)
3. $\angle M \cong \angle P$ (Isosceles $\triangle$ Th.)

4. N is the midpoint of $\overline{MP}$. (Given)
5. $\overline{MN} \cong \overline{PN}$ (Def. of midpoint)
6. $\triangle MNL \cong \triangle PNL$ (SAS)
7. $\angle LNM \cong \angle LNP$ (CPCTC)
8. $m\angle LNM = m\angle LNP$ (Def. of $\cong$ ∡)
9. $\angle LNM$ and $\angle LNP$ are a linear pair. (Def. of a linear pair)
10. $m\angle LNM + m\angle LNP = 180$ (Sum of measure of linear pair of ∡ = 180)
11. $2m\angle LNM = 180$ (Substitution)
12. $m\angle LNM = 90$ (Division)
13. $\angle LNM$ is a right angle. (Def. of rt. $\angle$)
14. $\overline{LN} \perp \overline{MP}$ (Def. of $\perp$)

Lesson 5-2

1

$PC = \frac{2}{3}FC$	Centroid Theorem
$PC = \frac{2}{3}(PF + PC)$	Segment Addition and Substitution
$PC = \frac{2}{3}(6 + PC)$	$PF = 6$
$PC = 4 + \frac{2}{3}PC$	Distributive Property
$\frac{1}{3}PC = 4$	Subtract $\frac{2}{3}PC$ from each side.
$PC = 12$	Multiply each side by 3.

3. (5, 6) **5.** 4.5 **7.** 13.5 **9.** 6 **11.** (3, 6)

13 The centroid is the point of balance for a triangle. Use the Midpoint Theorem to find the midpoint M of the side with endpoints at (0, 8) and (6, 4). The centroid is two-thirds the distance from the opposite vertex to that midpoint.

$$M\left(\frac{0+6}{2}, \frac{8+4}{2}\right) = M(3, 6)$$

The distance from $M(3, 6)$ to the point at (3, 0) is $6 - 0$ or 6 units. If P is the centroid of the triangle, then $P = \frac{2}{3}(6)$ or 4 units up from the point at (3, 0). The coordinates of P are $(3, 0 + 4)$ or (3, 4).

15. $(-4, -4)$ **17.** median **19.** median **21.** 3 **23.** $\frac{1}{2}$

25

$\overline{AC} \cong \overline{DC}$	Definition of median
$AC = DC$	Definition of congruence
$4x - 3 = 2x + 9$	Substitution
$2x - 3 = 9$	Subtract $2x$ from each side.
$2x = 12$	Add 3 to each side.
$x = 6$	Divide each side by 2.

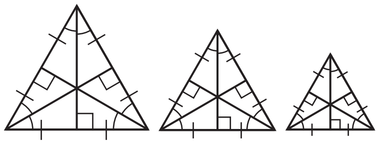
$m\angle ECA = 15x + 2$	Given
$= 15(6) + 2$	$x = 6$
$= 90 + 2$ or 92	Simplify.

$\overline{EC}$ is not an altitude of $\triangle AED$ because $m\angle ECA = 92$. If $\overline{EC}$ were an altitude, then $m\angle ECA$ must be 90.

27. altitude **29.** median

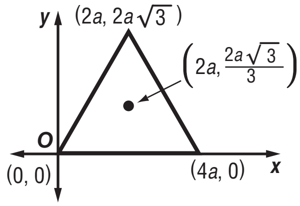
31. Given: $\triangle XYZ$ is isosceles. $\overline{WY}$ bisects $\angle Y$.
Prove: $\overline{WY}$ is a median.
Proof: Since $\triangle XYZ$ is isosceles, $\overline{XY} \cong \overline{ZY}$. By the definition of angle bisector, $\angle XYW \cong \angle ZYW$. $\overline{YW} \cong \overline{YW}$ by the Reflexive Property. So, by SAS, $\triangle XYW \cong \triangle ZYW$. By CPCTC, $\overline{XW} \cong \overline{ZW}$. By the definition of a midpoint, W is the midpoint of $\overline{XZ}$. By the definition of a median, $\overline{WY}$ is a median.

33a.



33b. Sample answer: The four points of concurrency of an equilateral triangle are all the same point.

33c.



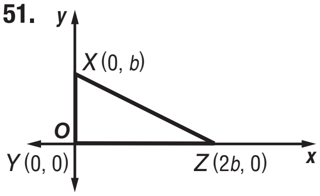
35. 7

37 Sample answer: Kareem is correct. According to the Centroid Theorem, $AP = \frac{2}{3}AD$. The segment lengths are transposed.

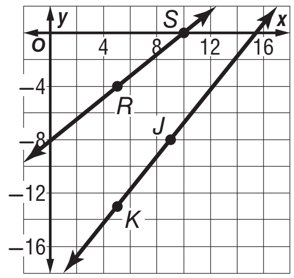
39. $\left(1, \frac{5}{3}\right)$; Sample answer: I found the midpoint of $\overline{AC}$ and used it to find the equation for the line that contains point B and the midpoint of $\overline{AC}$, $y = \frac{10}{3}x - \frac{5}{3}$. I also found the midpoint of $\overline{BC}$ and the equation for the line between point A and the midpoint of $\overline{BC}$, $y = -\frac{1}{3}x + 2$. I solved the system of two equations for x and y to get the coordinates of the centroid, $\left(1, \frac{5}{3}\right)$.

41. $2\sqrt{13}$ **43.** Sample answer: Each median divides the triangle into two smaller triangles of equal area, so the triangle can be balanced along any one of those lines. To balance the triangle on one point, you need to find the point where these three balance lines intersect. The balancing point for a rectangle is the intersection of the segments connecting the midpoints of the opposite sides, since each segment connecting these midpoints of a pair of opposite sides divides the rectangle into two parts with equal area. **45.** 3 **47.** B **49.** 5

51.



53. neither



55. Because the lines are perpendicular, the angles formed are right angles. All right angles are congruent. Therefore, $\angle 1$ is congruent to $\angle 2$.

Lesson 5-3

1. $\angle 1, \angle 2$ **3.** $\angle 4$ **5.** $\angle A, \angle C, \angle B$; $\overline{BC}, \overline{AB}, \overline{AC}$ **7.** $\overline{BC}$; Sample answer: Since the angle across from segment $\overline{BC}$ is larger than the angle across from $\overline{AC}$, $\overline{BC}$ is longer. **9.** $\angle 1, \angle 2$ **11.** $\angle 1, \angle 3, \angle 6, \angle 7$ **13.** $\angle 5, \angle 9$

15 The sides from shortest to longest are $\overline{RT}$, $\overline{RS}$, $\overline{ST}$. The angles opposite these sides are $\angle S$, $\angle T$, and $\angle R$, respectively. So the angles from smallest to largest are $\angle S$, $\angle T$, and $\angle R$.

17. $\angle L, \angle P, \angle M$; $\overline{PM}, \overline{ML}, \overline{PL}$ **19.** $\angle C, \angle D, \angle E$; $\overline{DE}, \overline{CE}, \overline{CD}$

21 To use Theorem 5.9, first show that the measure of the angle opposite $\overline{YZ}$ is greater than the measure of the angle opposite $\overline{XZ}$. If $m\angle X = 90$, then $m\angle Y + m\angle Z = 90$, so $m\angle Y < 90$ by the definition of inequality. So $m\angle X > m\angle Y$. According to Theorem 5.9, if $m\angle X > m\angle Y$, then the length of the side opposite $\angle X$ must be greater than the length of the side opposite $\angle Y$. Since $\overline{YZ}$ is opposite $\angle X$, and $\overline{XZ}$ is opposite $\angle Y$, then $YZ > XZ$. So YZ, the length of the top surface of the ramp, must be greater than the length of the ramp.

23. $\angle P, \angle Q, \angle M$; $\overline{MQ}, \overline{PM}, \overline{PQ}$ **25.** $\angle 2$ **27.** $\angle 3$ **29.** $\angle 8$ **31.** $m\angle BCF > m\angle CFB$ **33.** $m\angle DBF < m\angle BFD$ **35.** $RP > MP$ **37.** $RM > RQ$

39 Use the Distance Formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to find the lengths of the sides.

$AB = \sqrt{[-2 - (-4)]^2 + (1 - 6)^2}$ $x_1 = -4, x_2 = -2, y_1 = 6, y_2 = 1$

$= \sqrt{29}$ Simplify.

≈ 5.4 Use a calculator.

$BC = \sqrt{[5 - (-2)]^2 + (6 - 1)^2}$ $x_1 = -2, x_2 = 5, y_1 = 1, y_2 = 6$

$= \sqrt{74}$ Simplify.

≈ 8.6 Use a calculator.

$AC = \sqrt{[5 - (-4)]^2 + (6 - 6)^2}$ $x_1 = -4, x_2 = 5, y_1 = 6, y_2 = 6$

$= \sqrt{81}$ or 9 Simplify.

Since $AB < BC < AC$, $\angle C < \angle A < \angle B$. The angles in order from smallest to largest are $\angle C, \angle A, \angle B$.

41. AB, BC, AC, CD, BD; In $\triangle ABC$, $AB < BC < AC$ and in $\triangle BCD$, $BC < CD < BD$. By the figure $AC < CD$, so $BC < AC < CD$. **43.** Sample answer: $\angle R$ is an exterior angle to $\triangle PQR$, so by the Exterior Angle Inequality, $m\angle R$ must be greater than $m\angle Q$. The markings indicate that $\angle R \cong \angle Q$, indicating that $m\angle R = m\angle Q$. This is a contradiction of the Exterior Angle Inequality Theorem, so the markings are incorrect. **45.** Sample answer: 10; $m\angle C > m\angle B$, so if $AB > AC$, Theorem 5.10 is satisfied. Since $10 > 6$, $AB > AC$. **47.** $m\angle 1, m\angle 2 = m\angle 5, m\angle 4, m\angle 6, m\angle 3$; Sample answer: The side opposite $\angle 5$ is the smallest side in that triangle and $m\angle 2 = m\angle 5$, so we know that $m\angle 4$ and $m\angle 6$ are both greater than $m\angle 2$ and $m\angle 5$. The side opposite $\angle 6$ is greater than the side opposite $\angle 4$, Since the side opposite $\angle 2$ is greater than the side opposite $\angle 1$, we know that $m\angle 1 < m\angle 2$ and $m\angle 5$. Since $m\angle 2 = m\angle 5$, $m\angle 1 + m\angle 3 = m\angle 4 + m\angle 6$. Since $m\angle 1 < m\angle 4$, then $m\angle 3 > m\angle 6$. **49.** D

51a. $t = 2.5h + 198$

51b. 6 **51c.** \$180 **53.** 9 **55.** $y = -5x + 7$; The perpendicular bisector bisects the segment at the midpoint of the segment. The midpoint is $\left(\frac{1}{2}, \frac{9}{2}\right)$. The slope of the given segment is $\frac{1}{5}$, so the slope of the perpendicular bisector is -5.

57. Given: T is the midpoint of $\overline{SQ}$.
$\overline{SR} \cong \overline{QR}$

Prove: $\triangle SRT \cong \triangle QRT$

Proof:

Statements (Reasons)

1. T is the midpoint of $\overline{SQ}$. (Given)
2. $\overline{ST} \cong \overline{TQ}$ (Def. of midpoint)
3. $\overline{SR} \cong \overline{QR}$ (Given)
4. $\overline{RT} \cong \overline{RT}$ (Reflexive Prop.)
5. $\triangle SRT \cong \triangle QRT$ (SSS)

59. false **61.** true

Lesson 5-4

1. $\overline{AB} \ncong \overline{CD}$

3 The conclusion of the conditional statement is $x < 6$. If $x < 6$ is false, then x must be greater than or equal to 6. The negation of the conclusion is $x \geq 6$.

5. Given: $2x + 3 < 7$

Prove: $x < 2$

Indirect Proof: Step 1

Assume that $x > 2$ or $x = 2$ is true.

Step 2

x	2	3	4	5	6
$2x + 3$	7	9	11	13	15

When $x > 2$, $2x + 3 > 7$ and when $x = 2$, $2x + 3 = 7$.

Step 3 In both cases, the assumption leads to the contradiction of the given information that $2x + 3 < 7$. Therefore, the assumption that $x \geq 2$ must be false, so the original conclusion that $x < 2$ must be true.

7. Use a = average or $\frac{\text{number of points scored}}{\text{number of games played}}$.

Proof:

Indirect Proof: Step 1 Assume that Christina's average points per game was greater than or equal to 3, $a \geq 3$.

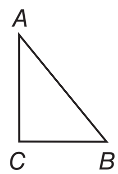
Step 2 CASE 1 | CASE 2

CASE 1	CASE 2
$a = 3$	$a > 3$
$3 \stackrel{?}{=} \frac{13}{6}$	$\frac{13}{6} \stackrel{?}{>} 3$
$3 \neq 2.2$	$2.2 \ngtr 3$

Step 3 The conclusions are false, so the assumption must be false. Therefore, Christina's average points per game was less than 3.

9. Given: $\triangle ABC$ is a right triangle; $\angle C$ is a right angle.

Prove: $AB > BC$ and $AB > AC$



Indirect Proof: Step 1 Assume that the hypotenuse of a right triangle is not the longest side. That is, $AB < BC$ and $AB < AC$.

Step 2 If $AB < BC$, then $m\angle C < m\angle A$. Since $m\angle C = 90$, $m\angle A > 90$. So, $m\angle C + m\angle A > 180$.

By the same reasoning, $m\angle C + m\angle B > 180$.
Step 3 Both relationships contradict the fact that the sum of the measures of the angles of a triangle equals 180. Therefore, the hypotenuse must be the longest side of a right triangle.

11. $x \leq 8$ **13.** The lines are not parallel. **15.** The triangle is equiangular.

17 To write an indirect proof, first identify the conclusion. Find the negation of the conclusion and assume that it is true. Make a table of values to show that the negation of the conclusion is false. Since the assumption leads to a contradiction, you can conclude that the original conclusion must be true.
Given: $2x - 7 > -11$
Prove: $x > -2$
Indirect Proof: Step 1 The negation of $x > -2$ is $x \leq -2$. So, assume that $x \leq -2$ is true.
Step 2 Make a table with several possibilities for x assuming $x < -2$ or $x = -2$.

x	−6	−5	−4	−3	−2
$2x - 7$	−19	−17	−15	−13	−11

When $x < -2$, $2x - 7 < -11$ and when $x = -2$, $2x - 7 = -11$.
Step 3 In both cases, the assumption leads to the contradiction of the given information that $2x - 7 > -11$. Therefore, the assumption that $x \leq -2$ must be false, so the original conclusion that $x > -2$ must be true.

19. Given: $-3x + 4 < 7$
Prove: $x > -1$
Indirect Proof: Step 1 Assume that $x \leq -1$ is true.
Step 2 When $x < -1$, $-3x + 4 > 7$ and when $x = -1$, $-3x + 4 = 7$.

x	−5	−4	−3	−2	−1
$-3x + 4$	19	16	13	10	7

Step 3 In both cases, the assumption leads to the contradiction of the given information that $-3x + 4 < 7$. Therefore, the assumption that $x \leq -1$ must be false, so the original conclusion that $x > -1$ must be true.

21. Let the cost of one game be x and the other be y.
Given: $x + y > 80$
Prove: $x > 40$ or $y > 40$
Indirect Proof: Step 1 Assume that $x \leq 40$ and $y \leq 40$.
Step 2 If $x \leq 40$ and $y \leq 40$, then $x + y \leq 40 + 40$ or $x + y \leq 80$. This is a contradiction because we know that $x + y > 80$.
Step 3 Since the assumption that $x \leq 40$ and $y \leq 40$ leads to a contradiction of a known fact, the assumption must be false. Therefore, the conclusion that $x > 40$ or $y > 40$ must be true. Thus, at least one of the games had to cost more than \$40.

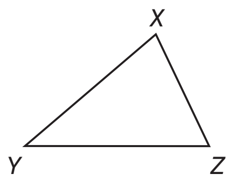
23. Given: xy is an odd integer.
Prove: x and y are odd integers.
Indirect Proof: Step 1 Assume that x and y are not both odd integers. That is, assume that either x or y is an even integer.
Step 2 You only need to show that the assumption that x is an even integer leads to a contradiction, since the argument for y is an even integer follows the same reasoning. So, assume that x is an even integer and y is an odd integer. This means that $x = 2k$ for some integer k and $y = 2m + 1$ for some integer m.

$xy = (2k)(2m + 1)$	Subst. of assumption
$= 4km + 2k$	Dist. Prop.
$= 2(km + k)$	Dist. Prop.

Since k and m are integers, $km + k$ is also an integer. Let p represent the integer $km + k$. So xy can be represented by $2p$, where p is an integer. This means that xy is an even integer, but this contradicts the given that xy is an odd integer.
Step 3 Since the assumption that x is an even integer and y is an odd integer leads to a contradiction of the given, the original conclusion that x and y are both odd integers must be true.

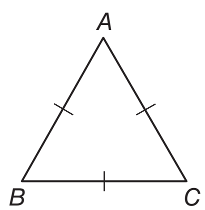
25. Given: x is an odd number.
Prove: x is not divisible by 4.
Indirect Proof: Step 1 Assume x is divisible by 4. In other words, 4 is a factor of x.
Step 2 Let $x = 4n$, for some integer n.
$x = 2(2n)$
So, 2 is a factor of x which means x is an even number, but this contradicts the given information.
Step 3 Since the assumption that x is divisible by 4 leads to a contradiction of the given, the original conclusion x is not divisible by 4 must be true.

27. Given: $XZ > YZ$
Prove: $\angle X \ncong \angle Y$
Indirect Proof: Step 1
Assume that $\angle X \cong \angle Y$.
Step 2 $\overline{XZ} \cong \overline{YZ}$ by the converse of the isosceles △ theorem.



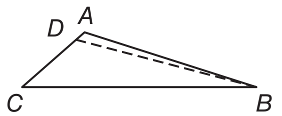
Step 3 This contradicts the given information that $XZ > YZ$. Therefore, the assumption $\angle X \cong \angle Y$ must be false, so the original conclusion $\angle X \ncong \angle Y$ must be true.

29. Given: $\triangle ABC$ is isosceles.
Prove: Neither of the base angles is a right angle.
Indirect Proof: Step 1
Assume that $\angle B$ is a right angle.
Step 2 By the Isosceles △ Theorem, $\angle C$ is also a right angle.



Step 3 This contradicts the fact that a triangle can have no more than one right angle. Therefore, the assumption that $\angle B$ is a right angle must be false, so the original conclusion neither of the base angles is a right angle must be true.

31. Given: $m\angle A > m\angle ABC$
Prove: $BC > AC$



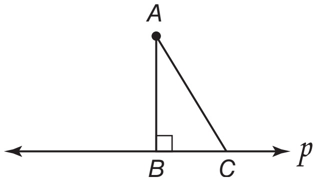
Proof:
Assume $BC \not> AC$. By the Comparison Property, $BC = AC$ or $BC < AC$.
Case 1: If $BC = AC$, then $\angle ABC \cong \angle A$ by the Isosceles Triangle Theorem. (If two sides of a triangle are congruent, then the angles opposite those sides are congruent.) But, $\angle ABC \cong \angle A$ contradicts the given statement that $m\angle A > m\angle ABC$. So, $BC \neq AC$.
Case 2: If $BC < AC$, then there must be a point D between A and C so that $\overline{DC} \cong \overline{BC}$. Draw the auxiliary segment $\overline{BD}$. Since $DC = BC$, by the Isosceles Triangle Theorem $\angle BDC \cong \angle DBC$. Now $\angle BDC$ is an exterior angle of $\triangle BAD$ and by the Exterior Angles Inequality Theorem (the measure of an exterior angle of a triangle is greater than the measure of either corresponding remote interior angle) $m\angle BDC > m\angle A$. By the Angle Addition Postulate, $m\angle ABC = m\angle ABD + m\angle DBC$. Then by the definition of inequality, $m\angle ABC > m\angle DBC$. By Substitution and the Transitive Property of Inequality, $m\angle ABC > m\angle A$. But this contradicts the given statement that $m\angle A > m\angle ABC$. In both cases, a contradiction was found, and hence our assumption must have been false. Therefore, $BC > AC$.

33. We know that the other team scored 3 points, and Katsu thinks that they made a three point shot. We also know that a player can score 3 points by making a basket and a foul shot.
Step 1 Assume that a player for the other team made a two-point basket and a foul shot.
Step 2 The other team's score before Katsu left was 26, so their score after a two-point basket and a foul shot would be 26 + 3 or 29.
Step 3 The score is correct when we assume that the other team made a two-point basket and a foul shot, so Katsu's assumption may not be correct. The other team could have made a three-point basket or a two-point basket and a foul shot.

35 **a.** To write an indirect proof, first identify the conclusion. Find the negation of the conclusion and assume that it is true. Then use the data to prove that the assumption is false.
Step 1 50% is half, and the statement says more than half of the teens polled said that they recycle, so assume that less than 50% recycle.
Step 2 The data shows that 51% of teens said that they recycle, and 51% > 50%, so the number of teens that recycle is not less than half.
Step 3 This contradicts the data given. Therefore, the assumption is false, and the conclusion more than half of the teens polled said they recycle must be true.
b. According to the data, 23% of 400 teenagers polled said that they participate in Earth Day. Verify that 23% of 400 is 92.

$400 \cdot 23\% \stackrel{?}{=} 92$
$400 \cdot 0.23 \stackrel{?}{=} 92$
$92 = 92$

37 **Given:** $\overline{AB} \perp$ line p
Prove: $\overline{AB}$ is the shortest segment from A to line p.
Indirect Proof: Step 1
Assume $\overline{AB}$ is not the shortest segment from A to p.



Step 2 Since $\overline{AB}$ is not the shortest segment from A to p, there is a point C such that $\overline{AC}$ is the shortest distance. $\triangle ABC$ is a right triangle with hypotenuse $\overline{AC}$, the longest side of $\triangle ABC$ since it is across from the largest angle in $\triangle ABC$ by the Angle-Side Relationships in Triangles Theorem.
Step 3 This contradicts the fact that $\overline{AC}$ is the shortest side. Therefore, the assumption is false, and the conclusion, $\overline{AB}$ is the shortest side, must be true.

39a. $n^3 + 3$ **39b.** Sample answer:

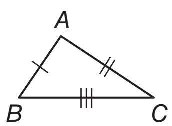
n	$n^3 + 3$
2	11
3	30
10	1003
11	1334
24	13,827
25	15,628
100	1,000,003
101	1,030,304
526	145,531,579
527	146,363,186

39c. Sample answer: When $n^3 + 3$ is even, n is odd.
39d. Indirect Proof: Step 1
Assume that n is even. Let $n = 2k$, where k is some integer.
Step 2

$n^3 + 3 = (2k)^3 + 3$	Substitute assumption
$= 8k^3 + 3$	Simplify.
$= (8k^3 + 2) + 1$	Replace 3 with 2 + 1 and group the first two terms.
$= 2(4k^3 + 1) + 1$	Distributive Property

Since k is an integer, $4k^3 + 1$ is also an integer. Therefore, $n^3 + 3$ is odd.
Step 3 This contradicts the given information that $n^3 + 3$ is even. Therefore, the assumption is false, so the conclusion that n is odd must be true.

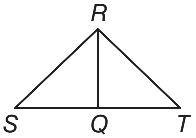
41. Sample answer: $\triangle ABC$ is scalene.
Given: $\triangle ABC$; $AB \neq BC$; $BC \neq AC$; $AB \neq AC$
Prove: $\triangle ABC$ is scalene.



Indirect Proof:
Step 1 Assume that $\triangle ABC$ is not scalene.
Case 1: $\triangle ABC$ is isosceles.
Step 2 If $\triangle ABC$ is isosceles, then $AB = BC$, $BC = AC$, or $AB = AC$.
Step 3 This contradicts the given information, so $\triangle ABC$ is not isosceles.

Case 2: $\triangle ABC$ is equilateral.
In order for a triangle to be equilateral, it must also be isosceles, and Case 1 proved that $\triangle ABC$ is not isosceles. Thus, $\triangle ABC$ is not equilateral. Therefore, $\triangle ABC$ is scalene. **43.** Neither; sample answer: Since the hypothesis is true when the conclusion is false, the statement is false. **45.** $y = 2x - 7$ **47.** J

49. Given: $\overline{RQ}$ bisects $\angle SRT$.
Prove: $m\angle SQR > mSRQ$
Proof:



Statements (Reasons)
1. $\overline{RQ}$ bisects $\angle SRT$. (Given)
2. $\angle SRQ \cong \angle QRT$ (Def. of bisector)
3. $m\angle QRS = m\angle QRT$ (Def. of $\cong$ $\angle$s)
4. $m\angle SQR = m\angle T + m\angle QRT$ (Exterior Angles Theorem)
5. $m\angle SQR > m\angle QRT$ (Def. of Inequality)
6. $m\angle SQR > m\angle SRQ$ (Substitution)

51. $\left(1\frac{2}{5}, 2\frac{3}{5}\right)$ **53.** 64 **55.** $\sqrt{5} \approx 2.2$ **57.** true **59.** false

Lesson 5-5

1 Yes; check each inequality.

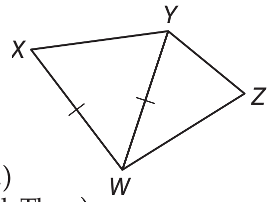
$5 + 7 \overset{?}{>} 10$ $\quad$ $5 + 10 \overset{?}{>} 7$ $\quad$ $7 + 10 \overset{?}{>} 5$

$12 > 10$ ✓ $\quad$ $15 > 7$ ✓ $\quad$ $17 > 5$ ✓

Since the sum of each pair of side lengths is greater than the third side length, sides with lengths 5 cm, 7 cm, and 10 cm will form a triangle.

3. yes; $6 + 14 > 10$, $6 + 10 > 14$, and $10 + 14 > 6$

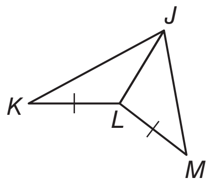
5. Given: $\overline{XW} \cong \overline{YW}$
Prove: $YZ + ZW > XW$
Proof:



Statements (Reasons)
1. $\overline{XW} \cong \overline{YW}$ (Given)
2. $XW = YW$ (Def. of $\cong$ segs.)
3. $YZ + ZW > YW$ ($\triangle$ Inequal. Thm.)
4. $YZ + ZW > XW$ (Subst.)

7. yes **9.** no; $2.1 + 4.2 \not> 7.9$ **11.** yes **13.** $6 \text{ m} < n < 16 \text{ m}$ **15.** $5.4 \text{ in.} < n < 13 \text{ in.}$ **17.** $5\frac{1}{3} \text{ yd} < n < 10 \text{ yd}$

19. Given: $\overline{JL} \cong \overline{LM}$
Prove: $KJ + KL > LM$
Proof:



Statements (Reasons)
1. $\overline{JL} \cong \overline{LM}$ (Given)
2. $JL = LM$ (Def. of $\cong$ segments)
3. $KJ + KL > JL$ ($\triangle$ Inequal. Thm.)
4. $KJ + KL > LM$ (Subst.)

21

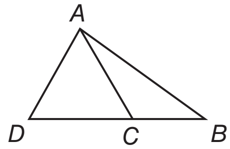
$$\begin{aligned} XY + YZ &> XZ \\ (4x - 1) + (2x + 7) &> x + 13 \\ 6x + 6 &> x + 13 \\ 5x &> 7 \\ x &> \frac{7}{5} \end{aligned}$$

$$\begin{aligned} XY + XZ &> YZ \\ (4x - 1) + (x + 13) &> 2x + 7 \\ 5x + 12 &> 2x + 7 \\ 3x &> -5 \\ x &> -\frac{5}{3} \end{aligned}$$

$$\begin{aligned} YZ + XZ &> XY \\ (2x + 7) + (x + 13) &> 4x - 1 \\ 3x + 20 &> 4x - 1 \\ 21 &> x \end{aligned}$$

Since x must be greater than $\frac{7}{5}$, greater than $-\frac{5}{3}$, and less than 21, possible values of x are $\frac{7}{5} < x < 21$.

23. Given: $\triangle ABC$
Prove: $AC + BC > AB$
Proof:



Statements (Reasons)
1. Construct $\overline{CD}$ so that C is between B and D and $\overline{CD} \cong \overline{AC}$. (Ruler Post.)
2. $CD = AC$ (Def. of $\cong$ segments)
3. $\angle CAD \cong \angle ADC$ (Isos. $\triangle$ Thm.)
4. $m\angle CAD = m\angle ADC$ (Def. of $\cong$ $\angle$s)
5. $m\angle BAC + m\angle CAD = m\angle BAD$ ($\angle$ Add. Post.)
6. $m\angle BAC + m\angle ADC = m\angle BAD$ (Subst.)
7. $m\angle ADC < m\angle BAD$ (Def. of inequality)
8. $AB < BD$ (Thoerem 5.10)
9. $BD = BC + CD$ (Seg. Add. Post.)
10. $AB < BC + CD$ (Subst.)
11. $AB < BC + AC$ (Subst. (Steps 2, 10))

25. $2 < x < 10$ **27.** $1 < x < 11$ **29.** $x > 0$ **31.** Yes; sample answer: The measurements on the drawing do not form a triangle. According to the Triangle Inequality Theorem, the sum of the lengths of any two sides of a triangle is greater than the length of the third side. The lengths in the drawing are 1 ft, $3\frac{7}{8}$ ft, and $6\frac{3}{4}$ ft. Since $1 + 3\frac{7}{8} \not> 6\frac{3}{4}$, the triangle is impossible. They should recalculate their measurements before they cut the wood.

33 A triangle 3 feet by 4 feet by x feet is formed. The length of the third side x must be less than the sum of the lengths of the other two sides. So, $x < 3 + 4$ or $x < 7$. Since the awning drapes 6 inches or 0.5 feet over the front, the total length should be less than $7 + 0.5$ or 7.5 feet. She should buy no more than 7.5 feet.

35. Yes; $\sqrt{99} \approx 9.9$ since $\sqrt{100} = 10$, $\sqrt{48} \approx 6.9$ since $\sqrt{49} = 7$, and $\sqrt{65} \approx 8.1$ since $\sqrt{64} = 8$. $6.9 + 8.1 > 9.9$, so it is possible.

37. no; $\sqrt{122} \approx 11.1$ since $\sqrt{121} = 11$, $\sqrt{5} \approx 2.1$ since $\sqrt{4} = 2$, and $\sqrt{26} \approx 5.1$ since $\sqrt{25} = 5$. So, $2.1 + 5.1 \not> 11.1$.

39 Use the Distance Formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to find the lengths of the sides.

$FG = \sqrt{[3 - (-4)]^2 + (-3 - 3)^2}$ $\quad$ $x_1 = -4, x_2 = 3, y_1 = 3, y_2 = -3$

$= \sqrt{85}$ $\quad$ Simplify.

≈ 9.2 $\quad$ Use a calculator.

$GH = \sqrt{(4 - 3)^2 + [6 - (-3)]^2}$ $\quad$ $x_1 = 3, x_2 = 4, y_1 = -3, y_2 = 6$

$= \sqrt{82}$ Simplify.
≈ 9.1 Use a calculator.

$FH = \sqrt{[4 - (-4)]^2 + (6 - 3)^2}$ $x_1 = -4, x_2 = 4, y_1 = 3, y_2 = 6$
$= \sqrt{73}$ Simplify.
≈ 8.5 Use a calculator.

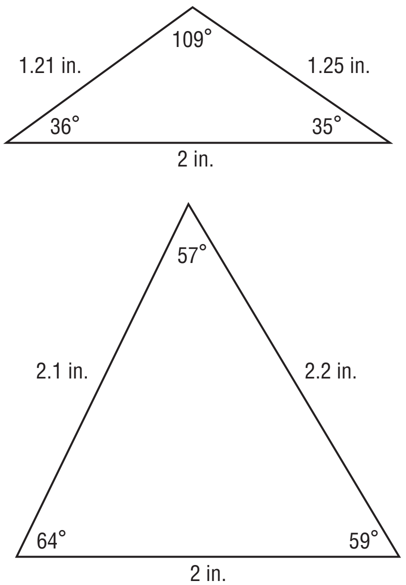
$FG + GH \overset{?}{>} FH$	$FG + FH \overset{?}{>} GH$	$GH + FH \overset{?}{>} FG$
$9.2 + 9.1 \overset{?}{>} 8.5$	$9.2 + 8.5 \overset{?}{>} 9.1$	$9.1 + 8.5 \overset{?}{>} 9.2$
$18.3 > 8.5$	$17.7 > 9.1$	$17.6 > 9.2$

Since $FG + GH > FH$, $FG + FH > GH$, and $GH + FH > FG$, the coordinates are the vertices of a triangle.

41. yes; $QR + QS > RS$, $QR + RS > QS$, and $QS + RS > QR$ **43.** The perimeter is greater than 36 and less than 64. Sample answer: From the diagram we know that $\overline{AC} \cong \overline{EC}$ and $\overline{DC} \cong \overline{BC}$, and $\angle ACB \cong \angle ECD$ because vertical angles are congruent, so $\triangle ACB \cong \triangle ECD$. Using the Triangle Inequality Theorem, the minimum value of AB and ED is 2 and the maximum value is 16. Therefore, the minimum value of the perimeter is greater than $2(2 + 7 + 9)$ or 36, and the maximum value of the perimeter is less than $2(16 + 7 + 9)$ or 64.

45. Sample answers: whether or not the side lengths actually form a triangle, what the smallest and largest angles are, whether the triangle is equilateral, isosceles, or scalene

47.



49. B **51.** H **53.** $y > 6$ or $y < 6$ **55.** 132 mi $< d <$ 618 mi **57.** 15; Alt. Ext. ∠s Thm. **59.** $x = \frac{4}{3} \approx 1.3$; $JK = 4$ **61.** $x = 2$; $JK = KL = JL = 14$ **63.** $x = 7$; $SR = RT = 24$, $ST = 19$

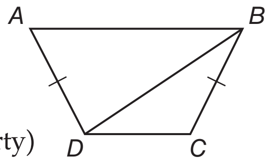
Lesson 5-6

1. $m\angle ACB > m\angle GDE$

3 $\overline{QR} \cong \overline{SR}$, $\overline{TR} \cong \overline{TR}$, and $m\angle QRT < m\angle SRT$. By the Hinge Theorem, $QT < ST$.

5a. $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$ **5b.** $\angle D$; Sample answer: Since $EF > BC$, according to the converse of the Hinge Theorem, $m\angle D > m\angle A$ **7.** $\frac{5}{3} < x < B$

9. Given: $\overline{AD} \cong \overline{CB}$, $DC < AB$
Prove: $m\angle CBD < m\angle ADB$



Statements (Reasons)
1. $\overline{AD} \cong \overline{CB}$ (Given)
2. $\overline{DB} \cong \overline{DB}$ (Reflexive Property)
3. $DC < AB$ (Given)
4. $m\angle CBD < m\angle ADB$ (SSS Inequality)

11. $m\angle MLP < m\angle TSR$

13 $\overline{TU} \cong \overline{VU}$, $\overline{WU} \cong \overline{WU}$, and $WT < WV$. By the Converse of the Hinge Theorem, $m\angle TUW > m\angle VUW$.

15. $JK > HJ$ **17.** $2 < x < 6$

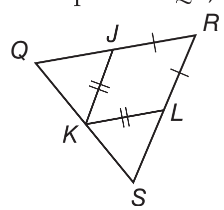
19 Two sides of one triangle are congruent to two sides of the other triangle and the third side of the first triangle is less than the third side of the second triangle. So, by the Converse of the Hinge Theorem, the included angle of the first triangle is less than the included angle of the second triangle.

$0 < x + 20 < 41$ Converse of Hinge Theorem
$-20 < x < 21$ Subtract 20 from each.

The range of values containing x is $-20 < x < 21$.

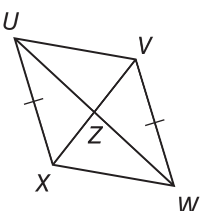
21. $\overline{RS}$; sample answer: The height of the crane is the same and the length of the crane arm is fixed, so according to the Hinge Theorem, the side opposite the smaller angle is shorter. Since 29° < 52°, $RS < MN$.

23. Given: $\overline{LK} \cong \overline{JK}$, $\overline{RL} \cong \overline{RJ}$, K is the midpoint of $\overline{QS}$, $m\angle SKL > m\angle QKJ$
Prove: $RS > QR$



Statements (Reasons)
1. $\overline{LK} \cong \overline{JK}$, $\overline{RL} \cong \overline{RJ}$, K is the midpoint of $\overline{QS}$, $m\angle SKL > m\angle QKJ$ (Given)
2. $SK = QK$ (Def. of midpoint)
3. $SL > QJ$ (Hinge Thm.)
4. $RL = RJ$ (Def. of ≅ segs.)
5. $SL + RL > RL + RJ$ (Add. Prop.)
6. $SL + RL > QJ + RJ$ (Subst.)
7. $RS = SL + RL$, $QR = QJ + RJ$ (Seg. Add. Post.)
8. $RS > QR$ (Subst.)

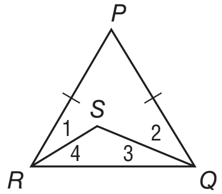
25. Given: $\overline{XU} \cong \overline{VW}$, $VW > XW$, $\overline{XU} \parallel \overline{VW}$
Prove: $m\angle XZU > m\angle UZV$



Statements (Reasons)
1. $\overline{XU} \cong \overline{VW}$, $\overline{XU} \parallel \overline{VW}$ (Given)
2. $\angle UXV \cong \angle XVW$, $\angle XUW \cong \angle UWV$ (Alt. Int. ∠s Thm.)
3. $\triangle XZU \cong \triangle VZW$ (ASA)
4. $\overline{XZ} \cong \overline{VZ}$ (CPCTC)
5. $\overline{WZ} \cong \overline{WZ}$ (Refl. Prop.)
6. $VW > XW$ (Given)
7. $m\angle VZW > m\angle XZW$ (Converse of Hinge Thm.)
8. $\angle VZW \cong \angle XZU$, $\angle XZW \cong \angle VZU$ (Vert. ∠s are ≅.)
9. $m\angle VZW = m\angle XZU$, $m\angle XZW = m\angle VZU$ (Def. of ≅ ∠s)
10. $m\angle XZU > m\angle UZV$ (Subst.)

27 **a.** Sample answer: Use a ruler to measure the distance from her shoulder to her fist for each position. The distance is 1.6 cm for Position 1 and 2 cm for Position 2. Therefore, the distance from her shoulder to her fist is greater in Position 2.
b. Sample answer: In each position, a triangle formed. The distance from her shoulder to her elbow and from her elbow to her wrist is the same in both triangles. Using the measurements in part **a** and the Converse of the Hinge Theorem, you know that the measure of the angle opposite the larger side is larger, so the angle formed by Anica's elbow is greater in Position 2.

29. Given: $\overline{PR} \cong \overline{PQ}$, $SQ > SR$



Prove: $m\angle 1 < m\angle 2$

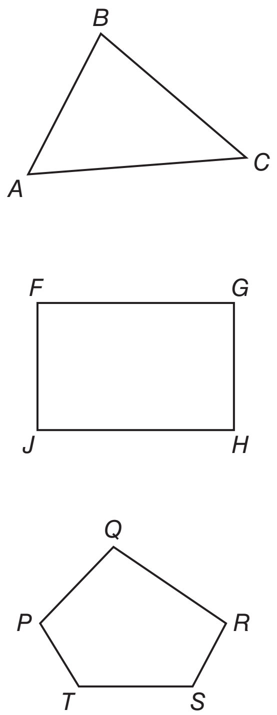
Statements (Reasons)

1. $\overline{PR} \cong \overline{PQ}$ (Given)
2. $\angle PRQ \cong \angle PQR$ (Isos. △ Thm.)
3. $m\angle PRQ = m\angle 1 + m\angle 4$, $m\angle PQR = m\angle 2 + m\angle 3$ (Angle Add. Post.)
4. $m\angle PRQ = m\angle PQR$ (Def. of ≅ ∆s)
5. $m\angle 1 + m\angle 4 = m\angle 2 + m\angle 3$ (Subst.)
6. $SQ > SR$ (Given)
7. $m\angle 4 > m\angle 3$ (Angle Side Relationship Thm.)
8. $m\angle 4 = m\angle 3 + x$ (Def. of inequality)
9. $m\angle 1 + m\angle 4 - m\angle 4 = m\angle 2 + m\angle 3 - (m\angle 3 + x)$ (Subt. Prop.)
10. $m\angle 1 = m\angle 2 - x$ (Subst.)
11. $m\angle 1 + x = m\angle 2$ (Add. Prop.)
12. $m\angle 1 < m\angle 2$ (Def. of inequality)

31. $CB < AB$ **33.** $m\angle BGC < m\angle FBA$

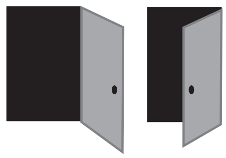
35 $\overline{WZ} \cong \overline{YZ}$, $\overline{ZU} \cong \overline{ZU}$, and $m\angle WZU > m\angle YZU$. By the Hinge Theorem, $WU > YU$.

37a.

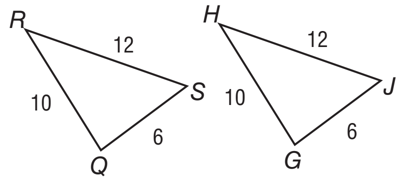


37b. ∠ measures: 59, 76, 45; 90, 90, 90, 90; 105, 100, 96, 116, 123; Sum of ∆s: 180, 360, 540
37c. Sample answer: The sum of the angles of the polygon is equal to 180 times two less than the number of sides of the polygon.
37d. Inductive; sample answer: Since I used a pattern to determine the relationship, the reasoning I used was inductive.
37e. $(n - 2)180$

39.



A door; as the door opens, the door opening increases as the angle made by the hinge increases. As the door closes, the door opening decreases as the angle made by the hinge decreases. This is similar to the side opposite the angle in a triangle, because as the side opposite an angle increases the measure of the angle also increases. As the side decreases, the angle also decreases. **41.** Never; from the Converse of the Hinge Theorem, $\angle ADB < \angle BDC$. $\angle ADB < \angle BDC$ form a linear pair. So, $m\angle ADB + m\angle BDC = 180$. Since, $m\angle BDC > m\angle ADB$, $m\angle BDC$ must be greater than 90 and $m\angle ADB$ must be smaller than 90. So, by the definition of obtuse and acute angles, $m\angle BDC$ is always obtuse and $m\angle ADB$ is always acute. **43.** $2.8 < x < 12$
45. F **47.** 1.2 cm $< n <$ 7.6 cm **49.** 6 m $< n <$ 12 m
51. $x = 8$



53. A
55. $\angle 3$, $\angle ACB$
57. $x = 66$ by the Consecutive Interior Angles Theorem; $y = 35$ by the Consecutive Interior Angles Theorem

Chapter 5 Study Guide and Review

1. false; orthocenter **3.** true **5.** false; median **7.** false; false **9.** false; the vertex opposite that side **11.** 5 **13.** 34 **15.** (2, 3) **17.** $\angle S$, $\angle R$, $\angle T$; $\overline{RT}$, $\overline{TS}$, $\overline{SR}$ **19.** The shorter path is for Sarah to get Irene and then go to Anna's house. **21.** $\triangle FGH$ is not congruent to $\triangle MNO$. **23.** $y \geq 4$
25. Let the cost of one DVD be x, and the cost of the other DVD be y.
Given: $x + y > 50$
Prove: $x > 25$ or $y > 25$
Indirect proof: Step 1 Assume that $x \leq 25$ and $y \leq 25$.
Step 2 If $x \leq 25$ and $y \leq 25$, then $x + y \leq 25 + 25$, or $x + y \leq 50$. This is a contradiction because we know that $x + y > 50$.
Step 3 Since the assumption that $x \leq 25$ and $y \leq 25$ leads to a contradiction of a known fact, the assumption must be false. Therefore, the conclusion that $x > 25$ or $y > 25$ must be true. Thus, at least one DVD had to be over $25.
27. no; $3 + 4 < 8$ **29.** Let x be the length of the third side. 6.5 cm $< x <$ 14.5 cm **31.** $m\angle ABC > m\angle DEF$ **33.** Rose

CHAPTER 6
Quadrilaterals

Chapter 6 Get Ready

1. 150 **3.** 54 **5.** 137 **7.** $x = 1$, $WX = XY = YW = 9$
9. Des Moines to Phoenix = 1153 mi, Des Moines to Atlanta = 738 mi, Phoenix to Atlanta = 1591 mi

Lesson 6-1

1. 1440 **3.** $m\angle X = 36$, $m\angle Y = 72$, $m\angle Z = 144$, $m\angle W = 108$

5 $(n - 2) \cdot 180 = (16 - 2) \cdot 180$ $n = 6$
$= 14 \cdot 180$ or 2520   Simplify.

The sum of the interior angle measures is 2520. So, the measure of one interior angle is $2520 \div 16$ or 157.5.

7. 36 **9.** 68 **11.** 45 **13.** 3240 **15.** 5400

17 $(n - 2) \cdot 180 = (4 - 2) \cdot 180$ $n = 4$
$= 2 \cdot 180$ or 360   Simplify.

The sum of the interior angle measures is 360.

$360 = m\angle J + m\angle K + m\angle L + m\angle M$	Sum of interior angle measures
$360 = (3x - 6) + (x + 10) + x + (2x - 8)$	Substitution
$360 = 7x - 4$	Combine like terms.
$364 = 7x$	Add 4 to each side.
$52 = x$	Simplify.

$m\angle J = 3x - 6 = 3(52) - 6$ or 150 $m\angle K = x + 10 = 52 + 10$ or 62

$m\angle L = x = 52$ $m\angle M = 2x - 8 = 2(52) - 8$ or 96

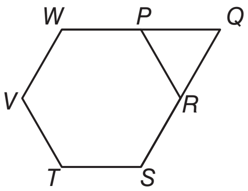
19. $m\angle U = 60$, $m\angle V = 193$, $m\angle W = 76$, $m\angle Y = 68$, $m\angle Z = 143$ **21.** 150 **23.** 144 **25a.** 720 **25b.** Yes, 120; sample answer: Since the measures of the sides of the hexagon are equal, it is regular and the measures of the angles are equal. That means each angle is $720 \div 6$ or 120. **27.** 4 **29.** 15

31 $21 + 42 + 29 + (x + 14) + x + (x - 10) + (x - 20) = 360$
$4x + 76 = 360$
$4x = 284$
$x = 71$

33. 37 **35.** 72 **37.** 24 **39.** 51.4, 128.6 **41.** 25.7, 154.3
43. Consider the sum of the measures of the exterior angles N for an n-gon.
N = sum of measures of linear pairs − sum of measures of interior angles
$= 180n - 180(n - 2)$
$= 180n - 180n + 360$
$= 360$
So, the sum of the exterior angle measures is 360 for any convex polygon. **45.** 105, 110, 120, 130, 135, 140, 160, 170, 180, 190

47 **a.** 60 ft $\div$ 8 = 7.5 ft   Perimeter $\div$ number of sides
b. $(n - 2) \cdot 180 = (8 - 2) \cdot 180$ $n = 8$
$= 6 \cdot 180$ or 1080   Simplify.
Sample answer: The sum of the interior angle measures is 1080. So, the measure of each angle of a regular octagon is $1080 \div 8$ or 135. So if each side of the board makes up half of the angle, each one measures $135 \div 2$ or 67.5.

49. Liam; by the Exterior Angle Sum Theorem, the sum of the measures of any convex polygon is 360.



51. Always; by the Exterior Angle Sum Theorem, $m\angle QPR = 60$ and $m\angle QRP = 60$. Since the sum of the interior angle measures of a triangle is 180, the measure of $\angle PQR = 180 - m\angle QPR - m\angle QRP = 180 - 60 - 60 = 60$. So, $\triangle PQR$ is an equilateral triangle. **53.** The Interior Angles Sum Theorem is derived from the pattern between the number of sides in a polygon and the number of triangles. The formula is the product of the sum of the measures of the angles in a triangle, 180, and the number of triangles in the polygon. **55.** 72 **57** C **59.** $ML < JM$ **61.** 3 **63.** $\angle E \cong \angle G$; $\angle EFH \cong \angle GHF$; $\angle EHF \cong \angle GFH$; $\overline{EF} \cong \overline{GH}$; $\overline{EH} \cong \overline{GF}$; $\overline{FH} \cong \overline{HF}$; $\triangle EFH \cong \triangle GHF$ **65.** $\angle 1$ and $\angle 5$, $\angle 4$ and $\angle 6$, $\angle 2$ and $\angle 8$, $\angle 3$ and $\angle 7$

Lesson 6-2

1a. 148 **1b.** 125 **1c.** 4 **3.** 15 **5.** $w = 5$, $b = 4$
7. Given: $\square ABCD$, $\angle A$ is a right angle.
Prove: $\angle B$, $\angle C$, and $\angle D$ are right angles. (Theorem 6.6)
Proof: By definition of a parallelogram, $\overline{AB} \parallel \overline{CD}$. Since $\angle A$ is a right angle, $\overline{AC} \perp \overline{AB}$. By the Perpendicular Transversal Theorem, $\overline{AC} \perp \overline{CD}$. $\angle C$ is a right angle, because perpendicular lines form a right angle. $\angle B \cong \angle C$ and $\angle A \cong \angle D$ because opposite angles in a parallelogram are congruent. $\angle C$ and $\angle D$ are right angles, since all right angles are congruent.

9

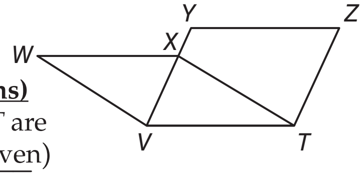
$m\angle R + m\angle Q = 180$	Consecutive angles are supplementary.
$m\angle R + 128 = 180$	Substitution
$m\angle R = 52$	Subtract 128 from each side.

11. 5

13

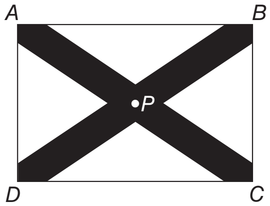
a. $\overline{JH} \cong \overline{FG}$	Opposite sides are congruent.
$JH = FG$	Definition of congruence
$= 1$ in.	Substitution
b. $\overline{GH} \cong \overline{FJ}$	Opposite sides are congruent.
$GH = FJ$	Definition of congruence
$= \frac{3}{4}$ in.	Substitution
c. $\angle JFG \cong \angle JHG$	Opposite angles are congruent.
$m\angle JFG = m\angle JHG$	Definition of congruence
$= 62$	Substitution
d. $m\angle FJH + m\angle JHG = 180$	Consecutive angles are supplementary.
$m\angle FJH + 62 = 180$	Substitution
$m\angle FJH = 118$	Subtract 62 from each side.

15. $a = 7$, $b = 11$ **17.** $x = 5$, $y = 17$ **19.** $x = 58$, $y = 63.5$ **21.** (2.5, 2.5)
23. Given: $WXTV$ and $ZYVT$ are parallelograms.
Prove: $\overline{WX} \cong \overline{ZY}$



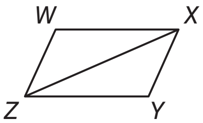
Proof:
Statements (Reasons)
1. $WXTV$ and $ZYVT$ are parallelograms. (Given)
2. $\overline{WX} \cong \overline{VT}$, $\overline{VT} \cong \overline{YZ}$ (Opp. sides of a $\square$ are $\cong$.)
3. $\overline{WX} \cong \overline{ZY}$ (Trans. Prop.)

25. Given: $\triangle ACD \cong \triangle CAB$
Prove: $\overline{DP} \cong \overline{PB}$



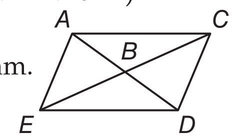
Proof:
Statements (Reasons)
1. $\triangle ACD \cong \triangle CAB$ (Given)
2. $\angle ACD \cong \angle CAB$ (CPCTC)
3. $\angle DPC \cong \angle BPA$ (Vert. ∡ are ≅.)
4. $\overline{AB} \cong \overline{CD}$ (CPCTC)
5. $\triangle ABP \cong \triangle CDP$ (AAS)
6. $\overline{DP} \cong \overline{PB}$ (CPCTC)

27. Given: ▱$WXYZ$
Prove: $\triangle WXZ \cong \triangle YZX$ (Theorem 6.8)
Proof:



Statements (Reasons)
1. ▱$WXYZ$ (Given)
2. $\overline{WX} \cong \overline{ZY}$, $\overline{WZ} \cong \overline{XY}$ (Opp. sides of a ▱ are ≅.)
3. $\angle ZWX \cong \angle XYZ$ (Opp. ∡ of a ▱ are ≅.)
4. $\triangle WXZ \cong \triangle YZX$ (SAS)

29. Given: $ACDE$ is a parallellogram.
Prove: $\overline{EC}$ bisects $\overline{AD}$. (Theorem 6.7)



Proof: It is given that $ACDE$ is a parallelogram. Since opposite sides of a parallelogram are congruent, $\overline{EA} \cong \overline{DC}$. By definition of a parallelogram, $\overline{EA} \parallel \overline{DC}$. $\angle AEB \cong \angle DCB$ and $\angle EAB \cong \angle CDB$ because alternate interior angles are congruent. $\triangle EBA \cong \triangle CBD$ by ASA. $\overline{EB} \cong \overline{BC}$ and $\overline{AB} \cong \overline{BD}$ by CPCTC. By the definition of segment bisector, $\overline{EC}$ bisects $\overline{AD}$ and $\overline{AD}$ bisects $\overline{EC}$.

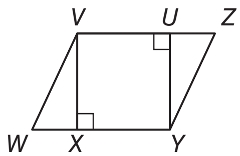
31. 3

33 $\angle AFB$ and $\angle BFC$ form a linear pair.

$\angle AFB + \angle BFC = 180$	Supplement Theorem
$\angle AFB + 49 = 180$	Substitution
$\angle AFB = 131$	Subtract 49 from each side.

35. 29 **37.** $(-1, -1)$; Sample answer: Opposite sides of a parallelogram are parallel. Since the slope of $\overline{BC} = \frac{-6}{2}$, the slope of $\overline{AD}$ must also be $\frac{-6}{2}$. To locate vertex D, start from vertex A and move down 6 and right 2.

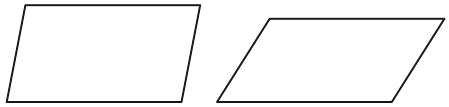
39 First, use properties involving opposite angles and opposite sides of a parallelogram to help prove that $\triangle YUZ$ and $\triangle VXW$ are right angles. Then use the Hypotenuse-Angle Congruence Theorem to prove the triangles are congruent.
Given: ▱$YWVZ$, $\overline{VX} \perp \overline{WY}$, $\overline{YU} \perp \overline{VZ}$
Prove: $\triangle YUZ \cong \triangle VXW$
Proof:



Statements (Reasons)
1. ▱$YWVZ$, $\overline{VX} \perp \overline{WY}$, $\overline{YU} \perp \overline{VZ}$ (Given)
2. $\angle Z \cong \angle W$ (Opp. ∡ of a ▱ are ≅.)
3. $\overline{WV} \cong \overline{ZY}$ (Opp. sides of a ▱ are ≅.)
4. $\angle VXW$ and $\angle YUZ$ are rt. ∡. (⊥ lines form four rt. ∡.)
5. $\triangle VXW$ and $\triangle YUZ$ are rt. △s. (Def. of rt. △s)
6. $\triangle YUZ \cong \triangle VXW$ (HA)

41. 7

43.



45. Sample answer: In a parallelogram, the opposite sides and angles are congruent. Two consecutive angles in a parallelogram are supplementary. If one angle of a parallelogram is right, then all the angles are right. The diagonals of a parallelogram bisect each other. **47.** 13 **49.** B **51.** 9 **53.** 18 **55.** 100
57. not a polyhedron; cylinder
59. not a polyhedron; cone
61. diagonal; $-\frac{4}{5}$

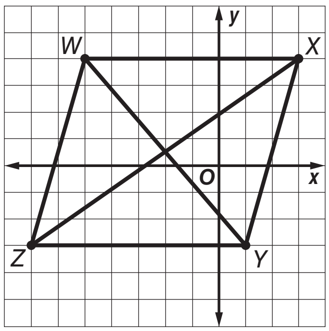
Lesson 6-3

1. Yes; each pair of opposite angles are congruent.
3. $AP = CP$, $BP = DP$; sample answer: If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram, so if $AP = CP$ and $BP = DP$, then the string forms a parallelogram.

5 If both pairs of opposite sides are congruent, then the quadrilateral is a parallelogram.

$2x + 3 = x + 7$	Congruent sides have equal measures.
$x + 3 = 7$	Subtract x from each side.
$x = 4$	Subtract 3 from each side.
$3y - 5 = y + 11$	Congruent sides have equal measures.
$2y - 5 = 11$	Subtract y from each side.
$2y = 16$	Add 5 to each side.
$y = 8$	Divide each side by 2.

7



If the diagonals bisect each other, then it is a parallelogram.

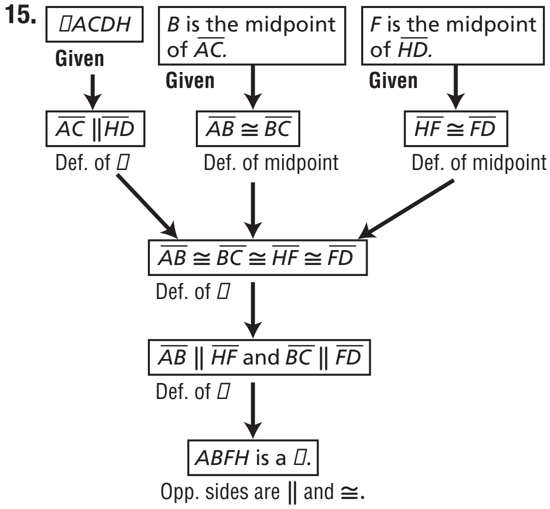
midpoint of $\overline{WY}$: $\left(\frac{-5+1}{2}, \frac{4+(-3)}{2}\right) = \left(-2, \frac{1}{2}\right)$

midpoint of $\overline{XZ}$: $\left(\frac{3+(-7)}{2}, \frac{4+(-3)}{2}\right) = \left(-2, \frac{1}{2}\right)$

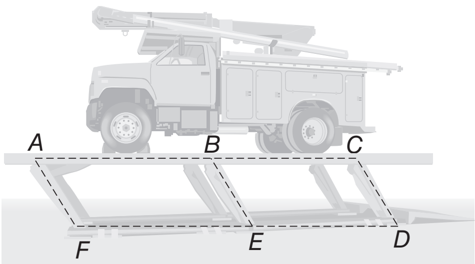
The midpoint of $\overline{WY}$ and $\overline{XZ}$ is $\left(-2, \frac{1}{2}\right)$. Since the diagonals bisect each other, $WXYZ$ is a parallelogram.

9. Yes; both pairs of opposite sides are congruent.
11. No; none of the tests for parallelograms are fulfilled. **13.** Yes; the diagonals bisect each other.

15.



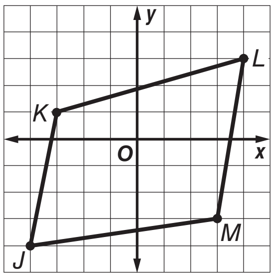
17. Given: *ABEF* is a parallelogram; *BCDE* is a parallelogram.
Prove: *ACDF* is a parallelogram.



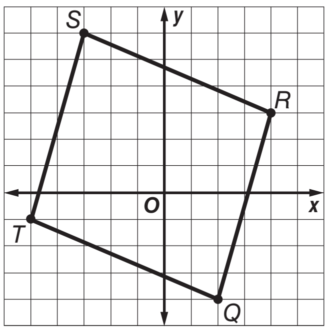
Proof:
Statements (Reasons)
1. *ABEF* is a parallelogram; *BCDE* is a parallelogram. (Given)
2. $\overline{AF} \cong \overline{BE}$, $\overline{BE} \cong \overline{CD}$, $\overline{AF} \parallel \overline{BE}$, $\overline{BE} \parallel \overline{CD}$ (Def. of ▱)
3. $\overline{AF} \cong \overline{CD}$, $\overline{AF} \parallel \overline{CD}$ (Trans. Prop.)
4. *ACDF* is a parallelogram. (If one pair of opp. sides is ≅ and ∥, then the quad. is a ▱.)

19. $x = 8, y = 9$ **21.** $x = 11, y = 7$ **23.** $x = 4, y = 3$

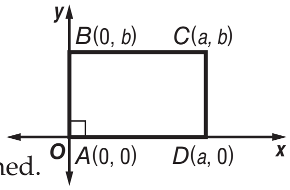
25. No; both pairs of opposite sides must be congruent. The distance between *K* and *L* is $\sqrt{53}$. The distance between *L* and *M* is $\sqrt{37}$. The distance between *M* and *J* is $\sqrt{50}$. The distance between *J* and *K* is $\sqrt{26}$. Since, both pairs of opposite sides are not congruent, *JKLM* is not a parallelogram.



27. Yes; a pair of opposite sides must be parallel and congruent. Slope of $\overline{QR} = \frac{7}{2}$ = slope of $\overline{ST}$, so $\overline{QR} \parallel \overline{ST}$. The distance between *Q* and $R = \sqrt{53}$ = distance between *S* and *T*, so $\overline{QR} \cong \overline{ST}$. So, *QRST* is a parallelogram.



29. Given: *ABCD* is a parallelogram. ∠*A* is a right angle.
Prove: ∠*B*, ∠*C*, and ∠*D* are right angles.



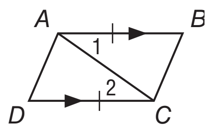
Proof:
slope of $\overline{BC} = \left(\frac{b-b}{a-0}\right)$ or 0
The slope of $\overline{CD}$ is undefined.
slope of $\overline{AD} = \left(\frac{0-0}{a-0}\right)$ or 0
The slope of $\overline{AB}$ is undefined.
Therefore, $\overline{BC} \perp \overline{CD}$, $\overline{CD} \perp \overline{AD}$, and $\overline{AB} \perp \overline{BC}$. So, ∠*B*, ∠*C*, and ∠*D* are right angles.

31 **a.** Use the Segment Addition Postulate to rewrite *AC* and *CF* each as the sum of two measures. Then use substitution, the Subtraction Property, and the Transitive Property to show that $\overline{BC} \cong \overline{DE}$. By proving that *BCDE* is a parallelogram, you can prove that $\overline{BE} \parallel \overline{CD}$.
Given: $\overline{AC} \cong \overline{CF}$, $\overline{AB} \cong \overline{CD} \cong \overline{BE}$, and $\overline{DF} \cong \overline{DE}$
Prove: $\overline{BE} \parallel \overline{CD}$
Proof: We are given that $\overline{AC} \cong \overline{CF}$, $\overline{AB} \cong \overline{CD} \cong \overline{BE}$, and $\overline{DF} \cong \overline{DE}$. $AC = CF$ by the definition of congruence. $AC = AB + BC$ and $CF = CD + DF$ by the Segment Addition Postulate and $AB + BC = CD + DF$ by substitution. Using substitution again, $AB + BC = AB + DF$, and $BC = DF$ by the Subtraction Property. $\overline{BC} \cong \overline{DF}$ by the definition of congruence, and $\overline{BC} \cong \overline{DE}$ by the Transitive Property. If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram, so *BCDE* is a parallelogram. By the definition of a parallelogram, $\overline{BE} \parallel \overline{CD}$.
b. Let x = width of the copy. If $AB = 12$, then $CD = BE = 12$. So, $CF = 12 + 8$ or 20.

$\frac{CF}{BE} = \frac{\text{width of copy}}{\text{width of original}}$	
$\frac{20}{12} = \frac{x}{5.5}$	Substitution
$20(5.5) = 12x$	Cross multiply.
$110 = 12x$	Simplify.
$9.2 \approx x$	Divide each side by 12.

The width of the original object is about 9.2 in.

33. Given: $\overline{AB} \cong \overline{DC}$, $\overline{AB} \parallel \overline{DC}$
Prove: *ABCD* is a parallelogram.



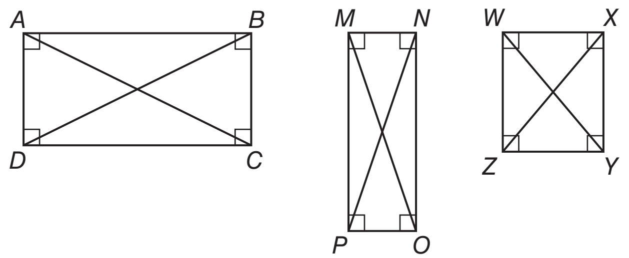
Proof:
Statements (Reasons)
1. $\overline{AB} \cong \overline{DC}$, $\overline{AB} \parallel \overline{DC}$ (Given)
2. Draw $\overline{AC}$. (Two points determine a line.)
3. ∠1 ≅ ∠2 (If two lines are ∥, then alt. int. ∠s are ≅.)
4. $\overline{AC} \cong \overline{AC}$ (Refl. Prop.)
5. △*ABC* ≅ △*CDA* (SAS)
6. $\overline{AD} \cong \overline{BC}$ (CPCTC)
7. *ABCD* is a parallelogram. (If both pairs of opp. sides are ≅, then the quad. is ▱.)

35 Opposite sides of a parallelogram are parallel and congruent. Since the slope of $\overline{AB}$ is 0, the slope of $\overline{DC}$ must be 0. The y-coordinate of vertex *D* must be *c*. The length of $\overline{AB}$ is $(a + b) - 0$ or $a + b$. So the length of $\overline{DC}$

must be $a + b$. If the x-coordinate of D is $-b$ and the x-coordinate of C is a, then the length of $\overline{DC}$ is $a - (-b)$ or $a + b$. So, the coordinates are $C(a, c)$ and $D(-b, c)$.

37 Sample answer: Since the two vertical rails are both perpendicular to the ground, he knows that they are parallel to each other. If he measures the distance between the two rails at the top of the steps and at the bottom of the steps, and they are equal, then one pair of sides of the quadrilateral formed by the handrails is both parallel and congruent, so the quadrilateral is a parallelogram. Since the quadrilateral is a parallelogram, the two hand rails are parallel by definition.

39a. Sample answer:



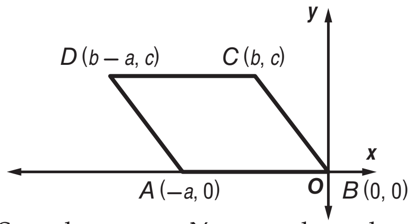
39b. Sample answer:

Rectangle	Side	Length
ABCD	$\overline{AC}$	3.3 cm
	$\overline{BD}$	3.3 cm
MNOP	$\overline{MO}$	2.8 cm
	$\overline{NP}$	2.8 cm
WXYZ	$\overline{WY}$	2.0 cm
	$\overline{XZ}$	2.0 cm

39c. Sample answer: The diagonals of a rectangle are congruent.

41. Sample answer: The theorems are converses of each other. The hypothesis of Theorem 6-3 is "a figure is a parallelogram," and the hypothesis of 6-9 is "both pairs of opposite sides of a quadrilateral are congruent." The conclusion of Theorem 6-3 is "opposite sides are congruent," and the conclusion of 6-9 is "the quadrilateral is a parallelogram."

43.



45. Sample answer: You can show that: both pairs of opposite sides are congruent or parallel, both pairs of opposite angles are congruent, diagonals bisect each other, or one pair of opposite sides is both congruent and parallel. **47.** 4 **49.** E **51.** (4.5, 1.5) **53.** 35

55. Given: $P + W > 2$ (P is time spent in the pool; W is time spent lifting weights.)

Prove: $P > 1$ or $W > 1$

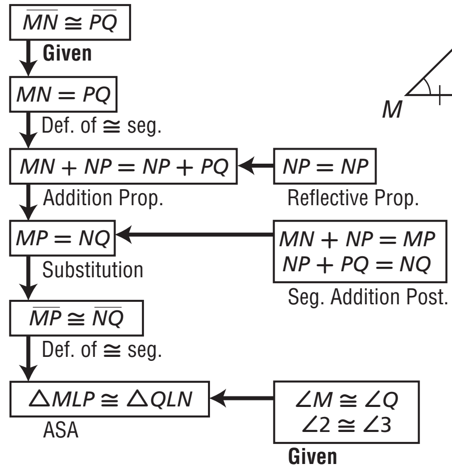
Proof:

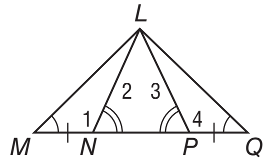
Step 1: Assume $P \leq 1$ and $W \leq 1$.

Step 2: $P + W \leq 2$

Step 3: This contradicts the given statement. Therefore he did at least one of these activities for more than an hour.

57.





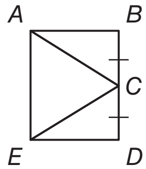
59. not perpendicular

Lesson 6-4

1. 7 ft **3.** 33.5 **5.** 11

7. Given: $ABDE$ is a rectangle; $\overline{BC} \cong \overline{DC}$.

Prove: $\overline{AC} \cong \overline{EC}$



Proof:

Statements (Reasons)

1. $ABDE$ is a rectangle; $\overline{BC} \cong \overline{DC}$. (Given)

2. $ABDE$ is a parallelogram. (Def. of rectangle)

3. $\overline{AB} \cong \overline{DE}$ (Opp. sides of a ▱ are ≅.)

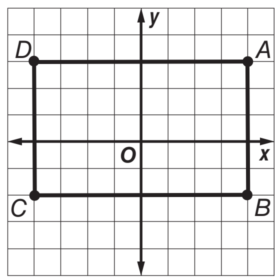
4. $\angle B$ and $\angle D$ are right angles. (Def. of rectangle)

5. $\angle B \cong \angle D$ (All rt. ∠s are ≅.)

6. $\triangle ABC \cong \triangle EDC$ (SAS)

7. $\overline{AC} \cong \overline{EC}$ (CPCTC)

9. Yes; $AB = 5 = CD$ and $BC = 8 = AD$. So, $ABCD$ is a parallelogram. $BD = \sqrt{89} = AC$, so the diagonals are congruent. Thus, $ABCD$ is a rectangle.



11 Since all angles in a rectangle are right angles, $\angle ABD$ is a right triangle.

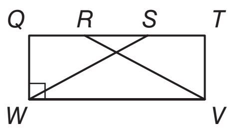
$AD^2 + AB^2 = BD^2$	Pythagorean Theorem
$2^2 + 6^2 = BD^2$	Substitution
$40 = BD^2$	Simplify.
$6.3 \approx BD$	Take the positive square root of each side.

$BD \approx 6.3$ ft

13. 25 **15.** 43 **17.** 38 **19.** 46

21. Given: $QTVW$ is a rectangle; $\overline{QR} \cong \overline{ST}$.

Prove: $\triangle SWQ \cong \triangle RVT$



Proof:

Statements (Reasons)

1. $QTVW$ is a rectangle; $\overline{QR} \cong \overline{ST}$. (Given)

2. $QTVW$ is a parallelogram. (Def. of rectangle)

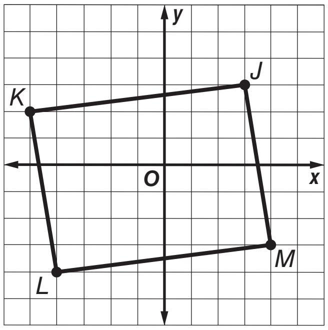
3. $\overline{WQ} \cong \overline{VT}$ (Opp sides of a ▱ are ≅.)

4. $\angle Q$ and $\angle T$ are right angles. (Def. of rectangle)

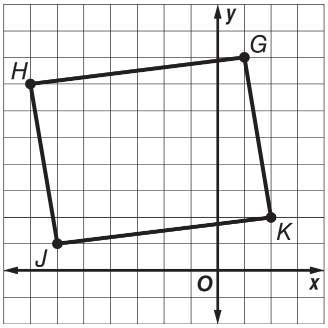
5. $\angle Q \cong \angle T$ (All rt. ∠s are ≅.)

6. $QR = ST$ (Def. of ≅ segs.)
7. $\overline{RS} \cong \overline{RS}$ (Refl. Prop.)
8. $RS = RS$ (Def. of ≅ segs.)
9. $QR + RS = RS + ST$ (Add. prop.)
10. $QS = QR + RS$, $RT = RS + ST$ (Seg. Add. Post.)
11. $QS = RT$ (Subst.)
12. $\overline{QS} \cong \overline{RT}$ (Def. of ≅ segs.)
13. $\triangle SWQ \cong \triangle RVT$ (SAS)

23. No; $JK = \sqrt{65} = LM$, $KL = \sqrt{37} = MJ$, so $JKLM$ is a parallelogram. $KM = \sqrt{106}$; $JL = \sqrt{98}$. $KM \neq JL$, so the diagonals are not congruent. Thus, $JKLM$ is not a rectangle.



25. No; slope of $\overline{GH} = \frac{1}{8}$ = slope of $\overline{JK}$ and slope of $\overline{HJ} = -6$ = slope of $\overline{KG}$. So, $GHJK$ is a parallelogram. The product of the slopes of consecutive sides $\neq -1$, so the consecutive sides are not perpendicular. Thus, $GHJK$ is not a rectangle. **27.** 40

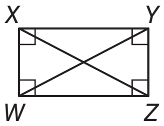


29 A rectangle has four right angles, so $m\angle CDB = 90$. Since a rectangle is a parallelogram, opposite sides are parallel. Alternate interior angles of parallel lines are congruent. So, $\angle 3 \cong \angle 2$ and $m\angle 3 = m\angle 2 = 40$. $m\angle 4 = 90 - 40$ or 50. Since diagonals of a rectangle are congruent and bisect each other, the triangle with angles 4, 5, and 6 is isosceles with $m\angle 6 = m\angle 4$.

$m\angle 4 + m\angle 5 + m\angle 6 = 180$	Triangle Angle-Sum Theorem
$m\angle 4 + m\angle 5 + m\angle 4 = 180$	$m\angle 6 = m\angle 4$
$50 + m\angle 5 + 50 = 180$	Substitution
$100 + m\angle 5 = 180$	Simplify.
$m\angle 5 = 80$	Subtract 100 from each side.

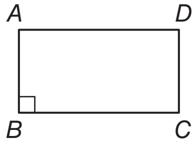
31. 100

33. Given: $WXYZ$ is a rectangle with diagonals $\overline{WY}$ and $\overline{XZ}$.
Prove: $\overline{WY} \cong \overline{XZ}$
Proof:

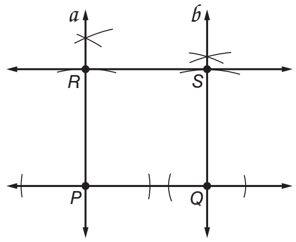


__Statements (Reasons)__
1. $WXYZ$ is a rectangle with diagonals $\overline{WY}$ and $\overline{XZ}$. (Given)
2. $\overline{WX} \cong \overline{ZY}$ (Opp. sides of a ▱ are ≅.)
3. $\overline{WZ} \cong \overline{WZ}$ (Refl. Prop.)
4. $\angle XWZ$ and $\angle YZW$ are right angles. (Def. of ▭)
5. $\angle XWZ \cong \angle YZW$ (All right ∠s are ≅.)
6. $\triangle XWZ \cong \triangle YZW$ (SAS)
7. $\overline{WY} \cong \overline{XZ}$ (CPCTC)

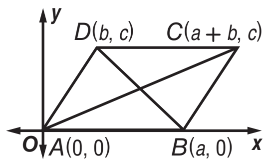
35. $ABCD$ is a parallelogram, and $\angle B$ is a right angle. Since $ABCD$ is a parallelogram and has one right angle, then it has four right angles. So by the definition of a rectangle, $ABCD$ is a rectangle.



37. Sample answer: Since $\overline{RP} \perp \overline{PQ}$ and $\overline{SQ} \perp \overline{PQ}$, $m\angle P = m\angle Q = 90$. Lines that are perpendicular to the same line are parallel, so $\overline{RP} \parallel \overline{SQ}$. The same compass setting was used to locate points R and S, so $\overline{RP} \cong \overline{SQ}$. If one pair of opposite sides of a quadrilateral is both parallel and congruent, then the quadrilateral is a parallelogram. A parallelogram with right angles is a rectangle. Thus, $PRSQ$ is a rectangle. **39.** 5 **41.** No; sample answer: Both pairs of opposite sides are congruent, so the sign is a parallelogram, but no measure is given that can be used to prove that it is a rectangle.



43 Draw ▱ $ABCD$ on the coordinate plane. Find the slopes of $\overline{AD}$ and $\overline{AB}$ and show that the lines are perpendicular.



Given: ▱$ABCD$ and $\overline{AC} \cong \overline{BD}$
Prove: ▱$ABCD$ is a rectangle.
Proof:
$AC = \sqrt{(a + b - 0)^2 + (c - 0)^2}$
$BD = \sqrt{(b - a)^2 + (c - 0)^2}$
But $AC = BD$ and
$$\sqrt{(a + b - 0)^2 + (c - 0)^2} = \sqrt{(b - a)^2 + (c - 0)^2}.$$
$$(a + b - 0)^2 + (c - 0)^2 = (b - a)^2 + (c - 0)^2$$
$$(a + b)^2 + c^2 = (b - a)^2 + c^2$$
$$a^2 + 2ab + b^2 + c^2 = b^2 - 2ab + a^2 + c^2$$
$$2ab = -2ab$$
$$4ab = 0$$
$$a = 0 \text{ or } b = 0$$
Because A and B are different points, $a \neq 0$. Then $b = 0$. The slope of $\overline{AD}$ is undefined and the slope of $\overline{AB} = 0$. Thus, $\overline{AD} \perp \overline{AB}$. $\angle DAB$ is a right angle and $ABCD$ is a rectangle.

45. $x = 6$, $y = -10$ **47.** 6 **49.** Sample answer: All rectangles are parallelograms because, by definition, both pairs of opposite sides are parallel. Parallelograms with right angles are rectangles, so some parallelograms are rectangles, but others with non-right angles are not. **51.** J **53.** E **55.** $x = 8$, $y = 22$ **57.** (2.5, 0.5) **59.** $\overline{AH}$ and $\overline{AJ}$ **61.** $\angle AJK$ and $\angle AKJ$ **63.** $\sqrt{101}$

Lesson 6-5

1. 32
3. Given: $ABCD$ is a rhombus with diagonal $\overline{DB}$.
Prove: $\overline{AP} \cong \overline{CP}$
Proof:

Statements (Reasons)

1. $ABCD$ is a rhombus with diagonal $\overline{DB}$. (Given)
2. $\angle ABP \cong \angle CBP$ (Diag. of rhombus bisects ∠)
3. $\overline{PB} \cong \overline{PB}$ (Refl. Prop.)
4. $\overline{AB} \cong \overline{CB}$ (Def. of rhombus)
5. $\triangle APB \cong \triangle CPB$ (SAS)
6. $\overline{AP} \cong \overline{CP}$ (CPCTC)

5. Rectangle, rhombus, square; consecutive sides are perpendicular, all sides are congruent. **7.** 14 **9.** 28

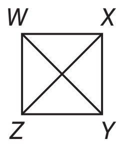
11

$m\angle ABC + m\angle BCD = 180$	Consecutive ∡ are supp.
$(2x - 7) + (2x + 3) = 180$	Substitution
$4x - 4 = 180$	Simplify.
$4x = 184$	Add 4 to each side.
$x = 46$	Divide each side by 4.

$m\angle BCD = 2x + 3$	Given
$= 2(46) + 3$ or 95	Substitution

$\angle DAB \cong \angle BCD$	Opposite angles are congruent.
$m\angle DAB = m\angle BCD$	Definition of congruence
$= 95$	Substitution

13. Given: $\overline{WZ} \parallel \overline{XY}$, $\overline{WX} \parallel \overline{ZY}$, $\overline{WZ} \cong \overline{ZY}$
Prove: $WXYZ$ is a rhombus.

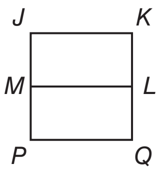


Proof:

Statements (Reasons)

1. $\overline{WZ} \parallel \overline{XY}$, $\overline{WX} \parallel \overline{ZY}$, $\overline{WZ} \cong \overline{ZY}$ (Given)
2. $WXYZ$ is a ▱. (Both pairs of opp. sides are ∥.)
3. $WXYZ$ is a rhombus. (If one pair of consecutive sides of a ▱ are ≅, the ▱ is a rhombus.)

15. Given: $JKQP$ is a square. $\overline{ML}$ bisects $\overline{JP}$ and $\overline{KQ}$.
Prove: $JKLM$ is a parallelogram.



Proof:

Statements (Reasons)

1. $JKQP$ is a square. $\overline{ML}$ bisects $\overline{JP}$ and $\overline{KQ}$. (Given)
2. $JKQP$ is a parallelogram. (All squares are parallelograms.)
3. $\overline{JM} \parallel \overline{KL}$ (Def. of ▱)
4. $\overline{JP} \cong \overline{KQ}$ (Opp. Sides of ▱ are ≅)
5. $JP = KQ$ (Def of ≅ segs.)
6. $JM = MP$, $KL = LQ$ (Def. of bisects)
7. $JP = JM + MP$, $KQ = KL + LQ$ (Seg. Add Post.)
8. $JP = 2JM$, $KQ = 2KL$ (Subst.)
9. $2JM = 2KL$ (Subst.)
10. $JM = KL$ (Division Prop.)
11. $\overline{KL} \cong \overline{JM}$ (Def. of ≅ segs.)
12. $JKLM$ is a parallelogram. (If one pair of opp. sides is ≅ and ∥, then the quad. is a ▱.)

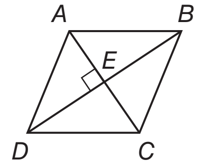
17. Rhombus; Sample answer: The measure of angle formed between the two streets is 29, and vertical angles are congruent, so the measure of one angle of the quadrilateral is 29. Since the crosswalks are the same length, the sides of the quadrilateral are congruent. Therefore, they form a rhombus. **19.** Rhombus; the diagonals are perpendicular. **21.** None; the diagonals are not congruent or perpendicular.

23 The diagonals of a rhombus are perpendicular, so $\triangle ABP$ is a right triangle.

$AP^2 + PB^2 = AB^2$	Pythagorean Theorem
$AP^2 + 12^2 = 15^2$	Substitution
$AP^2 + 144 = 225$	Simplify.
$AP^2 = 81$	Subtract 144 from each side.
$AP = 9$	Take the square root of each side.

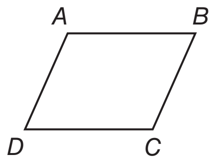
25. 24 **27.** 6 **29.** 90 **31.** square **33.** rectangle

35. Given: $ABCD$ is a parallelogram; $\overline{AC} \perp \overline{BD}$.
Prove: $ABCD$ is a rhombus.



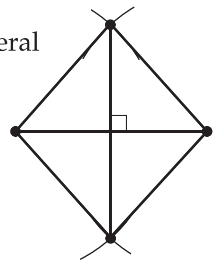
Proof: We are given that $ABCD$ is a parallelogram. The diagonals of a parallelogram bisect each other, so $\overline{AE} \cong \overline{EC}$. $\overline{BE} \cong \overline{BE}$ because congruence of segments is reflexive. We are also given that $\overline{AC} \perp \overline{BD}$. Thus, $\angle AEB$ and $\angle BEC$ are right angles by the definition of perpendicular lines. Then $\angle AEB \cong \angle BEC$ because all right angles are congruent. Therefore, $\triangle AEB \cong \triangle CEB$ by SAS. $\overline{AB} \cong \overline{CB}$ by CPCTC. Opposite sides of parallelograms are congruent, so $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$. Then since congruence of segments is transitive, $\overline{AB} \cong \overline{CD} \cong \overline{BC} \cong \overline{AD}$. All four sides of $ABCD$ are congruent, so $ABCD$ is a rhombus by definition.

37. Given: $ABCD$ is a parallelogram; $\overline{AB} \cong \overline{BC}$.
Prove: $ABCD$ is a rhombus.

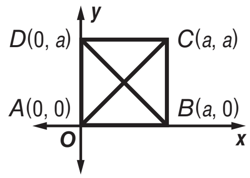


Proof: Opposite sides of a parallelogram are congruent, so $\overline{BC} \cong \overline{AD}$ and $\overline{AB} \cong \overline{CD}$. We are given that $\overline{AB} \cong \overline{BC}$. So, by the Transitive Property, $\overline{BC} \cong \overline{CD}$. So, $\overline{BC} \cong \overline{CD} \cong \overline{AB} \cong \overline{AD}$. Thus, $ABCD$ is a rhombus by definition.

39. Sample answer: The diagonals bisect each other, so the quadrilateral is a parallelogram. Since the diagonals of the parallelogram are perpendicular to each other, the parallelogram is a rhombus.



41. Given: $ABCD$ is a square.
Prove: $\overline{AC} \perp \overline{DB}$



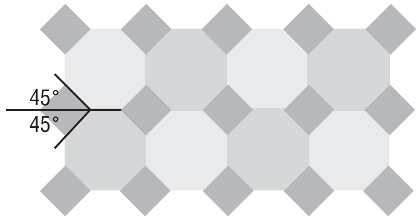
Proof:

slope of $\overline{DB} = \frac{0 - a}{a - 0}$ or -1

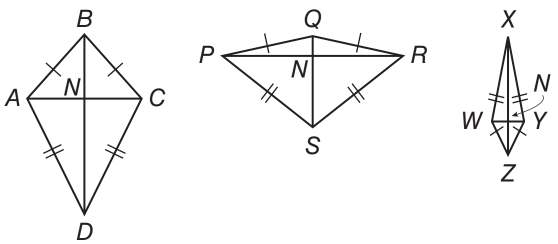
slope of $\overline{AC} = \frac{0 - a}{0 - a}$ or 1

The slope of $\overline{AC}$ is the negative reciprocal of the slope of $\overline{DB}$, so they are perpendicular.

43 Use the properties of exterior angles to classify the quadrilaterals. Sample answer: Since the octagons are regular, each side is congruent, and the quadrilaterals share common sides with the octagons, so the quadrilaterals are either rhombuses or squares. The vertices of the quadrilaterals are formed by the exterior angles of the sides of the octagons adjacent to the vertices. The sum of the measures of the exterior angles of a polygon is always 360 and since a regular octagon has 8 congruent exterior angles, each one measures 360 ÷ 8 or 45. As shown in the diagram, each angle of the quadrilaterals in the pattern measures 45 + 45 or 90. Therefore, the quadrilateral is a square.



45a.



45b.

Figure	Distance from N to each vertex along shorter diagonal		Distance from N to each vertex along longer diagonal	
ABCD	0.8 cm	0.8 cm	0.9 cm	1.5 cm
PQRS	1.2 cm	1.2 cm	0.3 cm	0.9 cm
WXYZ	0.2 cm	0.2 cm	1.1 cm	0.4 cm

45c. Sample answer: The shorter diagonal of a kite is bisected by the longer diagonal. **47.** True; sample answer: A rectangle is a quadrilateral with four right angles and a square is both a rectangle and a rhombus, so a square is always a rectangle. Converse: If a quadrilateral is a rectangle then it is a square. False; sample answer: A rectangle is a quadrilateral with four right angles. It is not necessarily a rhombus, so it is not necessarily a square. Inverse: If a quadrilateral is not a square, then it is not a rectangle. False; sample answer: A quadrilateral that has four right angles and two pairs of congruent sides is not a square, but it is a rectangle. Contrapositive: If a quadrilateral is not a rectangle, then it is not a square. True; sample answer: If a quadrilateral is not a rectangle, it is also not a square by definition. **49.** Sample answer: (0, 0), (6, 0), (0, 6), (6, 6) ; the diagonals are perpendicular, and any four points on the lines equidistant from the intersection of the lines will be the vertices of a square. **51.** B **53.** H **55.** 52 **57.** 38 **59.** Yes; both pairs of opposite sides are congruent. **61.** No; the Triangle Inequality Theorem states that the sum of the lengths of any two sides of a triangle must be greater than the length of the third side. Since 22 + 23 = 45, the sides of Monifa's backyard cannot be 22 ft, 23 ft and 45 ft. **63.** 2 **65.** $\frac{5}{4}$

Lesson 6-6

1. 101 **3.** $\overline{BC} \parallel \overline{AD}$, $\overline{AB} \nparallel \overline{CD}$; *ABCD* is a trapezoid. **5.** 1.2 **7.** 70 **9.** 70

11

$\overline{XZ} \cong \overline{YW}$	Diagonals are congruent.
$XZ = YW$	Definition of congruence.
$18 = YW$	Substitution
$18 = YP + PW$	Segment Addition Postulate
$18 = 3 + PW$	Substitution
$15 = PW$	Subtract 3 from each side.

13. $\overline{JK} \parallel \overline{LM}$, $\overline{KL} \nparallel \overline{JM}$; *JKLM* is a trapezoid, but not isosceles since $KL = \sqrt{26}$ and $JM = 5$. **15.** $\overline{XY} \parallel \overline{WZ}$, $\overline{WX} \nparallel \overline{YZ}$; *WXYZ* is a trapezoid, but not isosceles since $XZ = \sqrt{74}$ and $WY = \sqrt{68}$. **17.** 10 **19.** 8 **21.** 17

23 The G key represents the midsegment of the trapezoid. Use the Trapezoid Midsegment Theorem to find the length of the key.

length of G key $= \frac{1}{2}$(length of C key + length of D key)

$= \frac{1}{2}(6 + 1.8)$ Substitution

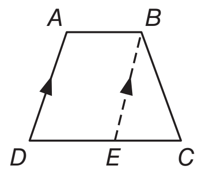
$= \frac{1}{2}(7.8)$ or 3.9 Simplify.

The length of the G key is 3.9 in.

25. $\sqrt{20}$ **27.** 75

29. Given: *ABCD* is a trapezoid; $\angle D \cong \angle C$.

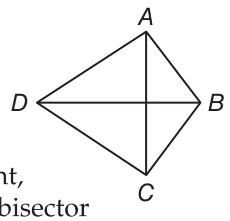
Prove: Trapezoid *ABCD* is isosceles.



Proof: By the Parallel Postulate, we can draw the auxillary line $\overline{EB} \parallel \overline{AD}$. $\angle D \cong \angle BEC$, by the Corr. ∠s Thm. We are given that $\angle D \cong \angle C$, so by the Trans. Prop, $\angle BEC \cong \angle C$. So, $\triangle EBC$ is isosceles and $\overline{EB} \cong \overline{BC}$. From the definition of a trapezoid, $\overline{AB} \parallel \overline{DE}$. Since both pairs of opposite sides are parallel, *ABED* is a parallelogram. So, $\overline{AD} \cong \overline{EB}$. By the Transitive Property, $\overline{BC} \cong \overline{AD}$. Thus, *ABCD* is an isosceles trapezoid.

31. Given: *ABCD* is a kite with $\overline{AB} \cong \overline{BC}$ and $\overline{AD} \cong \overline{DC}$.

Prove: $\overline{BD} \perp \overline{AC}$

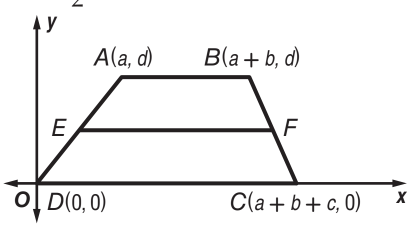


Proof: We know that $\overline{AB} \cong \overline{BC}$ and $\overline{AD} \cong \overline{DC}$. So, *B* and *D* are both equidistant from *A* and *C*. If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment. The line that contains *B* and *D* is the perpendicular bisector of $\overline{AC}$, since only one line exists through two points. Thus, $\overline{BD} \perp \overline{AC}$.

33. Given: *ABCD* is a trapezoid with median $\overline{EF}$.

Prove: $\overline{EF} \parallel \overline{AB}$ and $\overline{EF} \parallel \overline{DC}$ and

$EF = \frac{1}{2}(AB + DC)$



Proof:
By the definition of the median of a trapezoid, E is the midpoint of $\overline{AD}$ and F is the midpoint of $\overline{BC}$.

Midpoint E is $\left(\frac{a+0}{2}, \frac{d+0}{2}\right)$ or $\left(\frac{a}{2}, \frac{d}{2}\right)$.

Midpoint F is $\left(\frac{a+b+a+b+c}{2}, \frac{d+0}{2}\right)$ or $\left(\frac{2a+2b+c}{2}, \frac{d}{2}\right)$.

The slope of $\overline{AB} = 0$, the slope of $\overline{EF} = 0$, and the slope of $\overline{DC} = 0$. Thus, $\overline{EF} \parallel \overline{AB}$ and $\overline{EF} \parallel \overline{DC}$.

$$AB = \sqrt{[(a+b)-a]^2 + (d-d)^2} = \sqrt{b^2} \text{ or } b$$

$$DC = \sqrt{[(a+b+c)-0]^2 + (0-0)^2}$$
$$= \sqrt{(a+b+c)^2} \text{ or } a+b+c$$

$$EF = \sqrt{\left(\frac{2a+2b+c-a}{2}\right)^2 + \left(\frac{d}{2} - \frac{d}{2}\right)^2}$$
$$= \sqrt{\left(\frac{a+2b+c}{2}\right)^2} \text{ or } \frac{a+2b+c}{2}$$

$$\frac{1}{2}(AB + DC) = \frac{1}{2}[b + (a+b+c)]$$
$$= \frac{1}{2}(a+2b+c)$$
$$= \frac{a+2b+c}{2}$$
$$= EF$$

Thus, $\frac{1}{2}(AB + DC) = EF$.

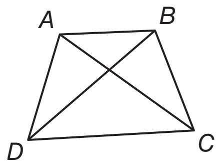
35. 15 **37.** 28 ft **39.** 70 **41.** 2 **43.** 20 **45.** 10 in. **47.** 105

49 $\angle ZWX \cong \angle ZYX$ because one pair of opposite sides of a kite is congruent and $\angle ZWX \cong \angle ZYX$. So, $m\angle ZYX = m\angle ZWX = 10x$.

$m\angle ZWX + m\angle WXY + m\angle ZYX + m\angle WZY = 360$	Polygon Interior Angles Sum Theorem
$10x + 120 + 10x + 4x = 360$	Substitution
$24x + 120 = 360$	Simplify.
$24x = 240$	Subtract 120 from each side.
$x = 10$	Divide each side by 24.

So, $m\angle ZYX = 10x$ or 100.

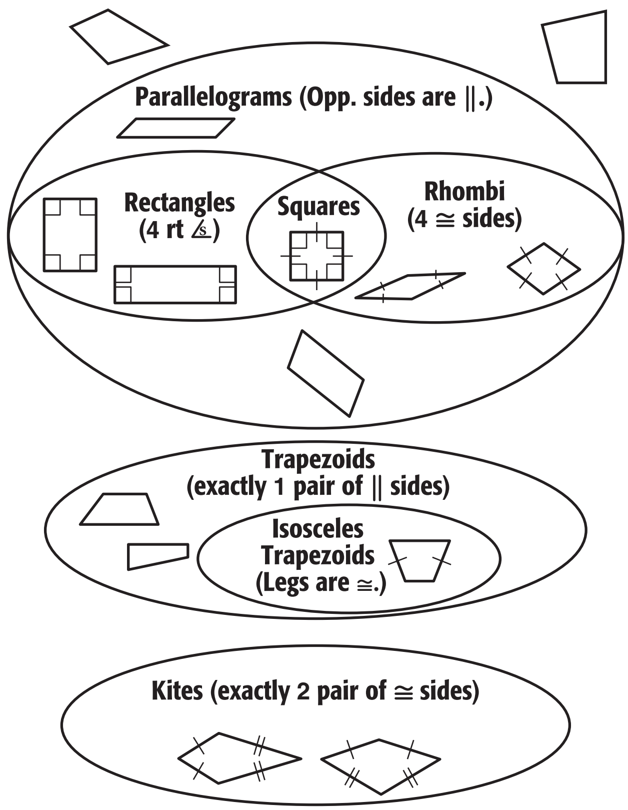
51. Given: $ABCD$ is an isosceles trapezoid.
Prove: $\angle DAC \cong \angle CBD$



Proof:
Statements (Reasons)
1. $ABCD$ is an isosceles trapezoid. (Given)
2. $\overline{AD} \cong \overline{BC}$ (Def. of isos. trap.)
3. $\overline{DC} \cong \overline{DC}$ (Refl. Prop.)
4. $\overline{AC} \cong \overline{BD}$ (Diags. of isos. trap. are ≅.)
5. $\triangle ADC \cong \triangle BCD$ (SSS)
6. $\angle DAC \cong \angle CBD$ (CPCTC)

53. Sometimes; opp ⦣s are supplementary in an isosceles trapezoid. **55.** Always; by def. a square is a quadrilateral with 4 rt. ⦣s and 4 ≅ sides. Since by def., a rhombus is a quadrilateral with 4 ≅ sides, a square is always a rhombus. **57.** Sometimes; only if the parallelogram has 4 rt. ⦣s and/or congruent diagonals, is it a rectangle.

59.



61 slope of $\overline{WX} = \frac{4-4}{3-(-3)} = 0$

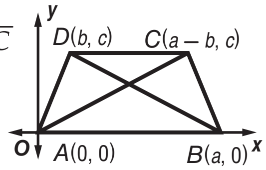
slope of $\overline{XY} = \frac{3-4}{5-3} = -\frac{1}{2}$

slope of $\overline{YZ} = \frac{1-3}{-5-5} = \frac{1}{5}$

slope of $\overline{WZ} = \frac{1-4}{-5-(-3)} = \frac{3}{2}$

Since the figure has no parallel sides, it is just a quadrilateral.

63. Given: isosceles trapezoid $ABCD$ with $\overline{AD} \cong \overline{BC}$
Prove: $\overline{BD} \cong \overline{AC}$



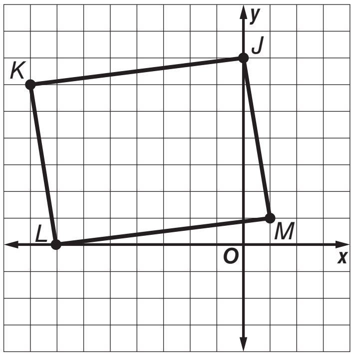
Proof:
$DB = \sqrt{(a-b)^2 + (0-c)^2}$ or $\sqrt{(a-b)^2 + c^2}$

$AC = \sqrt{[(a-b)-0]^2 + (c-0)^2}$ or $\sqrt{(a-b)^2 + c^2}$

$BD = AC$ and $\overline{BD} \cong \overline{AC}$

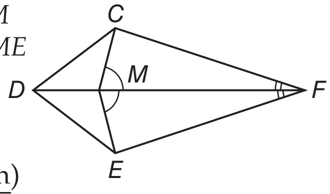
65. Belinda; $m\angle D = m\angle B$. So $m\angle A + m\angle B + m\angle C + m\angle D = 360$ or $m\angle A + 100 + 45 + 100 = 360$. So, $m\angle A = 115$. **67.** Never; a square has all 4 sides ≅, while a kite does not have any opposite sides congruent. **69.** A quadrilateral must have exactly one pair of sides parallel to be a trapezoid. If the legs are congruent, then the trapezoid is an isosceles trapezoid. If a quadrilateral has exactly two pairs of consecutive congruent sides with the opposite sides not congruent, the quadrilateral is a kite. A trapezoid and a kite both have four sides. In a trapezoid and isosceles trapezoid, both have exactly one pair of parallel sides. **71.** 76 **73.** B **75.** 18 **77.** 9

79. No; slope of $\overline{JK} = \frac{1}{8}$ = slope of $\overline{LM}$ and slope of $\overline{KL} = -6$ = slope of $\overline{MJ}$. So, *JKLM* is a parallelogram. The product of the slopes of consecutive sides $\neq -1$, so the consecutive sides are not perpendicular. Thus, *JKLM* is not a rectangle.



81. Given: $\angle CMF \cong \angle EMF$, $\angle CFM \cong \angle EFM$

Prove: $\triangle DMC \cong \triangle DME$



Proof:

__Statements (Reasons)__

1. $\angle CMF \cong \angle EMF$, $\angle CFM \cong \angle EFM$ (Given)
2. $\overline{MF} \cong \overline{MF}$, $\overline{DM} \cong \overline{MF}$ (Reflexive Property)
3. $\triangle CMF \cong \triangle EMF$ (ASA)
4. $\overline{CM} \cong \overline{EM}$ (CPCTC)
5. $\angle DMC$ and $\angle CMF$ are supplementary and $\angle DME$ and $\angle EMF$ are supplementary. (Supplement Th.)
6. $\angle DMC \cong \angle DME$ (∡ suppl. to ≅ ∡ are ≅.)
7. $\triangle DMC \cong \triangle DME$ (SAS)

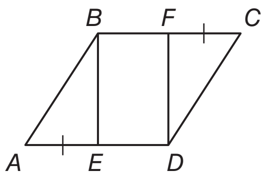
83. 1

Chapter 6 Study Guide and Review

1. false, both pairs of base angles **3.** false, diagonal **5.** true **7.** false, is always **9.** true **11.** 1440 **13.** 720 **15.** 26 **17.** 18 **19.** 115 **21.** $x = 37, y = 6$ **23.** yes, Theorem 6.11

25. Given: ▱*ABCD*, $\overline{AE} \cong \overline{CF}$

Prove: Quadrilateral *EBFD* is a parallelogram



__Statements (Reasons)__

1. *ABCD*, $\overline{AE} \cong \overline{CF}$ (Given)
2. $AE = CF$ (Def. of ≅ segs)
3. $\overline{BC} \cong \overline{AD}$ (Opp. sides of a ▱ are ≅)
4. $BC = AD$ (Def. of ≅ segs)
5. $BC = BF + CF$, $AD = AE + ED$ (Seg. Add. Post.)
6. $BF + CF = AE + ED$ (Subst.)
7. $BF + AE = AE + ED$ (Subst.)
8. $BF = ED$ (Subt. Prop.)
9. $\overline{BF} \cong \overline{ED}$ (Def. of ≅ segs)
10. $\overline{BF} \parallel \overline{ED}$ (Def. of ▱)
11. Quadrilateral *EBFD* is a parallelogram. (If one pair of opposite sides is parallel and congruent then it is a parallelogram.)

27. $x = 5, y = 12$ **29.** 33 **31.** 64 **33.** 6 **35.** 55 **37.** 35 **39.** Rectangle, rhombus, square; all sides are ≅, consecutive sides are ⊥. **41.** 19.2 **43a.** Sample answer: The legs of the trapezoids are part of the diagonals of the square. The diagonals of a square bisect opposite angles, so each base angle of a trapezoid measures 45°. One pair of sides is parallel and the base angles are congruent. **43b.** $16 + 8\sqrt{2} \approx$ 27.3 in.

CHAPTER 7

Proportions and Similarity

Chapter 7 Get Ready

1. 4 or –4 **3.** –37 **5.** 64 **7.** 64.5

Lesson 7-1

1. 23:50 **3.** 30, 75, 60 **5.** 16 **7.** 8 **9.** 18 **11.** Movie A; 1:2

13 The ratio of the sides can be written as $9x:7x:5x$.

$9x + 7x + 5x = 191.1$	Perimeter of a triangle
$21x = 191.1$	Combine like terms.
$x = 9.1$	Divide each side by 21.

The measures of the sides are 9(9.1) or 81.9 inches, 7(9.1) or 63.7 inches, and 5(9.1) or 45.5 inches.

15. 2.2 ft **17.** 54, 108, 18 **19.** 75, 60, 45 **21.** $\frac{15}{8}$ **23.** 3 **25.** 8 **27.** 3

29

$\frac{7}{500} = \frac{x}{350}$	← 13- to 17-year-old vegetarians ← 13- to 17-year olds
$7 \cdot 350 = 500 \cdot x$	Cross Products Property
$2450 = 500x$	Simplify.
$4.9 = x$	Divide each side by 500.

About 5 13- to 17-year-olds would be vegetarian.

31. 3, –3.5 **33.** 12.9, –0.2 **35.** 2541 in^2 **37.** 48, 96, 144, 72 **39a.** No; the HDTV aspect ratio is 1.77778 and the standard aspect ratio is 1.33333. Neither television set is a golden rectangle since the ratios of the lengths to the widths are not the golden ratio. **39b.** 593 pixels and 367 pixels

41. Given: $\frac{a}{b} = \frac{c}{d}$, $b \neq 0$, $d \neq 0$

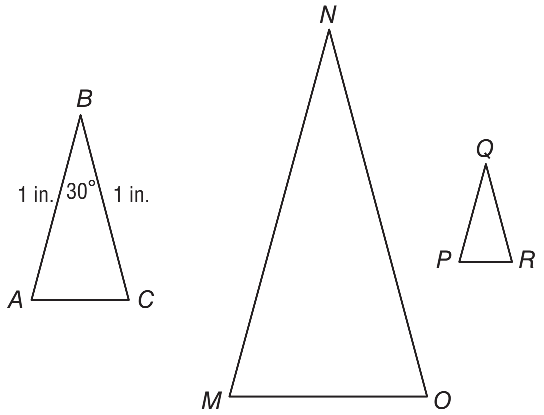
Prove: $ad = bc$

Proof:

__Statements (Reasons)__

1. $\frac{a}{b} = \frac{c}{d}$ $b \neq 0$, $d \neq 0$ (Given)
2. $(bd)\frac{a}{b} = (bd)\frac{c}{d}$ (Mult. Prop.)
3. $da = bc$ (Subst. Prop.)
4. $ad = bc$ (Comm. Prop.)

43 **a.**

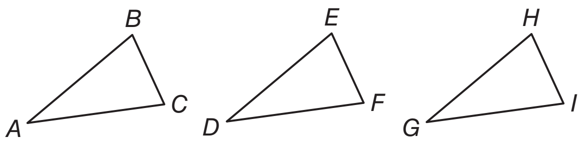


b.

Triangle	*ABC*	*MNO*	*PQR*
Leg length	1 in.	2 in.	0.5 in.
Perimeter	2.5 in.	5 in.	1.25 in.

c. Sample answer: When the vertex angle of an isosceles triangle is held constant and the leg length is increased or decreased by a factor, the perimeter of the triangle increases or decreases by the same factor.

45. 5 : 2 **47.** $\frac{2}{3} = \frac{5}{7.5}$; You need to multiply $\frac{2}{3}$ by a factor of 2.5 to get $\frac{5}{7.5}$. The factor of the other three proportions is 2.8. **49.** Both are written with fractions. A ratio compares two quantities with division, while a proportion equates two ratios. First, ratios are used to write proportions. Then the cross products are used to solve the proportion. **51.** F **53.** D **55.** 2.5 **57.** Since a square is a parallelogram, the diagonals bisect each other. Since a square is a rhombus, the diagonals are congruent. Therefore, the distance from first base to third base is equal to the distance between home plate and second base. Thus, the distance from home plate to the center of the infield is 127 ft $3\frac{3}{8}$ in. divided by 2 or 63 ft $7\frac{11}{16}$ in. This distance is longer than the distance from home plate to the pitcher's mound so the pitcher's mound is not located in the center of the field. It is about 3 feet closer to home. **59.** $-\frac{4}{7} < x < \frac{136}{7}$ **61.** $\angle 1, \angle 4, \angle 11$
63. $\angle 2, \angle 6, \angle 9, \angle 4$
65. Given: $\triangle ABC \cong \triangle DEF$
$\triangle DEF \cong \triangle GHI$
Prove: $\triangle ABC \cong \triangle GHI$



Proof:
You are given that $\triangle ABC \cong \triangle DEF$. Because corresponding parts of congruent triangles are congruent, $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$, $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\overline{AC} \cong \overline{DF}$. You are also given that $\triangle DEF \cong \triangle GHI$. So $\angle D \cong \angle G$, $\angle E \cong \angle H$, $\angle F \cong \angle I$, $\overline{DE} \cong \overline{GH}$, $\overline{EF} \cong \overline{HI}$, and $\overline{DF} \cong \overline{GI}$ by CPCTC. Therefore, $\angle A \cong \angle G$, $\angle B \cong \angle H$, $\angle C \cong \angle I$, $\overline{AB} \cong \overline{GH}$, $\overline{BC} \cong \overline{HI}$, and $\overline{AC} \cong \overline{GI}$ because congruence of angles and segments is transitive. Thus, $\triangle ABC \cong \triangle GHI$ by the definition of congruent triangles.

Lesson 7-2

1 Using the similarity statement, $\angle A$ and $\angle Z$ are corresponding angles, $\angle B$ and $\angle Y$ are corresponding angles, and $\angle C$ and $\angle X$ are corresponding angles. So, $\angle A \cong \angle Z$, $\angle B \cong \angle Y$, and $\angle C \cong \angle X$. $\overline{AC}$ and $\overline{ZX}$ are corresponding sides, $\overline{BC}$ and $\overline{YX}$ are corresponding sides, and $\overline{AB}$ and $\overline{ZY}$ are corresponding sides. So, $\frac{AC}{ZX} = \frac{BC}{YX} = \frac{AB}{ZY}$.

3. no; $\frac{NQ}{WZ} \neq \frac{QR}{WX}$ **5.** 6 **7.** 22 ft **9.** $\angle J \cong \angle P$, $\angle F \cong \angle S$, $\angle M \cong \angle T$; $\angle H \cong \angle Q$, $\frac{PQ}{JH} = \frac{TS}{MF} = \frac{SQ}{FH} = \frac{TP}{MJ}$
11. $\angle D \cong \angle K$, $\angle F \cong \angle M$, $\angle G \cong \angle J$; $\frac{DF}{KM} = \frac{FG}{MJ} = \frac{GD}{JK}$

13 Two sides and the included angle of $\triangle LTK$ are congruent to two sides and the included angle of $\triangle MTK$, so $\triangle LTK \cong \triangle MTK$. By CPCTC, the triangles have all corresponding angles congruent and all corresponding sides congruent. So, $\triangle LTK \sim \triangle MTK$, with a scale factor of 1.

15. no; $\frac{AD}{WM} \neq \frac{DK}{ML}$ **17.** Yes; sample answer: The ratio of the longer dimensions of the screens is approximately 1.1 and the ratio of the shorter dimensions of the screens is approximately 1.1.

19

$\frac{SB}{JH} = \frac{BP}{HT}$	Similarity proportion
$\frac{2}{3} = \frac{x+3}{2x+2}$	$SB = 2$, $JH = 3$, $BP = x + 3$, $HT = 2x + 2$
$2(2x + 2) = 3(x + 3)$	Cross Products Property
$4x + 4 = 3x + 9$	Distributive Property
$x + 4 = 9$	Subtract $3x$ from each side.
$x = 5$	Subtract 4 from each side.

21. 3 **23.** 10.8 **25.** 18.9 **27.** 40 m **29.** $\angle A \cong \angle V$, $\angle B \cong \angle X$, $\angle D \cong \angle Z$, $\angle F \cong \angle T$; $\frac{AB}{VX} = \frac{BD}{XZ} = \frac{DF}{ZT} = \frac{FA}{TV} = 2$ **31.** $\overline{AC}$, $\overline{AD}$ **33.** $\angle ABH$, $\angle ADF$

35

$\angle D \cong \angle P$	Corresponding angles of similar polygons are congruent.
$m\angle D = m\angle P$	Definition of congruence
$x + 34 = 97$	Substitution
$x = 63$	Subtract 34 from each side.
$\angle C \cong \angle R$	Corresponding angles of similar polygons are congruent.
$m\angle C = m\angle R$	Definition of congruence
$83 = 3y - 13$	Substitution
$96 = 3y$	Add 13 to each side.
$32 = y$	Divide each side by 3.

37. 52 in. by 37 in. **39.** no; $\frac{BC}{XY} \neq \frac{AB}{WX}$ **41.** Never; sample answer: Parallelograms have both pairs of opposite sides parallel. Trapezoids have exactly one pair of parallel legs. Therefore, the two figures cannot be similar because they can never be the same type of figure. **43.** Sometimes; sample answer: If corresponding angles are congruent and corresponding sides are proportional, two isosceles triangles are similar. **45.** Always; sample answer: Equilateral triangles always have three 60° angles, so the angles of one equilateral triangle are always congruent to the angles of a second equilateral triangle. The three sides of an equilateral triangle are always congruent, so the ratio of each pair of legs of one triangle to a second triangle will always be the same. Therefore, a pair of equilateral triangles is always similar.

47 Let ℓ = new length and w = new width.

$\frac{2\frac{1}{3}}{\ell} = \frac{2}{3}$	Use the lengths and scale factor to write a proportion.
$2\frac{1}{3} \cdot 3 = \ell \cdot 2$	Cross Products Property
$\frac{7}{3} \cdot 3 = 2\ell$	Write $2\frac{1}{3}$ as $\frac{7}{3}$.
$7 = 2\ell$	Multiply.
$\frac{7}{2} = \ell$	Divide each side by 7.

The new length is $\frac{7}{2}$ inches or $3\frac{1}{2}$ inches.

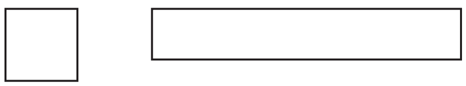
$\frac{1\frac{2}{3}}{w} = \frac{2}{3}$	Use the widths and scale factor to write a proportion.
$1\frac{2}{3} \cdot 3 = w \cdot 2$	Cross Products Property
$\frac{5}{3} \cdot 3 = 2w$	Write $1\frac{2}{3}$ as $\frac{5}{3}$.
$5 = 2w$	Multiply.
$\frac{5}{2} = w$	Divide each side by 2.

The new width is $\frac{5}{2}$ inches or $2\frac{1}{2}$ inches.

49a. $\frac{a}{3a} = \frac{b}{3b} = \frac{c}{3c} = \frac{a+b+c}{3(a+b+c)} = \frac{1}{3}$

49b. No; the sides are no longer proportional. **51.** 4

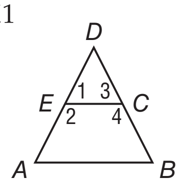
53. Sample answer:



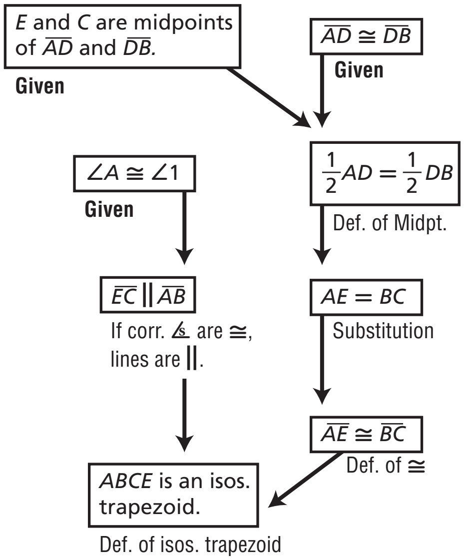
55. Sample answer: The figures could be described as congruent if they are the same size and shape, similar if their corresponding angles are congruent and their corresponding sides are proportional, and equal if they are the same exact figure. **57.** G **59.** E

61. Given: E and C are midpoints of $\overline{AD}$ and $\overline{DB}$. $\overline{AD} \cong \overline{DB}$; $\angle A \cong \angle 1$

Prove: $ABCE$ is an isosceles trapezoid.



Proof:



63. $x \leq 4$ **65.** The angle bisector of the vertex angle of an isosceles triangle is not an altitude of the triangle. **67.** 128 **69.** 68 **71.** $x = 2$, $RT = 8$, $RS = 8$

Lesson 7-3

1. Yes; $\triangle YXZ \sim \triangle VWZ$ by AA Similarity. **3.** No; corresponding sides are not proportional. **5.** C **7.** $\triangle QVS \sim \triangle RTS$; 20 **9.** Yes; $\triangle XUZ \sim \triangle WUY$ by SSS Similarity.

11 $\frac{CB}{DB} = \frac{10}{6}$ or $\frac{5}{3}$ and $\frac{BA}{BF} = \frac{9+6}{9} = \frac{15}{9}$ or $\frac{5}{3}$. $m\angle CBA = m\angle DBF = 38$, so $\angle CBA \cong \angle DBF$. Since the length of the sides that include the congruent angles are proportional, $\triangle CBA \sim \triangle DBF$ by SAS Similarity.

13. No; not enough information to determine. If $JH = 3$ or $WY = 24$, then $\triangle JHK \sim \triangle XWY$ by SSS Similarity. **15.** Yes; sample answer: Since $\overline{AB} \cong \overline{EB}$ and $\overline{CB} \cong \overline{DB}$, $\frac{AB}{CB} = \frac{EB}{DB}$. $\angle ABE \cong \angle CBD$ because vertical angles are congruent. Therefore, $\triangle ABE \sim \triangle CBD$ by SAS Similarity.

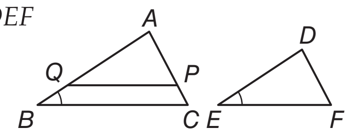
17 Since $\overline{RS} \parallel \overline{PT}$, $\angle QRS \cong \angle QPT$ and $\angle QSR \cong \angle QTP$ because they are corresponding angles. By AA Similarity, $\triangle QRS \sim \triangle QPT$.

$\frac{RS}{PT} = \frac{QS}{QT}$	Definition of similar polygons
$\frac{12}{16} = \frac{x}{20}$	$RS = 12$, $PT = 16$, $QS = x$, $QT = 20$
$12 \cdot 20 = 16 \cdot x$	Cross Products Property
$240 = 16x$	Simplify.
$15 = x$	Divide each side by 16.

Since $QS + ST = 20$ and $QS = 15$, then $ST = 5$.

19. $\triangle HJK \sim \triangle NQP$; 15, 10 **21.** $\triangle GHJ \sim \triangle GDH$; 14, 20 **23.** about 12.8 ft **25. Given:** $\angle B \cong \angle E$, $\overline{QP} \parallel \overline{BC}$; $\overline{QP} \cong \overline{EF}$, $\frac{AB}{DE} = \frac{BC}{EF}$

Prove: $\triangle ABC \sim \triangle DEF$

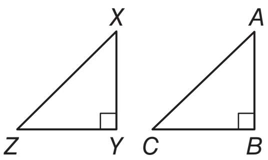


Proof:

Statements (Reasons)

1. $\angle B \cong \angle E$, $\overline{QP} \parallel \overline{BC}$; $\overline{QP} \cong \overline{EF}$, $\frac{AB}{DE} = \frac{BC}{EF}$ (Given)
2. $\angle APQ \cong \angle C$, $\angle AQP \cong \angle B$ (Corr. ∠s Post.)
3. $\angle AQP \cong \angle E$ (Trans. Prop.)
4. $\triangle ABC \sim \triangle AQP$ (AA Similarity)
5. $\frac{AB}{AQ} = \frac{BC}{QP}$ (Def. of ~ △s)
6. $AB \cdot QP = AQ \cdot BC$; $AB \cdot EF = DE \cdot BC$ (Cross products)
7. $QP = EF$ (Def. of ≅ segs.)
8. $AB \cdot EF = AQ \cdot BC$ (Subst.)
9. $AQ \cdot BC = DE \cdot BC$ (Subst.)
10. $AQ = DE$ (Div. Prop.)
11. $\overline{AQ} \cong \overline{DE}$ (Def. of ≅ segs.)
12. $\triangle AQP \cong \triangle DEF$ (SAS)
13. $\angle APQ \cong \angle F$ (CPCTC)
14. $\angle C \cong \angle F$ (Trans. Prop.)
15. $\triangle ABC \sim \triangle DEF$ (AA Similarity)

27. Given: $\triangle XYZ$ and $\triangle ABC$ are right triangles; $\frac{XY}{AB} = \frac{YZ}{BC}$.



Prove: $\triangle YXZ \sim \triangle BAC$

Proof:

Statements (Reasons)

1. $\triangle XYZ$ and $\triangle ABC$ are right triangles. (Given)
2. $\angle XYZ$ and $\angle ABC$ are right angles. (Def. of rt. △)
3. $\angle XYZ \cong \angle ABC$ (All rt. ∠s are ≅.)
4. $\frac{XY}{AB} = \frac{YZ}{BC}$ (Given)
5. $\triangle YXZ \sim \triangle BAC$ (SAS Similarity)

29. 20 in.

31 Use $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to find the side lengths of $\triangle XYZ$ and $\triangle WYV$.

$XY = \sqrt{[5 - (-1)]^2 + [3 - (-9)]^2} = 6\sqrt{5}$
$(x_1, y_1) = (-1, -9), (x_2, y_2) = (5, 3)$

$YZ = \sqrt{(-1 - 5)^2 + (6 - 3)^2} = 3\sqrt{5}$
$(x_1, y_1) = (5, 3), (x_2, y_2) = (-1, 6)$

$XZ = \sqrt{[-1 - (-1)]^2 + [6 - (-9)]^2} = 15$
$(x_1, y_1) = (-1, -9), (x_2, y_2) = (-1, 6)$

$WY = \sqrt{(5 - 1)^2 + [3 - (-5)]^2} = 2\sqrt{5}$
$(x_1, y_1) = (1, -5), (x_2, y_2) = (5, 3)$

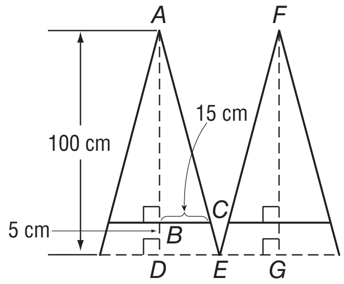
$YV = \sqrt{(1 - 5)^2 + (5 - 3)^2} = 2\sqrt{5}$
$(x_1, y_1) = (5, 3), (x_2, y_2) = (1, 5)$

$WV = \sqrt{(1 - 1)^2 + [5 - (-5)]^2} = 10$
$(x_1, y_1) = (1, -5), (x_2, y_2) = (1, 5)$

Since the perimeter of $\triangle XYZ = 6\sqrt{5} + 3\sqrt{5} + 15$ or $9\sqrt{5} + 15$ and the perimeter of $\triangle WYV = 4\sqrt{5} + 2\sqrt{5} + 10$ or $6\sqrt{5} + 10$, the ratio of the perimeters is $\frac{9\sqrt{5} + 15}{6\sqrt{5} + 10} = \frac{3(3\sqrt{5} + 5)}{2(3\sqrt{5} + 5)}$ or $\frac{3}{2}$.

33. $\angle C \cong \angle C'$, since all right angles are congruent. Line ℓ is a transversal of parallel segments $\overline{BC}$ and $\overline{B'C'}$, so $\angle ABC \cong \angle A'B'C'$ since corresponding angles of parallel lines are congruent. Therefore, by AA Similarity, $\triangle ABC \sim \triangle A'B'C'$. So $\frac{BC}{AC}$, the slope of line ℓ through points A and B, is equal to $\frac{B'C'}{A'C'}$, the slope of line ℓ through points A' and B'.

35 Since $\angle CAB \cong \angle CAB$ by the Reflexive Property and $\angle ABC \sim \angle ADE$ by the definition of right angles, $\triangle ABC \sim \triangle ADE$ by the AA Similarity Postulate.

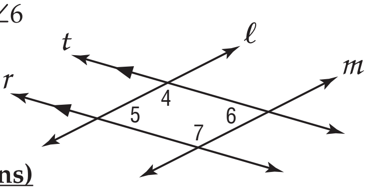


$\frac{AB}{BC} = \frac{AD}{DE}$	Definition of similar polygons
$\frac{100}{15} = \frac{100 + 5}{DE}$	Substitution
$100 \cdot DE = 105 \cdot 15$	Cross Products Property
$100DE = 1575$	Simplify.
$DE = 15.75$	Divide each side by 100.

Since the △s formed by the laser sources are ≅, $\overline{DE} \cong \overline{GE}$. So, $DG = 15.75 + 15.75$ or 31.5. Since $\overline{AD}$ and $\overline{FG}$ are ||, $AF = DG$. So, $AF = 31.5$. The laser sources should be placed 31.5 cm apart.

37. Sample answer: The AA Similarity Postulate, SSS Similarity Theorem, and SAS Similarity Theorem are all tests that can be used to determine whether two triangles are similar. The AA Similarity Postulate is used when two pairs of congruent angles on two triangles are given. The SSS Similarity Theorem is used when the corresponding side lengths of two triangles are given. The SAS Similarity Theorem is used when two proportional side lengths and the included angle on two triangles are given. **39.** 6
41. Sample answer: You could consider the amount of space that the actual object occupies and compare it to the amount of space that is available for the scale model or drawing. Then, you could determine the amount of detail that you want the scale model or drawing to have, and you could use these factors to choose an appropriate scale. **43a.** $\frac{6}{x - 2} = \frac{4}{5}$
43b. 9.5; 7.5 **45.** B **47.** $\angle X \cong \angle R$, $\angle W \cong \angle Q$, $\angle Y \cong \angle S$; $\angle Z \cong \angle T$, $\frac{WX}{QR} = \frac{ZY}{TS} = \frac{WZ}{QT} = \frac{XY}{RS}$
49. 12 **51.** 52.3 **53.** Sample answer: If one pair of opposite sides are congruent and parallel, the quadrilateral is a parallelogram. **55.** not possible

57. Given: $r \parallel t$; $\angle 5 \cong \angle 6$
Prove: $\ell \parallel m$



Proof:
Statements (Reasons)
1. $r \parallel t$; $\angle 5 \cong \angle 6$ (Given)
2. $\angle 4$ and $\angle 5$ are supplementary. (Consecutive Interior Angles Theorem)
3. $m\angle 4 + m\angle 5 = 180$ (Def. of Suppl. ∠s)
4. $m\angle 5 = m\angle 6$ (Def. of ≅ ∠s)
5. $m\angle 4 + m\angle 6 = 180$ (Substitution)
6. $\angle 4$ and $\angle 6$ are supplementary. (Def. of Suppl. ∠s)
7. $\ell \parallel m$ (If cons. int. ∠s are suppl., then lines are ∥.)

Lesson 7-4

1. 10 **3.** Yes; $\frac{AD}{DC} = \frac{BE}{EC} = \frac{2}{3}$, so $\overline{DE} \parallel \overline{AB}$. **5.** 11
7. 2360.3 ft **9.** $x = 20$; $y = 2$

11 If $AB = 12$ and $AC = 16$, then $BC = 4$.

$\frac{AB}{BC} = \frac{AE}{ED}$	Triangle Proportionality Theorem
$\frac{12}{4} = \frac{AE}{5}$	Substitute.
$12 \cdot 5 = 4 \cdot AE$	Cross Products Property
$60 = 4AE$	Multiply.
$15 = AE$	Divide each side by 4.

13. 10 **15.** yes; $\frac{ZV}{VX} = \frac{WY}{YX} = \frac{11}{5}$ **17.** no; $\frac{ZV}{VX} \neq \frac{WY}{YX}$

19

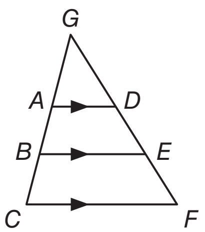
$m\angle PHM + m\angle PHJ + m\angle JHL = 180$	Definition of a straight angle
$44 + m\angle PHJ + 76 = 180$	Substitution
$120 + m\angle PHJ = 180$	Simplify.
$m\angle PHJ = 60$	Subtract 120 from each side.

By the Triangle Midsegment Theorem, $\overline{PH} \parallel \overline{KL}$.

$\angle PHJ \cong \angle JHL$	Alternate Interior Angles Theorem
$m\angle PHJ = m\angle JHL$	Definition of congruence
$60 = x$	Substitution

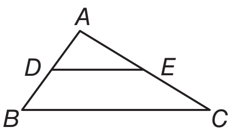
21. 1.35 **23.** 1.2 in. **25.** $x = 18$; $y = 3$ **27.** $x = 48$; $y = 72$

29. Given: $\overleftrightarrow{AD} \parallel \overleftrightarrow{BE} \parallel \overleftrightarrow{CF}$, $\overline{AB} \cong \overline{BC}$
Prove: $\overline{DE} \cong \overline{EF}$



Proof:
From Corollary 7.1, $\frac{AB}{BC} = \frac{DE}{EF}$.
Since $\overline{AB} \cong \overline{BC}$, $AB = BC$ by definition of congruence.
Therefore, $\frac{AB}{BC} = 1$.
By substitution, $1 = \frac{DE}{EF}$. Thus, $DE = EF$. By definition of congruence, $\overline{DE} \cong \overline{EF}$.

31. Given: $\frac{DB}{AD} = \frac{EC}{AE}$
Prove: $\overline{DE} \parallel \overline{BC}$



Proof:
Statements (Reasons)
1. $\frac{DB}{AD} = \frac{EC}{AE}$ (Given)
2. $\frac{AD}{AD} + \frac{DB}{AD} = \frac{AE}{AE} + \frac{EC}{AE}$ (Add. Prop.)
3. $\frac{AD + DB}{AD} = \frac{AE + EC}{AE}$ (Subst.)
4. $AB = AD + DB$, $AC = AE + EC$ (Seg. Add. Post.)
5. $\frac{AB}{AD} = \frac{AC}{AE}$ (Subst.)
6. $\angle A \cong \angle A$ (Refl. Prop.)
7. $\triangle ADE \sim \triangle ABC$ (SAS Similarity)
8. $\angle ADE \cong \angle ABC$ (Def. of ~ polygons)
9. $\overline{DE} \parallel \overline{BC}$ (If corr. ∠s are ≅, then the lines are ∥.)

33. 9

35 If $CA = 10$ and $CD = 2$, then $DA = 8$.

$\frac{CE}{EB} = \frac{CD}{DA}$	Triangle Proportionality Theorem
$\frac{t-2}{t+1} = \frac{2}{8}$	Substitute.
$(t-2)(8) = (t+1)(2)$	Cross Products Property
$8t - 16 = 2t + 2$	Distributive Property
$6t - 16 = 2$	Subtract $2t$ from each side.
$6t = 18$	Add 16 to each side.
$t = 3$	Divide each side by 6.

If $t = 3$, then $CE = 3 - 2$ or 1.

37. 8, 7.5
39. $\triangle ABC \sim \triangle ADE$ SAS Similarity **41.** 6

$\frac{AD}{AB} = \frac{DE}{BC}$	Def. ~ △s
$\frac{40}{100} = \frac{DE}{BC}$	Substitution
$\frac{2}{5} = \frac{DE}{BC}$	Simplify.
$\frac{2}{5}BC = DE$	Multiply.

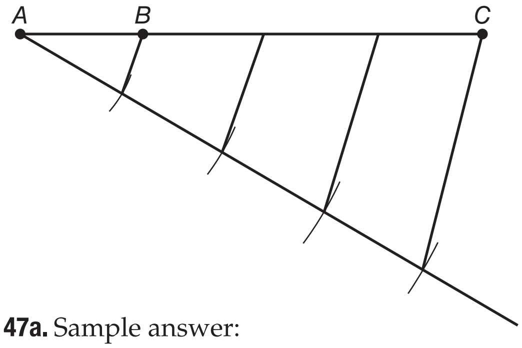
43 All the triangles are isosceles. Segment EH is the midsegment of triangle ABC. Therefore, segment EH is half of the length of AC, which is $35 \div 2$ or 17.5 feet. Similarly, FG is the midsegment of triangle BEH, so $FG = 17.5 \div 2$ or 8.75 feet. To find DJ, use the vertical altitude which is 12 feet. Let the altitude from B to the segment AC meet the segment DJ at K. Find BC using the Pythagorean Theorem.

$BC^2 = BK^2 + KC^2$
$BC^2 = 12^2 + 17.5^2$
$BC = \sqrt{12^2 + 17.5^2}$
$BC \approx 21.22$ in.

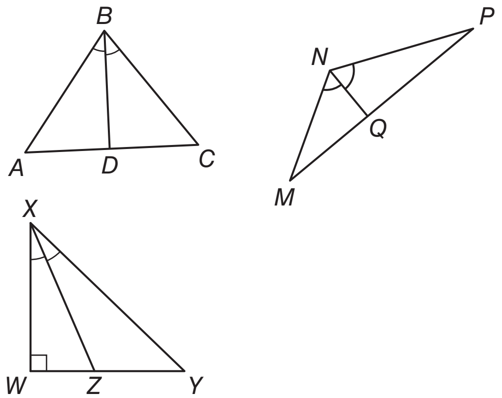
Since the width of each piece of siding is the same, $BJ = \frac{3}{4}BC$, which is about $\frac{3}{4}(21.22)$ or 15.92 in.
Now, use the Triangle Proportionality Theorem.

$\frac{AC}{BC} = \frac{DJ}{BJ}$
$\frac{35}{21.22} = \frac{DJ}{15.92}$
$21.22(DJ) = (15.92)(35)$
$21.22(DJ) = 557.2$
$DJ \approx 26.25$ in.

45. Sample answer:



47a. Sample answer:



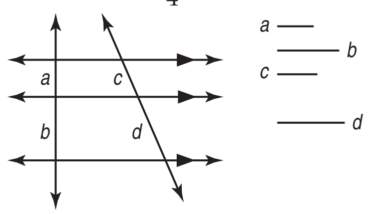
47b.

Triangle	Length		Ratio	
ABC	AD	1.1 cm	$\frac{AD}{CD}$	1.0
	CD	1.1 cm		
	AB	2.0 cm	$\frac{AB}{CB}$	1.0
	CB	2.0 cm		
MNP	MQ	1.4 cm	$\frac{MQ}{PQ}$	0.8
	PQ	1.7 cm		
	MN	1.6 cm	$\frac{MN}{PN}$	0.8
	PN	2.0 cm		
WXY	WZ	0.8 cm	$\frac{WZ}{YZ}$	0.7
	YZ	1.2 cm		
	WX	2.0 cm	$\frac{WX}{YX}$	0.7
	YX	2.9 cm		

47c. Sample answer: The proportion of the segments created by the angle bisector of a triangle is equal to the proportion of their respective consecutive sides.

49. Always; sample answer: $\overline{FH}$ is a midsegment. Let $BC = x$, then $FH = \frac{1}{2}x$. $FHCB$ is a trapezoid, so $DE = \frac{1}{2}(BC + FH) = \frac{1}{2}\left(x + \frac{1}{2}x\right) = \frac{1}{2}x + \frac{1}{4}x = \frac{3}{4}x$. Therefore, $DE = \frac{3}{4}BC$.

51.



By Corollary 7.1, $\frac{a}{b} = \frac{c}{d}$.

53. 8 **55.** G **57.** $\triangle ABE \sim \triangle CDE$ by AA Similarity; 6.25

59. $\triangle WZT \sim \triangle WXY$ by AA Similarity; 7.5

61. $\overline{QR} \parallel \overline{TS}$, $\overline{QT} \nparallel \overline{RS}$; $QRST$ is an isosceles trapezoid since $RS = \sqrt{26} = QT$. **63.** 6 **65.** 56 **67.** $\frac{2}{3}$

69. 2.1 **71.** 8.7

Lesson 7-5

1 The triangles are similar by AA Similarity.

$\frac{x}{10} = \frac{12}{15}$	~△s have corr. medians proportional to the corr. sides.
$x \cdot 15 = 10 \cdot 12$	Cross Products Property
$15x = 120$	Simplify.
$x = 8$	Divide each side by 15.

3. 35.7 ft **5.** 20 **7.** 8.5 **9.** 18

11

$\frac{15}{27} = \frac{28 - b}{b}$	Triangle Angle Bisector Theorem
$15 \cdot b = 27(28 - b)$	Cross Products Property
$15b = 756 - 27b$	Multiply.
$42b = 756$	Add 27b to each side.
$b = 18$	Divide each side by 42.

13. 15

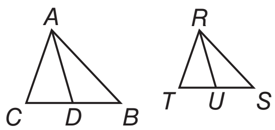
15

$\frac{AB}{JK} = \frac{AD}{JM}$	~△s have corr. altitudes proportional to the corr. sides.
$\frac{9}{21} = \frac{4x - 8}{5x + 3}$	Substitute.
$9(5x + 3) = 21(4x - 8)$	Cross Products Property
$45x + 27 = 84x - 168$	Distributive Property
$27 = 39x - 168$	Subtract 45x from each side.
$195 = 39x$	Add 168 to each side.
$5 = x$	Divide each side by 39.

17. 4

19. Given: $\triangle ABC \sim \triangle RST$
$\overline{AD}$ is a median of $\triangle ABC$.
$\overline{RU}$ is a median of $\triangle RST$.

Prove: $\frac{AD}{RU} = \frac{AB}{RS}$



Proof:

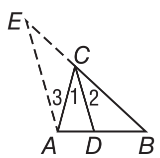
__Statements (Reasons)__

1. $\triangle ABC \sim \triangle RST$; $\overline{AD}$ is a median of $\triangle ABC$; $\overline{RU}$ is a median of $\triangle RST$. (Given)
2. $CD = DB$; $TU = US$ (Def. of median)
3. $\frac{AB}{RS} = \frac{CB}{TS}$ (Def. of ~ △s)
4. $CB = CD + DB$; $TS = TU + US$ (Seg. Add. Post.)
5. $\frac{AB}{RS} = \frac{CD + DB}{TU + US}$ (Subst.)
6. $\frac{AB}{RS} = \frac{DB + DB}{US + US}$ or $\frac{2(DB)}{2(US)}$ (Subst.)
7. $\frac{AB}{RS} = \frac{DB}{US}$ (Subst.)
8. $\angle B \cong \angle S$ (Def. of ~ △s)
9. $\triangle ABD \sim \triangle RSU$ (SAS Similarity)
10. $\frac{AD}{RU} = \frac{AB}{RS}$ (Def. of ~ △s)

21. 3 **23.** 70

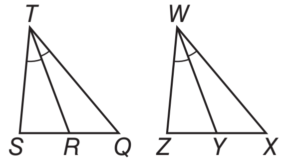
25. Given: $\overline{CD}$ bisects $\angle ACB$.
By construction, $\overline{AE} \parallel \overline{CD}$.

Prove: $\frac{AD}{DB} = \frac{AC}{BC}$



Proof:
Statements (Reasons)
1. $\overline{CD}$ bisects $\angle ACB$; By construction, $\overline{AE} \parallel \overline{CD}$. (Given)
2. $\frac{AD}{DB} = \frac{EC}{BC}$ ($\triangle$ Prop. Thm.)
3. $\angle 1 \cong \angle 2$ (Def. of $\angle$ Bisector)
4. $\angle 3 \cong \angle 1$ (Alt. Int. $\angle$s Thm.)
5. $\angle 2 \cong \angle E$ (Corr. $\angle$s Post.)
6. $\angle 3 \cong \angle E$ (Trans. Prop.)
7. $\overline{EC} \cong \overline{AC}$ (Converse of Isos. $\triangle$ Thm.)
8. $EC = AC$ (Def. of $\cong$ segs.)
9. $\frac{AD}{DB} = \frac{AC}{BC}$ (Subst.)

27. Given: $\triangle QTS \sim \triangle XWZ$, $\overline{TR}$, $\overline{WY}$ are $\angle$ bisectors.
Prove: $\frac{TR}{WY} = \frac{QT}{XW}$



Proof:
Statements (Reasons)
1. $\triangle STQ \sim \triangle ZWX$, $\overline{TR}$ and $\overline{WY}$ are angle bisectors. (Given)
2. $\angle STQ \cong \angle ZWX$, $\angle Q \cong \angle X$ (Def of $\sim \triangle$s)
3. $\angle STR \cong \angle QTR$, $\angle ZWY \cong \angle XWY$ (Def. $\angle$ bisector)
4. $m\angle STQ = m\angle STR + m\angle QTR$, $m\angle ZWX = m\angle ZWY + m\angle XWY$ ($\angle$ Add. Post.)
5. $m\angle STQ = 2m\angle QTR$, $m\angle ZWX = 2m\angle XWY$ (Subst.)
6. $2m\angle QTR = 2m\angle XWY$ (Subst.)
7. $m\angle QTR = m\angle XWY$ (Div. Prop.)
8. $\angle QTR \cong \angle XWY$ (Def. of $\cong$ Angles)
9. $\triangle QTR \sim \triangle XWY$ (AA Similarity)
10. $\frac{TR}{WY} = \frac{QT}{XW}$ (Def. of $\sim \triangle$s)

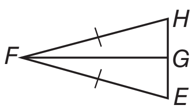
29 Since the segment from Trevor to Ricardo is an angle bisector, the segments from Ricardo to Craig and from Ricardo to Eli are proportional to the segments from Trevor to Craig and from Trevor to Eli. Since Craig is closer to Trevor than Eli is, Craig is also closer to Ricardo than Eli is. So, Craig will reach Ricardo first.

31. Chun; by the Angle Bisector Theorem, the correct proportion is $\frac{5}{8} = \frac{15}{x}$. **33.** $PS = 18.4$, $RS = 24$

35. Both theorems have a segment that bisects an angle and have proportionate ratios. The Triangle Angle Bisector Theorem pertains to one triangle, while Theorem 7.9 pertains to similar triangles. Unlike the Triangle Angle Bisector Theorem, which separates the opposite side into segments that have the same ratio as the other two sides, Theorem 7.9 relates the angle bisector to the measures of the sides. **37.** 2.2 **39.** C

41. $x = 2$; $y = 3$ **43.** $KP = 5$, $KM = 15$, $MR = 13\frac{1}{3}$, $ML = 20$, $MN = 12$, $PR = 16\frac{2}{3}$

45. Given: $\overline{EF} \cong \overline{HF}$
G is the midpoint of $\overline{EH}$.
Prove: $\triangle EFG \cong \triangle HFG$



Proof:
Statements (Reasons)
1. $\overline{EF} \cong \overline{HF}$; G is the midpoint of $\overline{EH}$. (Given)
2. $\overline{EG} \cong \overline{GH}$ (Def. of midpoint)
3. $\overline{FG} \cong \overline{FG}$ (Reflexive Prop.)
4. $\triangle EFG \cong \triangle HFG$ (SSS)

47. 5 **49.** $\sqrt{137} \approx 11.7$ **51.** $\sqrt{340} \approx 18.4$

Lesson 7-6

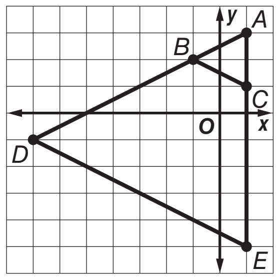
1. enlargement; 2

3 The ratio comparing the widths is $\frac{152.5}{27} \approx 5.6$.
The ratio comparing the lengths is $\frac{274}{78} \approx 3.5$.
Sample answer: Since $\frac{152.5}{27} \neq \frac{274}{78}$, a table tennis table is not a dilation of a tennis court.

5. $\frac{RJ}{KJ} = \frac{SJ}{LJ} = \frac{RS}{KL} = \frac{1}{2}$, so $\triangle RSJ \sim \triangle KLJ$ by SSS Similarity. **7.** reduction; $\frac{1}{2}$ **9.** enlargement; 2

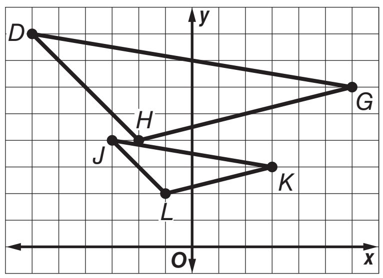
11. reduction **13.** No; sample answer: Since $\frac{1.2}{2.5} \neq \frac{1.25}{3}$, the design and the actual tattoo are not proportional. Therefore, the tattoo is not a dilation of the design.

15.



$\angle A \cong \angle A$ and $\frac{AB}{AD} = \frac{AC}{AE} = \frac{1}{4}$, so $\triangle ABC \sim \triangle ADE$ by SAS Similarity.

17.



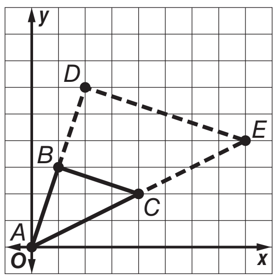
$\frac{JK}{DG} = \frac{KL}{GH} = \frac{JL}{DH} = \frac{1}{2}$, so $\triangle JKL \sim \triangle DGH$ by SSS Similarity.

19

$\frac{AC}{AB} = \frac{AZ}{AY}$	Definition of similar polygons
$\frac{4}{AB} = \frac{12}{6}$	$AC = 4$, $AZ = 12$, $AY = 6$
$4 \cdot 6 = AB \cdot 12$	Cross Products Property
$24 = 12AB$	Simplify.
$2 = AB$	Divide each side by 12.

Since $AB = 2$, the coordinates of B are (0, –2).

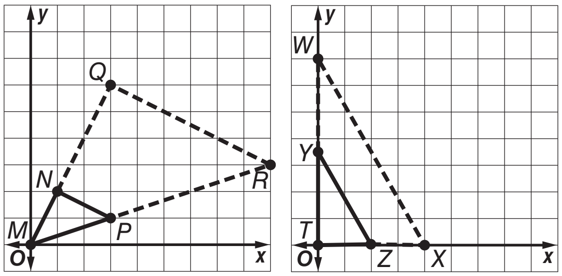
21 **a.** Sample answer: $\triangle ABC$: $A(0, 0)$, $B(1, 3)$, $C(4, 2)$
Draw points D and E so that $AD = 2AB$ and $AE = 2AC$.
Connect the points to form $\triangle ADE$: $A(0, 0)$, $D(2, 6)$, $E(8, 4)$.



b. Sample answer: $\triangle MNP$: $M(0, 0)$, $N(1, 2)$, $P(3, 1)$ Draw points Q and R so that $MQ = 3MN$ and $MR = 3MP$. Connect the points to form $\triangle MQR$: $M(0, 0)$, $Q(3, 6)$, $R(9, 3)$.

$\triangle TWX$: $T(0, 0)$, $W(0, 7)$, $X(4, 0)$

Draw points Y and Z so that $TY = 0.5TW$ and $TZ = 0.5TX$. Connect the points to form $\triangle TYZ$: $T(0, 0)$, $Y(0, 3.5)$, $Z(2, 0)$.



c.

Coordinates											
$\triangle ABC$		$\triangle ADE$		$\triangle MNP$		$\triangle MQR$		$\triangle TWX$		$\triangle TYZ$	
A	(0, 0)	A	(0, 0)	M	(0, 0)	M	(0, 0)	T	(0, 0)	T	(0, 0)
B	(1, 3)	D	(2, 6)	N	(1, 2)	Q	(3, 6)	W	(0, 7)	Y	(0, 3.5)
C	(4, 2)	E	(8, 4)	P	(3, 1)	R	(9, 3)	X	(4, 0)	Z	(2, 0)

d. Sample answer: Multiply the coordinates of the given triangle by the scale factor to get the coordinates of the dilated triangle.

$\triangle ADE$: $A(0 \cdot 2, 0 \cdot 2)$ or $A(0, 0)$, $D(1 \cdot 2, 3 \cdot 2)$ or $D(2, 6)$, $E(4 \cdot 2, 2 \cdot 2)$ or $E(8, 4)$

$\triangle MQR$: $M(0 \cdot 3, 0 \cdot 3)$ or $M(0, 0)$, $Q(1 \cdot 3, 2 \cdot 3)$ or $Q(3, 6)$, $R(3 \cdot 3, 1 \cdot 3)$ or $R(9, 3)$

$\triangle TYZ$: $T(0 \cdot 0.5, 0 \cdot 0.5)$ or $T(0, 0)$, $Y(0 \cdot 0.5, 7 \cdot 0.5)$ or $Y(0, 3.5)$, $Z(4 \cdot 0.5, 0 \cdot 0.5)$ or $Z(2, 0)$

23. No; sample answer: Since the x-coordinates are multiplied by 3 and the y-coordinates are multiplied by 2, $\triangle XYZ$ is 3 times as wide and only 2 times as tall as $\triangle PQR$. Therefore, the transformation is not a dilation. **25.** Sample answer: Architectural plans are reductions. **27.** Sample answer: If a transformation is an enlargement, the lengths of the transformed object will be greater than the original object, so the scale factor will be greater than 1. If a transformation is a reduction, the lengths of the transformed object will be less than the original object, so the scale factor will be less than 1, but greater than 0. If the transformation is a congruence transformation, the scale factor is 1, because the lengths of the transformed object are equal to the lengths of the original object. **29.** $\frac{1}{2}$ **31.** E

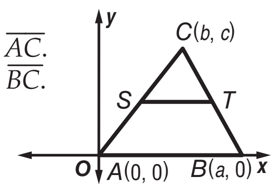
33. yes; $\frac{AC}{BD} = \frac{DE}{CE} = \frac{4}{3}$ **35.** no; $\frac{AB}{CD} \neq \frac{AE}{CE}$ **37.** 117

39. Given: $\triangle ABC$

S is the midpoint of $\overline{AC}$.

T is the midpoint of $\overline{BC}$.

Prove: $\overline{ST} \parallel \overline{AB}$



Proof:

Midpoint S is $\left(\frac{b+0}{2}, \frac{c+0}{2}\right)$ or $\left(\frac{b}{2}, \frac{c}{2}\right)$.

Midpoint T is $\left(\frac{a+b}{2}, \frac{0+c}{2}\right)$ or $\left(\frac{a+b}{2}, \frac{c}{2}\right)$.

Slope of $\overline{ST} = \frac{\frac{c}{2} - \frac{c}{2}}{\frac{a+b}{2} - \frac{b}{2}}, = \frac{0}{\frac{a}{2}}$ or 0.

Slope of $\overline{AB} = \frac{0-0}{a-0} = \frac{0}{a}$ or 0.

$\overline{ST}$ and $\overline{AB}$ have the same slope so, $\overline{ST} \parallel \overline{AB}$.

41. 8 **43.** 0.003 **45.** 0.17

Lesson 7-7

1. about 117 mi **3a.** 6 in. : 50 ft **3b.** $\frac{1}{100}$ **5.** 380 km **7.** 173 km

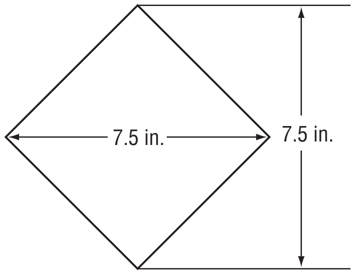
9 **a.** $\frac{\text{replica height}}{\text{statue height}} = \frac{10 \text{ in.}}{10 \text{ ft}} = \frac{1 \text{ in.}}{1 \text{ ft}}$

The scale of the replica is 1 inch : 1 foot.

b. $\frac{10 \text{ in.}}{10 \text{ ft}} = \frac{10 \text{ in.}}{10 \text{ ft}} \cdot \frac{1 \text{ ft}}{12 \text{ in.}} = \frac{1}{12}$

The scale factor is 1 : 12. So, the replica is $\frac{1}{12}$ times as tall as the actual sculpture.

11. Sample answer: 1 in. = 12 ft



13. about 2.7 h or 2 h and 42 min
15. 1.61 km

17 **a. Scale Model to Rocket**

$\frac{1 \text{ in.}}{12 \text{ ft}} = \frac{7 \text{ in.}}{x \text{ ft}}$	Write a proportion.
$1 \cdot x = 12 \cdot 7$	Cross Product Property
$x = 84$	Simplify.

The height of the rocket is 84 feet.

b. Since 12 ft = 144 in., $\frac{1 \text{ in.}}{12 \text{ ft}} = \frac{1 \text{ in.}}{144 \text{ in.}}$.

Scale Model to Rocket

$\frac{1 \text{ in.}}{144 \text{ in.}} = \frac{x \text{ in.}}{70 \text{ in.}}$	Write a proportion.
$1 \cdot 70 = 144 \cdot x$	Cross Product Property
$70 = 144x$	Simplify.
$0.5 \approx x$	Divide each side by 144.

The diameter of the model is about 0.5 inch.

19 **Scale Actual Tower to Replica**

$\frac{3}{1} = \frac{986 \text{ ft}}{x \text{ ft}}$	Write a proportion.
$3 \cdot x = 1 \cdot 986$	Cross Product Property
$3x = 986$	Simplify.
$x \approx 329$	Divide each side by 3.

The height of the ride is about 329 feet.

21. Felix; sample answer: The ratio of the actual high school to the replica is $\frac{75}{1.5}$ or 50 : 1. **23.** The first drawing will be larger. The second drawing will be $\frac{1}{6}$ the size of the first drawing, so the scale factor is 1 : 6. **25.** Both can be written as ratios comparing lengths. A scale factor must have the same unit of measure for both measurements. **27.** D **29.** C **31.** 12 **33.** 17.5 **35.** 29.3 **37.** 3.5 **39.** 10.5 **41.** 8 **43.** $JK = \sqrt{10}$, $KL = \sqrt{10}$, $JL = \sqrt{20}$, $XY = \sqrt{10}$, $YZ = \sqrt{10}$, and $XZ = \sqrt{20}$. Each pair of corresponding sides has the same measure so they are congruent. $\triangle JKL \cong \triangle XYZ$ by SSS. **45.** 8 **47.** 48 **49.** $3\sqrt{77}$

Chapter 7 Study Guide and Review

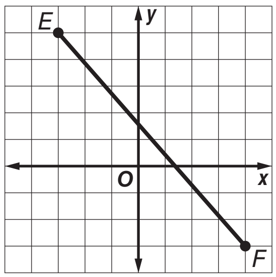
1. j; ratio **3.** g; AA Similarity Postulate **5.** d; extremes **7.** b; proportion **9.** 49 **11.** 10 or −10 **13.** 120 in. **15.** No, the polygons are not similar because the corresponding sides are not proportional. **17.** 16.5 **19.** Yes, $\triangle ABE \sim \triangle ADC$ by the SAS ~ Thm. **21.** No, the triangles are not similar because not all corresponding angles are congruent. **23.** 34.2 feet **25.** 22.5 **27.** 6 **29.** 633 mi **31.** enlargement; 2 **33.** 10.5 inches **35.** 95.8 mi

CHAPTER 8
Right Angles and Trigonometry

Chapter 8 Get Ready

1. $4\sqrt{7}$ **3.** $10\sqrt{3}$ **5.** $\frac{3}{4}$ **7.** 10 **9.** $x = 17.2$ in., 68.8 in. of trim

11.



Lesson 8-1

1. 10 **3.** $10\sqrt{6}$ or 24.5 **5.** $x = 6$; $y = 3\sqrt{5} \approx 6.7$; $z = 6\sqrt{5} \approx 13.4$ **7.** 18 ft 11 in.

9

$x = \sqrt{ab}$	Definition of geometric mean
$= \sqrt{16 \cdot 25}$	$a = 16$ and $b = 25$
$= \sqrt{(4 \cdot 4) \cdot (5 \cdot 5)}$	Factor.
$= 4 \cdot 5$ or 20	Simplify.

11. $12\sqrt{6} \approx 29.4$ **13.** $3\sqrt{3} \approx 5.2$ **15.** $\triangle WXY \sim \triangle XZY \sim \triangle WZX$ **17.** $\triangle HGF \sim \triangle HIG \sim \triangle GIF$

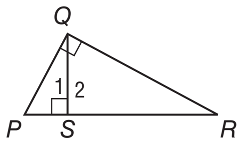
19

$17 = \sqrt{6 \cdot (y - 6)}$	Geometric Mean (Altitude) Theorem
$289 = 6 \cdot (y - 6)$	Square each side.
$\frac{289}{6} = y - 6$	Divide each side by 6.
$54\frac{1}{6} = y$	Add 6 to each side.
$54.2 \approx y$	Write as a decimal.
$x = \sqrt{6 \cdot y}$	Geometric Mean (Leg) Theorem
$= \sqrt{6 \cdot 54\frac{1}{6}}$	$y = 54\frac{1}{6}$
$= \sqrt{325}$	Multiply.
$= 5\sqrt{13}$	Simplify.
≈ 18.0	Use a calculator.
$z = \sqrt{(y - 6) \cdot y}$	Geometric Mean (Leg) Theorem
$= \sqrt{\left(54\frac{1}{6} - 6\right) \cdot 54\frac{1}{6}}$	$y = 54\frac{1}{6}$
$= \sqrt{48\frac{1}{6} \cdot 54\frac{1}{6}}$	Subtract.
≈ 51.1	Use a calculator.

21. $x \approx 4.7$; $y \approx 1.8$; $z \approx 13.1$ **23.** $x = 24\sqrt{2} \approx 33.9$; $y = 8\sqrt{2} \approx 11.3$; $z = 32$ **25.** 161.8 ft **27.** $\frac{\sqrt{30}}{7}$ or 0.8 **29.** $x = \frac{3\sqrt{3}}{2} \approx 2.6$; $y = \frac{3}{2}$; $z = 3$ **31.** 11 **33.** 3.5 ft **35.** 5 **37.** 4

39. Given: $\angle PQR$ is a right angle. $\overline{QS}$ is an altitude of $\triangle PQR$.

Prove: $\triangle PSQ \sim \triangle PQR$
$\triangle PQR \sim \triangle QSR$
$\triangle PSQ \sim \triangle QSR$



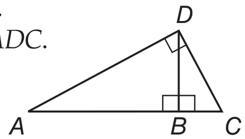
Proof:

Statements (Reasons)

1. $\angle PQR$ is a right angle. $\overline{QS}$ is an altitude of $\triangle PQR$. (Given)
2. $\overline{QS} \perp \overline{RP}$ (Definition of altitude)
3. $\angle 1$ and $\angle 2$ are right angles. (Definition of perpendicular lines)
4. $\angle 1 \cong \angle PQR$, $\angle 2 \cong \angle PQR$ (All right ∠s are ≅.)
5. $\angle P \cong \angle P$, $\angle R \cong \angle R$ (Congruence of angles is reflexive.)
6. $\triangle PSQ \sim \triangle PQR$, $\triangle PQR \sim \triangle QSR$ (AA Similarity Statements 4 and 5)
7. $\triangle PSQ \sim \triangle QSR$ (Similarity of triangles is transitive.)

41. Given: $\angle ADC$ is a right angle. $\overline{DB}$ is an altitude of $\triangle ADC$.

Prove: $\frac{AB}{AD} = \frac{AD}{AC}$
$\frac{BC}{DC} = \frac{DC}{AC}$



Proof:

Statements (Reasons)

1. $\angle ADC$ is a right angle. $\overline{DB}$ is an altitude of $\triangle ADC$. (Given)
2. $\triangle ADC$ is a right triangle. (Definition of right triangle)
3. $\triangle ABD \sim \triangle ADC$, $\triangle DBC \sim \triangle ADC$ (If the altitude is drawn from the vertex of the rt. ∠ to the hypotenuse of a rt. △, then the 2 △s formed are similar to the given △ and to each other.)
4. $\frac{AB}{AD} = \frac{AD}{AC}$, $\frac{BC}{DC} = \frac{DC}{AC}$ (Def. of similar triangles)

43

$x = \sqrt{ab}$	Definition of geometric mean
$= \sqrt{7 \cdot 12}$	$a = 7$ and $b = 12$

$= \sqrt{84}$ Multiply.

≈ 9 Simplify.

The average rate of return is about 9%.

45 Sample answer: The geometric mean of two consecutive integers is $\sqrt{x(x+1)}$ and the average of two consecutive integers is $\frac{x+(x+1)}{2}$.

$$\sqrt{x(x+1)} \stackrel{?}{=} \frac{x+(x+1)}{2}$$
$$\sqrt{x^2+x} \stackrel{?}{=} \frac{2x+1}{2}$$
$$\sqrt{x^2+x} \stackrel{?}{=} x+\frac{1}{2}$$
$$x^2+x \stackrel{?}{=} \left(x+\frac{1}{2}\right)^2$$
$$x^2+x \stackrel{?}{=} x^2+x+\frac{1}{4}$$
$$0 \neq \frac{1}{4}$$

If you set the two expressions equal to each other, the equation has no solution. So, the statement is never true.

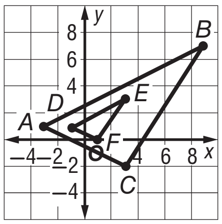
47. Sometimes; sample answer: When the product of the two integers is a perfect square, the geometric mean will be a positive integer. **49.** Neither; sample answer: On the similar triangles created by the altitude, the leg that is x units long on the smaller triangle corresponds with the leg that is 8 units long on the larger triangle, so the correct proportion is $\frac{4}{x} = \frac{x}{8}$ and x is about 5.7. **51.** Sample answer: 9 and 4, 8 and 8; In order for two whole numbers to result in a whole-number geometric mean, their product must be a perfect square. **53.** Sample answer: Both the arithmetic and the geometric mean calculate a value between two given numbers. The arithmetic mean of two numbers a and b is $\frac{a+b}{2}$, and the geometric mean of two numbers a and b is $\sqrt{ab}$. The two means will be equal when $a = b$.

Justification:

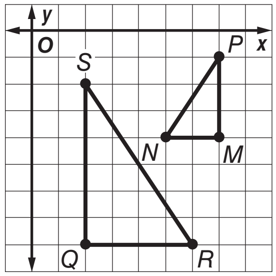
$$\frac{a+b}{2} = \sqrt{ab}$$
$$\left(\frac{a+b}{2}\right)^2 = ab$$
$$\frac{(a+b)^2}{4} = ab$$
$$(a+b)^2 = 4ab$$
$$a^2+2ab+b^2 = 4ab$$
$$a^2-2ab+b^2 = 0$$
$$(a-b)^2 = 0$$
$$a-b = 0$$
$$a = b$$

55. 10 **57.** C

59. $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = 3$, so $\triangle ABC \sim \triangle DEF$ by SSS Similarity.



61. $\angle M \cong \angle Q$ and $\frac{PM}{SQ} = \frac{MN}{QR} = \frac{1}{2}$, so $\triangle MNP \sim \triangle QRS$ by SAS Similarity.



63. octagon **65.** $x = 34, y = \pm 5$ **67.** hexagonal pyramid; base: $ABCDEF$, faces: $ABCDEF$, AGF, FGE, EGD, DGC, CGB, BGA; edges: $\overline{AF}$, $\overline{FE}$, $\overline{ED}$, $\overline{DC}$, $\overline{CB}$, $\overline{BA}$, $\overline{AG}$, $\overline{FG}$, $\overline{EG}$, $\overline{DG}$, $\overline{CG}$, and $\overline{BG}$; vertices: A, B, C, D, E, F, and G **69.** cone; base: circle Q; vertex: P

71. $\frac{16\sqrt{3}}{3}$ **73.** $\frac{3\sqrt{55}}{11}$

Lesson 8-2

1. 12

3 The side opposite the right angle is the hypotenuse, so $c = 16$.

$a^2 + b^2 = c^2$	Pythagorean Theorem
$4^2 + x^2 = 16^2$	$a = 4$ and $b = x$
$16 + x^2 = 256$	Simplify.
$x^2 = 240$	Subtract 16 from each side.
$x = \sqrt{240}$	Take the positive square root of each side.
$x = 4\sqrt{15}$	Simplify.
$x \approx 15.5$	Use a calculator.

5. D **7.** yes; obtuse
$26^2 \stackrel{?}{=} 16^2 + 18^2$
$676 > 256 + 324$
9. 20 **11.** $\sqrt{21} \approx 4.6$

13. $\frac{\sqrt{10}}{5} \approx 0.6$

15 16 and 30 are both multiples of 2: $16 = 2 \cdot 8$ and $30 = 2 \cdot 15$. Since 8, 15, 17 is a Pythagorean triple, the missing hypotenuse is $2 \cdot 17$ or 34.

17. 70 **19.** about 3 ft

21. yes; obtuse
$21^2 \stackrel{?}{=} 7^2 + 15^2$
$441 > 49 + 225$

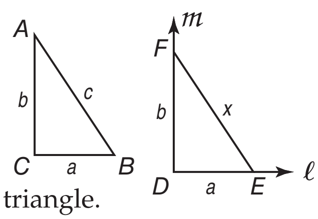
23. yes; right
$20.5^2 \stackrel{?}{=} 4.5^2 + 20^2$
$420.25 = 20.25 + 400$

25. yes; acute
$7.6^2 \stackrel{?}{=} 4.2^2 + 6.4^2$
$57.76 < 17.64 + 40.96$

27. 15 **29.** $4\sqrt{6} \approx 9.8$

31. acute; $XY = \sqrt{29}$, $YZ = \sqrt{20}$, $XZ = \sqrt{13}$; $(\sqrt{29})^2 < (\sqrt{20})^2 + (\sqrt{13})^2$ **33.** right; $XY = 6$, $YZ = 10$, $XZ = 8$; $6^2 + 8^2 = 10^2$

35. Given: $\triangle ABC$ with sides of measure a, b, and c, where $c^2 = a^2 + b^2$



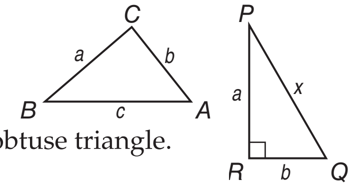
Prove: $\triangle ABC$ is a right triangle.

Proof:

Draw $\overline{DE}$ on line ℓ with measure equal to a. At D, draw line $m \perp \overline{DE}$. Locate point F on m so that $DF = b$. Draw $\overline{FE}$ and call its measure x. Because $\triangle FED$ is a right triangle, $a^2 + b^2 = x^2$. But $a^2 +$

$b^2 = c^2$, so $x^2 = c^2$ or $x = c$. Thus, $\triangle ABC \cong \triangle FED$ by SSS. This means $\angle C \cong \angle D$. Therefore, $\angle C$ must be a right angle, making $\triangle ABC$ a right triangle.

37. Given: In $\triangle ABC$, $c^2 > a^2 + b^2$ where c is the length of the longest side.



Prove: $\triangle ABC$ is an obtuse triangle.

Proof:

Statements (Reasons)

1. In $\triangle ABC$, $c^2 > a^2 + b^2$ where c is the length of the longest side. In $\triangle PQR$, $\angle R$ is a right angle. (Given)
2. $a^2 + b^2 = x^2$ (Pythagorean Theorem)
3. $c^2 > x^2$ (Substitution Property)
4. $c > x$ (A property of square roots)
5. $m\angle R = 90$ (Definition of a right angle)
6. $m\angle C > m\angle R$ (Converse of the Hinge Theorem)
7. $m\angle C > 90$ (Substitution Property of Equality)
8. $\angle C$ is an obtuse angle. (Definition of an obtuse angle)
9. $\triangle ABC$ is an obtuse triangle. (Definition of an obtuse triangle)

39. $P = 36$ units; $A = 60$ square units^2 **41.** 15

43

Scale	Width to Length
$\frac{16}{9} = \frac{41 \text{ in.}}{x \text{ in.}}$	Write a proportion.
$16 \cdot x = 9 \cdot 41$	Cross Product Property
$16x = 369$	Simplify.
$x = \frac{369}{16}$	Divide each side by 16.

The length of the television is about 23 inches.

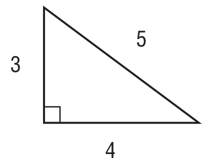
$a^2 + b^2 = c^2$	Pythagorean Theorem
$\left(\frac{369}{16}\right)^2 + 41^2 = c^2$	$a = \frac{369}{16}$ and $b = 41$
$\sqrt{\left(\frac{369}{16}\right)^2 + 41^2} = c$	Take the positive square root of each side.
$47.0 \approx c$	Use a calculator.

The screen size is about 47 inches.

45 The side opposite the right angle is the hypotenuse, so $c = x$.

$a^2 + b^2 = c^2$	Pythagorean Theorem
$8^2 + (x - 4)^2 = x^2$	$a = 8$ and $b = x - 4$
$64 + x^2 - 8x + 16 = x^2$	Find 8^2 and $(x - 4)^2$.
$-8x + 80 = 0$	Simplify.
$80 = 8x$	Add $8x$ to each side.
$10 = x$	Divide each side by 8.

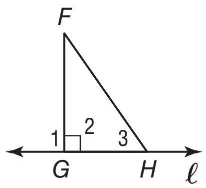
47. $\frac{1}{2}$ **49.** 5.4 **51.** Right; sample answer: If you double or halve the side lengths, all three sides of the new triangles are proportional to the sides of the original triangle. Using the Side-Side-Side Similarity Theorem, you know that both of the new triangles are similar to the original triangle, so they are both right.



53. D **55.** 250 units **57.** 6 **59.** $6\sqrt{5} \approx 13.4$

61. 1 in. = 2 ft; 6 in. × 4 in. **63.** yes; AA

65. Given: $\overline{FG} \perp \ell$; $\overline{FH}$ is any nonperpendicular segment from F to ℓ.



Prove: $FH > FG$

Proof:

Statements (Reasons)

1. $\overline{FG} \perp \ell$ (Given)
2. $\angle 1$ and $\angle 2$ are right angles. ($\perp$ lines form right angles.)
3. $\angle 1 \cong \angle 2$ (All right angles are congruent.)
4. $m\angle 1 = m\angle 2$ (Definition of congruent angles)
5. $m\angle 1 > m\angle 3$ (Exterior Angle Inequality Thm.)
6. $m\angle 2 > m\angle 3$ (Substitution Property.)
7. $FH > FG$ (If an $\angle$ of a $\triangle$ is > a second $\angle$, then the side opposite the greater $\angle$ is longer than the side opposite the lesser $\angle$.)

67. 50 **69.** 40 **71.** $\frac{7\sqrt{5}}{5}$ **73.** $2\sqrt{3}$ **75.** 2

Lesson 8-3

1. $5\sqrt{2}$ **3.** 22 **5.** $x = 14$; $y = 7\sqrt{3}$ **7.** Yes; sample answer: The height of the triangle is about $3\frac{1}{2}$ in., so since the height of the plaque is less than the diameter of the opening, it will fit. **9.** $\frac{15\sqrt{2}}{2}$ or $7.5\sqrt{2}$

11 In a 45°-45°-90° triangle, the length of the hypotenuse is $\sqrt{2}$ times the length of a leg.

$h = x\sqrt{2}$	Theorem 8.8
$= 18\sqrt{3} \cdot \sqrt{2}$	Substitution
$= 18\sqrt{6}$	$\sqrt{3} \cdot \sqrt{2} = \sqrt{6}$

13. $20\sqrt{2}$ **15.** $\frac{11\sqrt{2}}{2}$ **17.** $8\sqrt{2}$ or 11.3 cm **19.** $x = 10$; $y = 20$ **21.** $x = \frac{17\sqrt{3}}{2}$; $y = \frac{17}{2}$ **23.** $x = \frac{14\sqrt{3}}{3}$; $y = \frac{28\sqrt{3}}{3}$ **25.** $16\sqrt{3}$ or 27.7 ft **27.** 22.6 ft

29 In a 45°-45°-90° triangle, the length of the hypotenuse is 2 times the length of a leg.

$h = x\sqrt{2}$	Theorem 8.8
$6 = x\sqrt{2}$	Substitution
$\frac{6}{\sqrt{2}} = x$	Divide each side by $\sqrt{2}$.
$\frac{6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = x$	Rationalize the denominator.
$\frac{6 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = x$	Multiply.
$\frac{6\sqrt{2}}{2} = x$	$\sqrt{2} \cdot \sqrt{2} = 2$
$3\sqrt{2} = x$	Simplify.
$h = x\sqrt{2}$	Theorem 8.8
$y = 6\sqrt{2}$	Substitution

31. $x = 5$; $y = 10$ **33.** $x = 45$; $y = 12\sqrt{2}$

35 In a 30°-60°-90° triangle, the length of the hypotenuse is 2 times the length of the shorter leg.

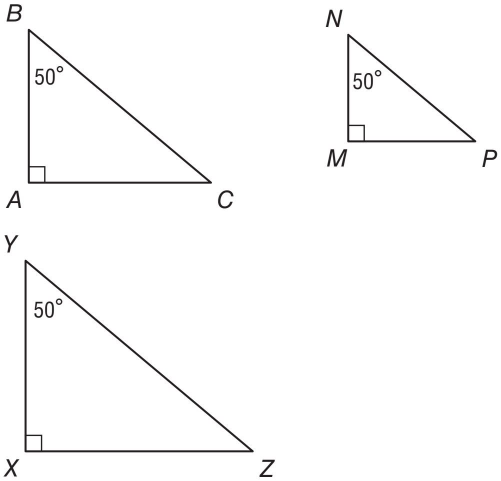
$h = 2s$	Theorem 8.9
$= 2(25)$ or 50	Substitution

The zip line's length is 50 feet.

37. $x = 9\sqrt{2}$; $y = 6\sqrt{3}$; $z = 12\sqrt{3}$

39. 7.5 ft; 10.6 ft; 13.0 ft **41.** (6, 9) **43.** (4, −2)

45 **a.**

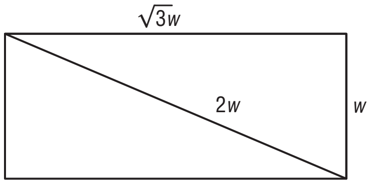


b. Measure the sides of the triangles to the nearest tenth of a centimeter. Find the ratios to the nearest tenth of a centimeter. Sample answer:

Triangle	Length				Ratio	
ABC	*AC*	2.4 cm	*BC*	3.2 cm	$\frac{BC}{AC}$	1.3
MNP	*MP*	1.7 cm	*NP*	2.2 cm	$\frac{NP}{MP}$	1.3
XYZ	*XZ*	3.0 cm	*YZ*	3.9 cm	$\frac{YZ}{XZ}$	1.3

c. Sample answer: In a right triangle with a 50° angle, the ratio of the leg opposite the 50° angle to the hypotenuse will always be the same, 1.3.

47. Sample answer: Let ℓ represent the length. $\ell^2 + w^2 = (2w)^2$; $\ell^2 = 3w^2$; $\ell = w\sqrt{3}$.



49. 37.9 **51.** B **53.** (−13, −3) **55.** 15 ft **57.** $x \approx 16.9$, $y \approx 22.6$, $z \approx 25.0$ **59.** 36, 90, 54 **61.** 45, 63, 72 **63.** ∠1, ∠4, ∠11 **65.** ∠2, ∠6, ∠9, ∠8, ∠7 **67.** 12.0

Lesson 8-4

1. $\frac{16}{20} = 0.80$ **3.** $\frac{12}{20} = 0.60$ **5.** $\frac{16}{20} = 0.80$ **7.** $\frac{\sqrt{3}}{2} \approx 0.87$ **9.** 27.44 **11.** about 1.2 ft **13.** 44.4 **15.** $RS \approx 6.7$; $m\angle R \approx 42$; $m\angle T \approx 48$

17

$$\sin J = \frac{\text{opp}}{\text{hyp}} = \frac{56}{65} \approx 0.86 \qquad \cos J = \frac{\text{adj}}{\text{hyp}} = \frac{33}{65} \approx 0.51$$

$$\tan J = \frac{\text{opp}}{\text{adj}} = \frac{56}{33} \approx 1.70 \qquad \sin L = \frac{\text{opp}}{\text{hyp}} = \frac{33}{65} \approx 0.51$$

$$\cos L = \frac{\text{adj}}{\text{hyp}} = \frac{56}{65} \approx 0.86 \qquad \tan L = \frac{\text{opp}}{\text{adj}} = \frac{33}{56} \approx 0.59$$

19. $\frac{84}{85} = 0.99$; $\frac{13}{85} = 0.15$; $\frac{84}{13} = 6.46$; $\frac{13}{85} = 0.15$; $\frac{84}{85} = 0.99$; $\frac{13}{84} = 0.15$ **21.** $\frac{\sqrt{3}}{2} = 0.87$; $\frac{2\sqrt{2}}{4\sqrt{2}} = 0.50$; $\frac{2\sqrt{6}}{2\sqrt{2}} = \sqrt{3} = 1.73$; $\frac{2\sqrt{2}}{4\sqrt{2}} = 0.50$; $\frac{\sqrt{3}}{2} = 0.87$; $\frac{\sqrt{3}}{3} = 0.58$

23. $\frac{\sqrt{3}}{2} \approx 0.87$ **25.** $\frac{1}{2}$ or 0.5 **27.** $\frac{1}{2}$ or 0.5 **29.** 28.7 **31.** 57.2 **33.** 17.4

35 Let $m\angle A = 55$ and let x be the height of the roller coaster.

$\sin A = \frac{\text{opp}}{\text{hyp}}$	Definition of sine ratio
$\sin 55 = \frac{x}{98}$	Substitution
$98 \cdot \sin 55 = x$	Multiply each side by 98.
$80 \approx x$	Use a calculator.

The height of the roller coaster is about 80 feet.

37. 61.4 **39.** 28.5 **41.** 21.8 **43.** $WX = 15.1$; $XZ = 9.8$; $m\angle W = 33$ **45.** $ST = 30.6$; $m\angle R = 58$; $m\angle T = 32$

47 $JL = \sqrt{[-2 - (-2)]^2 + [4 - (-3)]^2} = 7$

$KJ = \sqrt{[-2 - (-7)]^2 + [-3 - (-3)]^2} = 5$

$\tan K = \frac{\text{opp}}{\text{adj}}$	Definition of tangent ratio
$= \frac{7}{5}$	Substitution
$m\angle K = \tan^{-1}\left(\frac{7}{5}\right) \approx 54.5$	Use a calculator.

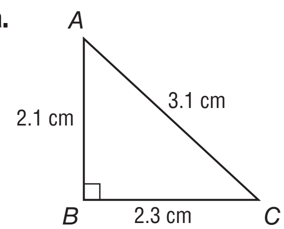
49. 51.3 **51.** 13.83 in.; 7.50 in^2 **53.** 8.74 ft; 3.41 ft^2 **55.** 0.92

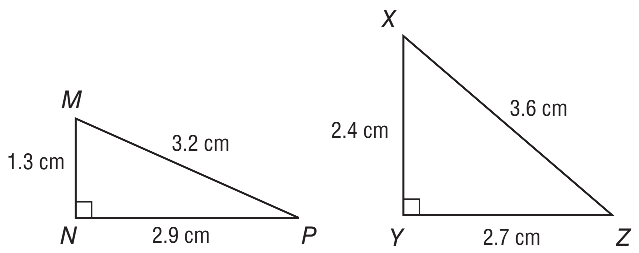
57 The triangle is isosceles, so two sides measure 32 and the two smaller triangles each have a side that measures x. Let $m\angle A = 54$.

$\cos A = \frac{\text{adj}}{\text{hyp}}$	Definition of cosine ratio
$\cos 54 = \frac{x}{32}$	Substitution
$32 \cdot \cos 54 = x$	Multiply each side by 32.
$18.8 \approx x$	Simplify.
$\sin A = \frac{\text{opp}}{\text{hyp}}$	Definition of sine ratio
$\sin 54 = \frac{y}{32}$	Substitution
$32 \cdot \sin 54 = y$	Multiply each side by 32.
$25.9 \approx y$	Simplify.

59. $x = 9.2$; $y = 11.7$

61a.





61b. Sample answer:

Triangle	Trigonometric Ratios				Sum of Ratios Squared	
ABC	cos *A*	0.677	sin *A*	0.742	$(\cos A)^2 + (\sin A)^2$	1
	cos *C*	0.742	sin *C*	0.677	$(\cos C)^2 + (\sin C)^2$	1
MNP	cos *M*	0.406	sin *M*	0.906	$(\cos M)^2 + (\sin M)^2$	1
	cos *P*	0.906	sin *P*	0.406	$(\cos P)^2 + (\sin P)^2$	1
XYZ	cos *X*	0.667	sin *X*	0.75	$(\cos X)^2 + (\sin X)^2$	1
	cos *Z*	0.75	sin *Z*	0.667	$(\cos Z)^2 + (\sin Z)^2$	1

61c. Sample answer: The sum of the cosine squared and the sine squared of an acute angle of a right triangle is 1. **61d.** $(\sin X)^2 + (\cos X)^2 = 1$

61e. Sample answer:

$(\sin A)^2 + (\cos A)^2 \stackrel{?}{=} 1$ Conjecture

$\left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2 \stackrel{?}{=} 1$ $\sin A = \frac{y}{r}$, $\cos A = \frac{x}{r}$

$\frac{y^2}{r^2} + \frac{x^2}{r^2} \stackrel{?}{=} 1$ Simplify.

$\frac{y^2 + x^2}{r^2} \stackrel{?}{=} 1$ Combine fractions with like denominators.

$\frac{r^2}{r^2} \stackrel{?}{=} 1$ Pythagorean Theorem

$1 = 1$ Simplify.

63. Sample answer: Yes; since the values of sine and cosine are both calculated by dividing one of the legs of a right triangle by the hypotenuse, and the hypotenuse is always the longest side of a right triangle, the values will always be less than 1. You will always be dividing the smaller number by the larger number. **65.** Sample answer: To find the measure of an acute angle of a right triangle, you can find the ratio of the leg opposite the angle to the hypotenuse and use a calculator to find the inverse sine of the ratio, you can find the ratio of the leg adjacent to the angle to the hypotenuse and use a calculator to find the inverse cosine of the ratio, or you can find the ratio of the leg opposite the angle to the leg adjacent to the angle and use a calculator to find the inverse tangent of the ratio. **67.** H **69.** E **71.** $x = 7\sqrt{2}$; $y = 14$

73. yes; right
$17^2 \stackrel{?}{=} 8^2 + 15^2$
$289 = 64 + 225$

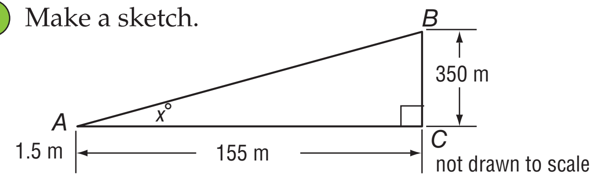
75. yes; obtuse
$35^2 \stackrel{?}{=} 30^2 + 13^2$
$1225 > 900 + 169$

77. no; $8.6 > 3.2 + 5.3$ **79.** $5\frac{7}{15}$ h or 5 h 28 min **81.** $x = 1$, $y = \frac{3}{2}$ **83.** 260 **85.** 18.9 **87.** 157.1

Lesson 8-5

1. 27.5 ft **3.** 14.2 ft

5 Make a sketch.



$\tan A = \frac{BC}{AC}$ $\tan = \frac{\text{opposite}}{\text{adjacent}}$

$\tan x° = \frac{348.5}{155}$ $m\angle A = x$, $BC = 350 - 1.5$ or 348.5, $AC = 155$

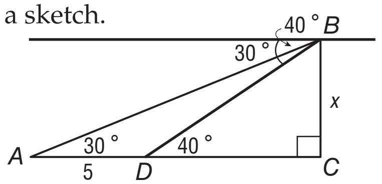
$x = \tan^{-1}\left(\frac{348.5}{155}\right)$ Solve for *x*.

$x \approx 66.0$ Use a calculator.

The angle of elevation is about 66°.

7. 14.8°

9 Make a sketch.



$\tan 30 = \frac{x}{5} + DC$ $\tan = \frac{\text{opposite}}{\text{adjacent}}$

$(5 + DC) \tan 30 = x$ Solve for *x*.

$\tan 40 = \frac{x}{DC}$ $\tan = \frac{\text{opposite}}{\text{adjacent}}$

$DC \tan 40 = x$ Solve for *x*.

$DC \tan 40 = (5 + DC) \tan 30$ Substitution

$DC \tan 40 = 5 \tan 30 + DC \tan 30$ Distributive Property

$DC \tan 40 - DC \tan 30 = 5 \tan 30$ Subtract *DC* tan 30 from each side.

$DC(\tan 40 - \tan 30) = 5 \tan 30$ Factor *DC*.

$DC = \frac{5 \tan 30}{\tan 40 - \tan 30}$ Divide each side by tan 40 − tan 30.

$DC \approx 11.0$ Use a calculator.

$\tan A = \frac{BC}{AC}$ $\tan = \frac{\text{opposite}}{\text{adjacent}}$

$\tan 30 = \frac{x}{16.0}$ $A = 30$, $BC = x$, $AC = 5 + 11.0$ or 16.0

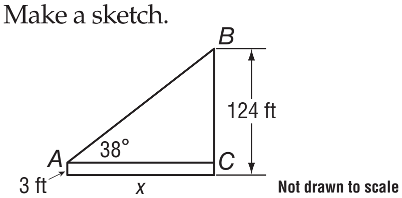
$16.0 \tan 30 = x$ Multiply each side by 16.0.

$9.3 \approx x$ Use a calculator.

The platform is about 9.3 feet high.

11. about 1309 ft **13.** 16.6° **15.** 240.2 ft

17 Make a sketch.



$$\tan A = \frac{BC}{AC} \qquad \tan = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 38° = \frac{121}{x} \qquad m\angle A = 38,\ BC = 124 - 3 \text{ or } 121,\ AC = x$$

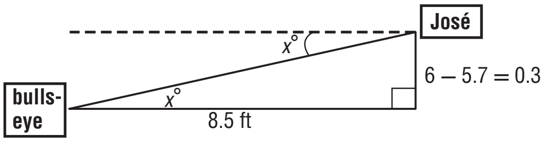
$$x = \frac{121}{\tan 38°} \qquad \text{Solve for } x.$$

$$x \approx 154.9 \qquad \text{Use a calculator.}$$

You should place the tripod about 154.9 feet from the monument.

19a. 74.8° **19b.** 110.1 m

21 Make two sketches.

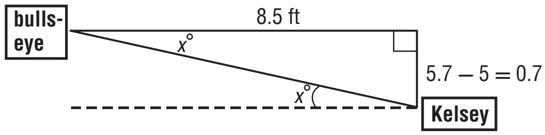


$$\tan x° = \frac{0.3}{8.5} \qquad \tan = \frac{\text{opp}}{\text{adj}}$$

$$x = \tan^{-1}\left(\frac{0.3}{8.5}\right) \qquad \text{Solve for } x.$$

$$x \approx 2.02 \qquad \text{Use a calculator.}$$

José throws at an angle of depression of 2.02°.



$$\tan x° = \frac{0.7}{8.5} \qquad \tan = \frac{\text{opp}}{\text{adj}}$$

$$x = \tan^{-1}\left(\frac{0.7}{8.5}\right) \qquad \text{Solve for } x.$$

$$x \approx 4.71 \qquad \text{Use a calculator.}$$

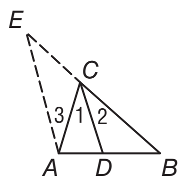
Kelsey throws at an angle of elevation of 4.71°.

23. Rodrigo; sample answer: Since your horizontal line of sight is parallel to the other person's horizontal line of sight, the angles of elevation and depression are congruent according to the Alternate Interior Angles Theorem. **25.** True; sample answer: As a person moves closer to an object, the horizontal distance decreases, but the height of the object is constant. The tangent ratio will increase, and therefore the measure of the angle also increases. **27.** Sample answer: If you sight something with a 45° angle of elevation, you don't have to use trigonometry to determine the height of the object. Since the legs of a 45°-45°-90° are congruent, the height of the object will be the same as your horizontal distance from the object. **29.** 6500 ft **31.** B

33. $\frac{20}{15} = 1.33$ **35.** $\frac{15}{20} = 0.75$ **37.** $\frac{20}{25} = 0.80$

39. Given: $\overline{CD}$ bisects $\angle ACB$.
By construction, $\overline{AE} \parallel \overline{CD}$.

Prove: $\frac{AD}{DB} = \frac{AC}{BC}$



Proof:

Statements (Reasons)

1. $\overline{CD}$ bisects $\angle ACB$. By construction, $\overline{AE} \parallel \overline{CD}$. (Given)
2. $\frac{AD}{DB} = \frac{EC}{BC}$ (Triangle Proportionality Theorem)
3. $\angle 1 \cong \angle 2$ (Definition of Angle Bisector)
4. $\angle 3 \cong \angle 1$ (Alternate Interior Angle Theorem)
5. $\angle 2 \cong \angle E$ (Corresponding Angle Postulate)
6. $\angle 3 \cong \angle E$ (Transitive Prop.)
7. $\overline{EC} \cong \overline{AC}$ (Isosceles △ Thm.)
8. $EC = AC$ (Def. of congruent segments)
9. $\frac{AD}{DB} = \frac{AC}{BC}$ (Substitution)

41. (7, 4) **43.** (–5, 6) **45.** 2 **47.** 2.1

Lesson 8-6

1. 6.1

3 By the Triangle Angle Sum Theorem, the remaining angle measures 180 – (60 + 55) or 65.

$$\frac{\sin A}{a} = \frac{\sin B}{b} \qquad \text{Law of Sines}$$

$$\frac{\sin 65°}{73} = \frac{\sin 60°}{x} \qquad m\angle A = 65,\ a = 73,\ m\angle B = 60,\ b = x$$

$$x \sin 65° = 73 \sin 60° \qquad \text{Cross Products Property}$$

$$x = \frac{73 \sin 60°}{\sin 65°} \qquad \text{Divide each side by } \sin 65°.$$

$$x \approx 69.8 \qquad \text{Use a calculator.}$$

5. 8.3 **7.** 47.1 ft **9.** $m\angle N = 42,\ MP \approx 35.8,\ NP \approx 24.3$
11. $m\angle D \approx 73,\ m\angle E \approx 62,\ m\angle F \approx 45$ **13.** 4.1 **15.** 22.8
17. 15.1 **19.** 2.0 **21.** 2.8 in.

23

$$a^2 = b^2 + c^2 - 2bc \cos A \qquad \text{Law of Cosines}$$

$$x^2 = 1.2^2 + 3.0^2 - 2(1.2)(3.0) \cos 123° \qquad \text{Substitution}$$

$$x^2 = 10.44 - 7.2 \cos 123° \qquad \text{Simplify.}$$

$$x = \sqrt{10.44 - 7.2 \cos 123°} \qquad \text{Take the square root of each side.}$$

$$x \approx 3.8 \qquad \text{Use a calculator.}$$

25. 98 **27.** 112 **29.** 126.2 ft **31.** $m\angle B = 34,\ AB \approx 9.5,\ CA \approx 6.7$

33

$$j^2 = k^2 + \ell^2 - 2k\ell \cos J \qquad \text{Law of Cosines}$$

$$29.7^2 = 30.0^2 + 24.6^2 - 2(30.0)(24.6) \cos J \qquad j = 29.7,\ k = 30.0,\ \ell = 24.6$$

$$882.09 = 1505.16 - 1476 \cos J \qquad \text{Simplify.}$$

$$-623.07 = -1476 \cos J \qquad \text{Subtract 1505.16 from each side.}$$

$$\frac{-623.07}{-1476} = \cos J \qquad \text{Divide each side by } -1476.$$

$$m\angle J = \cos^{-1}\left(\frac{623.07}{1476}\right) \qquad \text{Use the inverse cosine ratio.}$$

$$m\angle J \approx 65 \qquad \text{Use a calculator.}$$

$$\frac{\sin J}{j} = \frac{\sin K}{k} \qquad \text{Law of Sines}$$

$$\frac{\sin 65°}{29.7} = \frac{\sin K}{30.0} \qquad m\angle J \approx 65,\ j = 29.7, \text{ and } k = 30.0$$

$$30.0 \sin 65° = 29.7 \sin K \qquad \text{Cross Products Property}$$

$$\frac{30.0 \sin 65°}{29.7} = \sin K \qquad \text{Divide each side by 29.7.}$$

$$m\angle K = \sin^{-1}\left(\frac{30.0 \sin 65°}{29.7}\right) \qquad \text{Use the inverse sine ratio.}$$

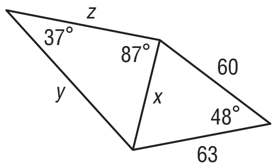
$$m\angle K \approx 66 \qquad \text{Use a calculator.}$$

By the Triangle Angle-Sum Theorem, $m\angle L \approx 180 - (65 + 66)$ or 49.

35. $m\angle G = 75,\ GH \approx 19.9,\ GJ \approx 11.8$ **37.** $m\angle P \approx 35,\ m\angle R \approx 75,\ RP \approx 14.6$ **39.** $m\angle C \approx 23,\ m\angle D \approx 67,$

$m\angle E \approx 90$ **41.** $m\angle A = 35, AB \approx 7.5, BC \approx 9.8$
43. ≈ 96.2 ft **45a.** Def. of sine **45b.** Mult. Prop.
45c. Subst. **45d.** Div. Prop. **47.** 24.3

49



$a^2 = b^2 + c^2 - 2bc \cos A$	Law of Cosines
$x^2 = 60^2 + 63^2 - 2(60)(63) \cos 48°$	Substitution
$x^2 = 7569 - 7560 \cos 48°$	Simplify.
$x = \sqrt{7569 - 7560 \cos 48°}$	Take the square root of each side.
$x \approx 50.1$	Use a calculator.
$\frac{\sin A}{a} = \frac{\sin B}{b}$	Law of Sines
$\frac{\sin 37°}{50.1} = \frac{\sin 87°}{y}$	$m\angle A = 37°, a = x = 50.1$, $m\angle B = 87°, b = y$
$y \sin 37° = 50.1 \sin 87°$	Cross Products Property
$y = \frac{50.1 \sin 87°}{\sin 37°}$	Divide each side by sin 37°.
$y \approx 83.1$	Use a calculator.

By the Triangle Angle-Sum Theorem, the angle opposite side z measures $180 - (37 + 87)$ or 56.

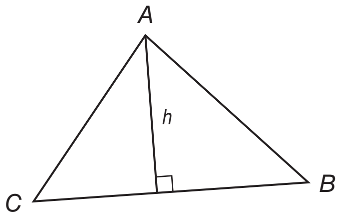
$\frac{\sin A}{a} = \frac{\sin B}{b}$	Law of Sines
$\frac{\sin 37°}{50.1} = \frac{\sin 56°}{z}$	$m\angle A = 37°, a = x = 50.1$, $m\angle B = 87°, b = z$
$z \sin 37° = 50.1 \sin 56°$	Cross Products Property
$z = \frac{50.1 \sin 56°}{\sin 37°}$	Divide each side by sin 37°.
$z \approx 69.0$	Use a calculator.

$$\text{perimeter} = 60 + 63 + y + z$$
$$\approx 60 + 63 + 83.1 + 69.0 \text{ or } 275.1$$

51

$a^2 = b^2 + c^2 - 2bc \cos A$	Law of Cosines
$x^2 = 8^2 + 6^2 - 2(8)(6) \cos 71.8°$	Substitution
$x^2 = 100 - 96 \cos 71.8°$	Simplify.
$x = \sqrt{100 - 96 \cos 71.8°}$	Take the square root of each side.
$x \approx 8.4$ in.	Use a calculator.

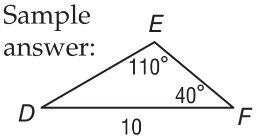
53a.



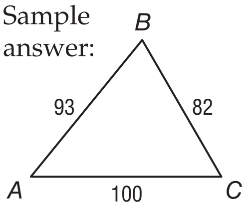
53b. $h = AB \sin B$ **53c.** $A = \frac{1}{2}(BC)(AB \sin B)$

53d. 57.2 units^2 **53e.** $A = \frac{1}{2}(BC)(CA \sin C)$ **55.** 5.6

57a. Sample answer:



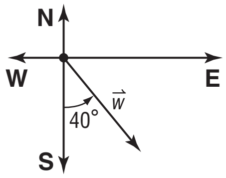
57b. Sample answer:



59. A **61.** 325.6 **63.** 269.6 ft **65.** 45 **67.** Never; sample answer: Since an equilateral triangle has three congruent sides and a scalene triangle has three non-congruent sides, the ratios of the three pairs of sides can never be equal. Therefore, an equilateral triangle and a scalene triangle can never be similar.
69. $Q(a, a), P(a, 0)$ **71.** $\sqrt{80} \approx 8.9$ **73.** 8.5

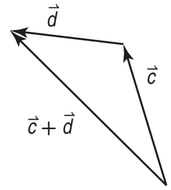
Lesson 8-7

1.



1 in. : 60 mi/h

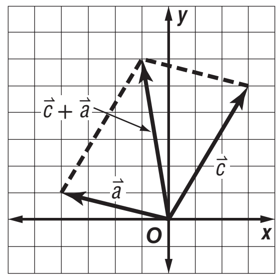
3.



5

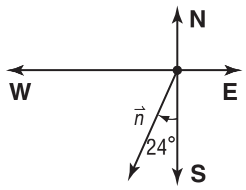
$\overrightarrow{YZ} = \langle x_2 - x_1, y_2 - y_1 \rangle$	Component form of a vector
$= \langle 5 - 0, 5 - 0 \rangle$	$(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (5, 5)$
$= \langle 5, 5 \rangle$	Simplify.

7. $\sqrt{20}$; $\approx 296.6°$
9. $\langle -1, 6 \rangle$



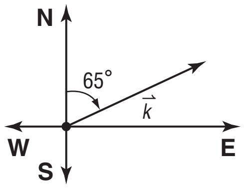
11. ≈ 354.3 mi/h at angle of 8.9° east of north

13.



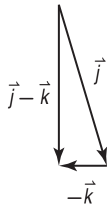
1 in. : 5 m

15.



1 in. : 50 km/h

17.



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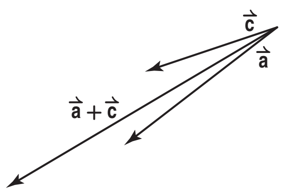

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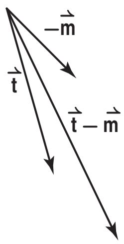


23. $\langle 5, 0 \rangle$ **25.** $\langle -6, -3 \rangle$ **27.** $\langle -3, -6 \rangle$
29. $\sqrt{34}$; $\approx 31.0°$ **31.** $\sqrt{50}$; $\approx 171.9°$

33 Use the distance formula to find the magnitude.

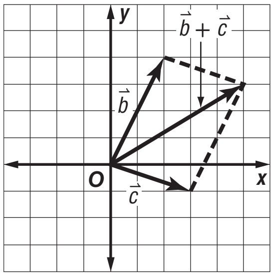
$\lvert \vec{r} \rvert = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance Formula
$= \sqrt{(-3 - 0)^2 + (-6 - 0)^2}$	$(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (-3, -6)$
$= \sqrt{45}$ or about 6.7	Simplify.

Use the inverse tangent function to find θ.

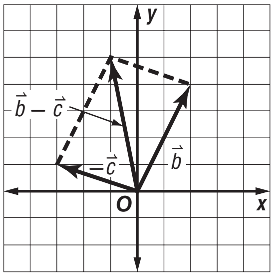
$\tan \theta = \frac{6}{3}$	$\tan \theta = \frac{\text{opp}}{\text{adj}}$
$\theta = \tan^{-1} \frac{6}{3} \approx 63.4°$	Def. of inverse tangent

The direction of $\vec{r}$ is about $180° + 63.4°$ or $243.4°$.

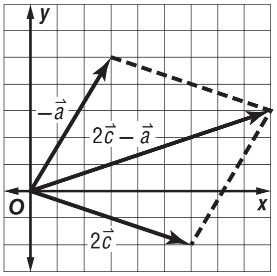
35. ⟨5, 3⟩



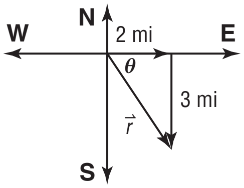
37. ⟨−1, 5⟩



39. ⟨9, 3⟩



41a.



41b. In component form, the vectors representing Amy's hikes due east and due south are ⟨2, 0⟩ and ⟨0, −3⟩, respectively. So, the resultant vector is ⟨2, 0⟩ + ⟨0, −3⟩ or ⟨2, −3⟩. The resultant distance is about 3.6 mi. The direction of $\vec{r}$ is about 90° − 56.3° or 33.7° east of south.

43. ⟨4, −4⟩ **45.** ⟨26, 5⟩ **47.** 2.3 ft/s **49.** Sometimes; sample answer: Parallel vectors can either have the same or opposite direction.

51.
$$\begin{aligned} k(\vec{a}+\vec{b}) &= k(\langle x_1, y_1\rangle + \langle x_2, y_2\rangle) \\ &= k\langle x_1 + x_2, y_1 + y_2\rangle \\ &= \langle k(x_1 + x_2), k(y_1 + y_2)\rangle \\ &= \langle kx_1 + kx_2, ky_1 + ky_2\rangle \\ &= \langle kx_1, ky_1\rangle + \langle kx_2, ky_2\rangle \\ &= k\langle x_1, y_1\rangle + k\langle x_2, y_2\rangle \\ &= k\vec{a} + k\vec{b} \end{aligned}$$

53. The initial point of the resultant starts at the initial point of the first vector in both methods. However, in the parallelogram method, both vectors start at the same initial point, whereas, in the triangle method, the resultant connects the initial point of the first vector and the terminal point of the second. The resultant is the diagonal of the parallelogram formed using the parallelogram method. **55.** B **57.** D **59.** 72.0 **61.** 376.4 ft **63.** 30 **65.** 30 **67.** 60 **69.** Δs 1–4, Δs 5–12, Δs 13–20

Chapter 8 Study Guide and Review

1. false, geometric **3.** false, sum **5.** true **7.** true
9. false, Law of Cosines **11.** 6 **13.** $\frac{8}{3}$ **15.** 50 ft
17. $9\sqrt{3} \approx 15.6$
19. yes; acute
$16^2 \stackrel{?}{=} 13^2 + 15^2$
$256 < 169 + 225$
21. 18.4 m **23.** $x = 4\sqrt{2}$, $y = 45°$ **25.** $\frac{5}{13}$, 0.38
27. $\frac{12}{13}$, 0.92 **29.** $\frac{5}{12}$, 0.42 **31.** 32.2 **33.** 63.4° and 26.6°
35. 86.6 feet **37.** 15.2 **39.** ⟨−6, −5⟩ **41.** ⟨−8, 1⟩

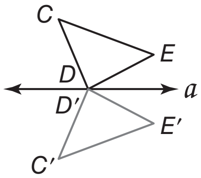
CHAPTER 9
Transformations and Symmetry

Chapter 9 Get Ready

1. rotation **3.** translation **5.** ⟨−16, −28⟩
7. reduction; $\frac{1}{2}$

Lesson 9-1

1.



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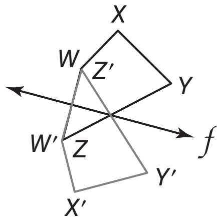

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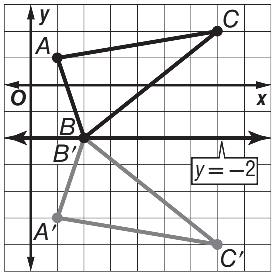

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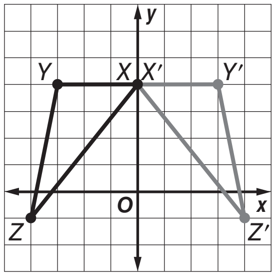

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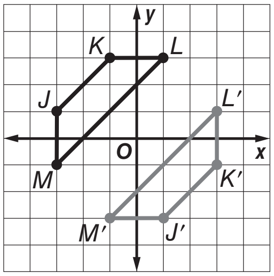
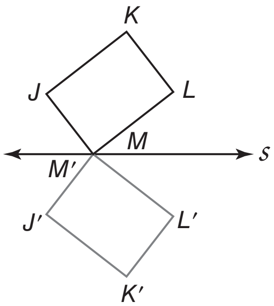
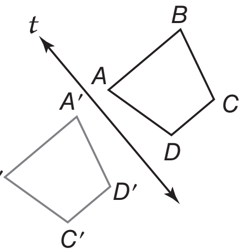


11.



13 Draw a line through each vertex that is perpendicular to line *t*. Measure the distance from point *A* to line *t*. Then locate *A*′ the same distance from line *t* on the opposite side. Repeat to locate points *B*′, *C*′, and *D*′. Then connect vertices *A*′, *B*′, *C*′ and *D*′ to form the reflected image.



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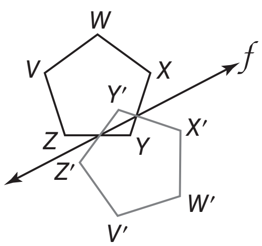
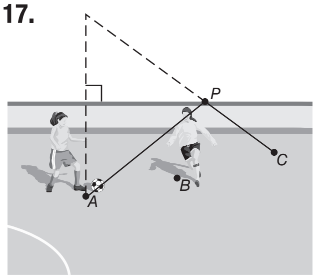


17.



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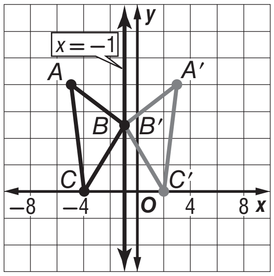

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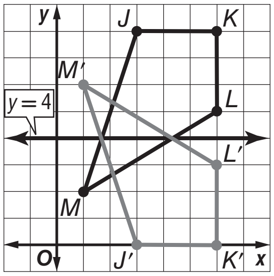

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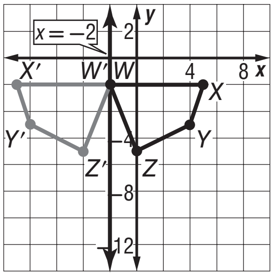
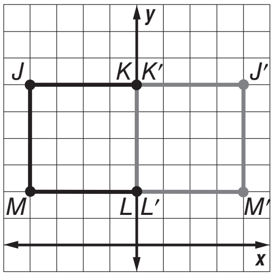


25 Multiply the x-coordinate of each vertex by −1.

(x, y)	→	$(-x, y)$
$J(-4, 6)$	→	$J'(4, 6)$
$K(0, 6)$	→	$K'(0, 6)$
$L(0, 2)$	→	$L'(0, 2)$
$M(-4, 2)$	→	$M'(4, 2)$

Graph $JKLM$ and its image $J'K'L'M'$



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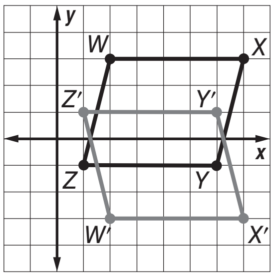

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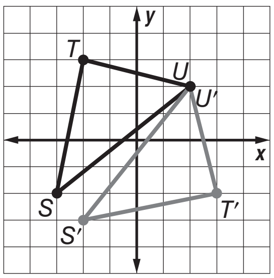

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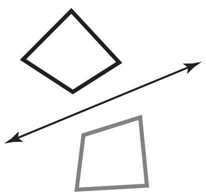

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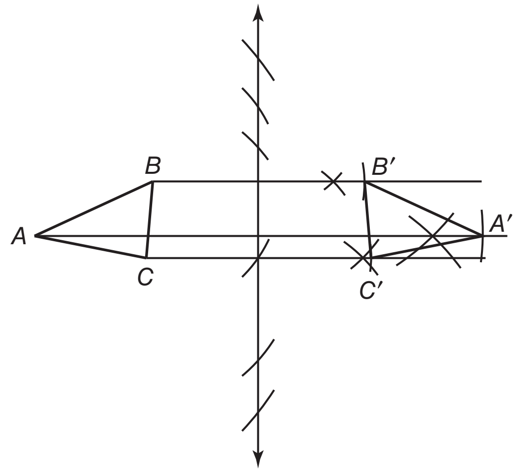
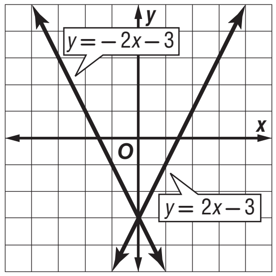


35 **a.** The reflections of the zebras are shown in the water. So, the water separates the zebras and their reflections. **b.** The water is a flat surface that extends in all directions. It represents a finite plane.

37.



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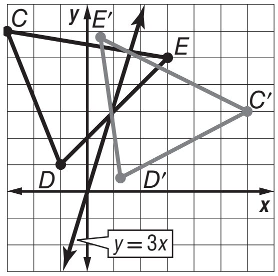
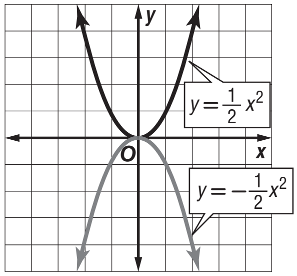


41 In the reflection, the x-coordinates stay the same, but the y-coordinates are opposites. The equation of the reflected image is $-y = \frac{1}{2}x^2$ or $y = -\frac{1}{2}x^2$.



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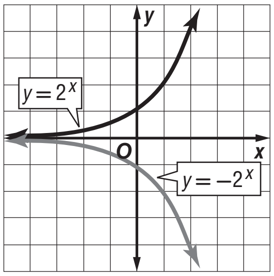
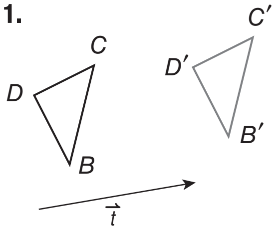


45. Jamil; sample answer: When you reflect a point across the x-axis, the reflected point is in the same place horizontally, but not vertically. When (2, 3) is reflected across the x-axis, the coordinates of the reflected point are (2, −3) since it is in the same location horizontally, but the other side of the x-axis vertically. **47.** (a, b) **49.** The slope of the line connecting the two points is $\frac{3}{5}$. The Midpoint Formula can be used to find the midpoint between the two points, which is $\left(\frac{3}{2}, \frac{3}{2}\right)$. Using the point-slope form, the equation of the line is $y = -\frac{5}{3}x + 4$. (The slope of the bisector is $-\frac{5}{3}$ because it is the negative reciprocal of the slope $\frac{3}{5}$.) **51.** Construct

P, Q, R collinear with Q between P and R. Draw line ℓ, then construct perpendicular lines from P, Q, and R to line ℓ. Show equidistance or similarity of slope.
53. B **55.** E **57.** $\langle 3, 2\rangle$ **59.** about 536 ft **61.** $AB > FD$
63. $m\angle FBA > m\angle DBF$ **65.** $2\sqrt{13} \approx 7.2$, 146.3°
67. $2\sqrt{122} \approx 22.1$, 275.2°

Lesson 9-2

1.



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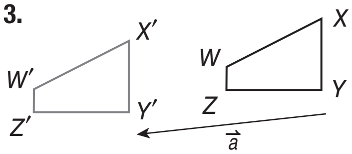
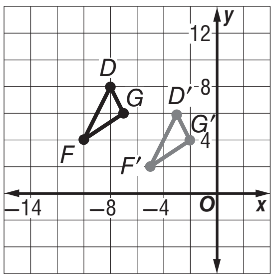
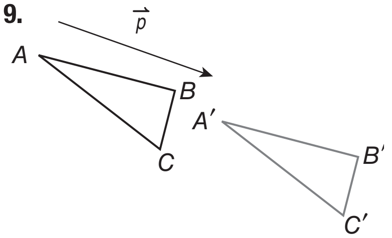


5.

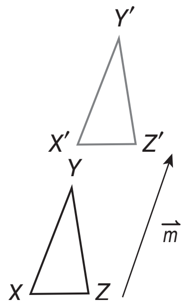


7. $(x, y) \to (x + 3, y - 5)$

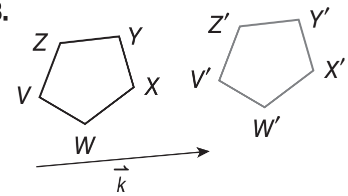
9.



11.



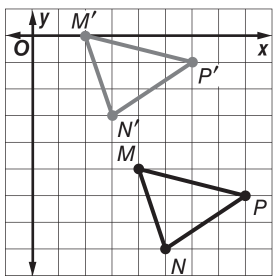
13.



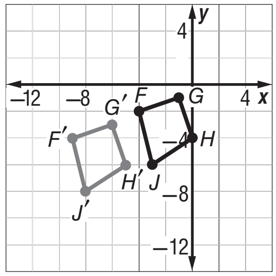
15 The vector indicates a translation 2 units left and 5 units up.

(x, y)	→	**(x – 2, y + 5)**
$M(4, -5)$	→	$M'(2, 0)$
$N(5, -8)$	→	$N'(3, -3)$
$P(8, -6)$	→	$P'(6, -1)$

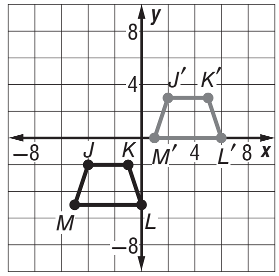
Graph $\triangle MNP$ and its image $\triangle M'N'P'$.



17.



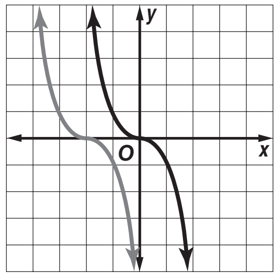
19.



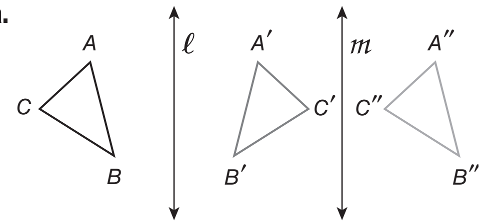
21 –12 represents a horizontal change of 12 yards left and 17 represents a vertical change of 17 yards up. A vector that indicates a translation 12 units left and 17 units up is $\langle -12, 17\rangle$.

23. $\langle 3, -5\rangle$ **25.** They move to the right 13 seats and back one row; $\langle 13, -1\rangle$.

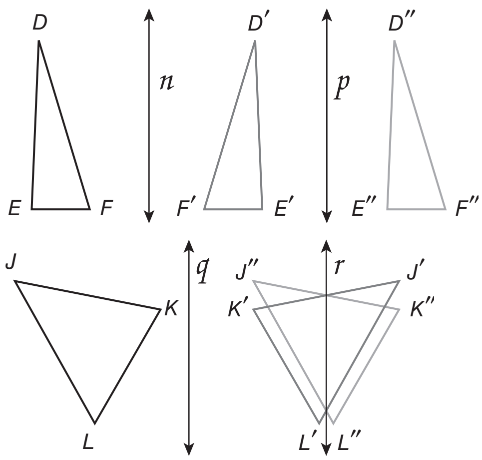
27 The translation vector $\langle -2, 0\rangle$ represents a horizontal shift 2 units left. The equation of the reflected image is $y = -[x - (-2)]^3$ or $y = -(x + 2)^3$.



29a.



29b.

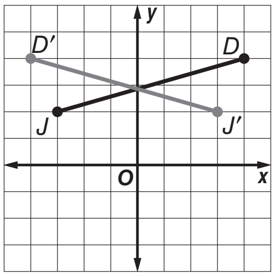


29c.

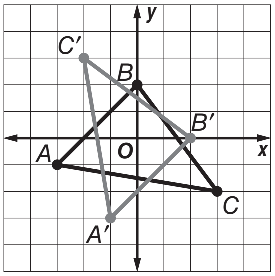
Distance Between Corresponding Points (cm)		Distance Between Parallel Lines (cm)	
A and A'', B and B'', C and C''	4.4	ℓ and m	2.2
D and D'', E and E'', F and F''	5.6	n and p	2.8
J and J'', K and K'', L and L''	2.8	q and r	1.4

29d. Sample answer: The composition of two reflections in vertical lines can be described by a horizontal translation that is twice the distance between the two vertical lines. **31.** $y = m(x - a) + 2b$; $2b - ma$ **33.** Sample answer: Both vector notation and function notation describe the distance a figure is translated in the horizontal and vertical directions. Vector notation does not give a rule in terms of initial location, but function notation does. For example, the translation a units to the right and b units up from the point (x, y) would be written $\langle a, b \rangle$ in vector notation and $(x, y) \longrightarrow (x + a, y + b)$ in function notation. **35.** D **37.** F

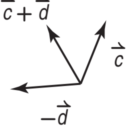
39.



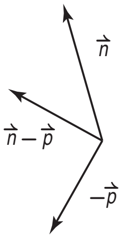
41.



43. $\vec{c} + \vec{d}$



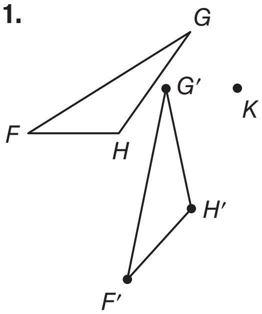
45.



47. 100 **49.** 80
51. obtuse; 110
53. obtuse; 140

Lesson 9-3

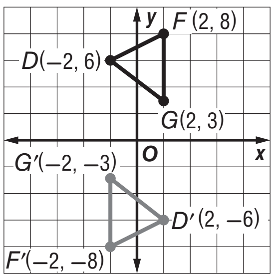
1.



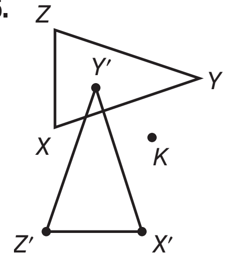
3 Multiply the x- and y-coordinates by -1.

(x, y)	$\rightarrow$	$(-x, -y)$
$D(-2, 6)$	$\rightarrow$	$D'(2, -6)$
$F(2, 8)$	$\rightarrow$	$F'(-2, -8)$
$G(2, 3)$	$\rightarrow$	$G'(-2, -3)$

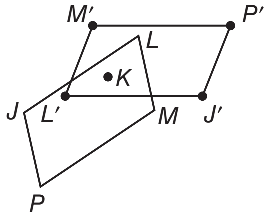
Graph $\triangle DFG$ and its image $\triangle D'F'G'$.



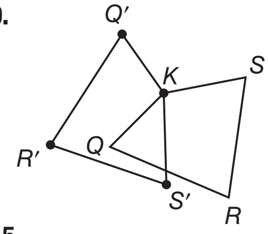
5.



7.

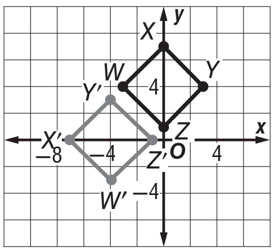


9.



11. 120°; 360° ÷ 6 petals = 60° per petal. Two petal turns is 2 • 60° or 120°. **13.** 154.2°; 360° ÷ 7 petals = 51.4° per petal. Three petal turns is 3 • 51.4° or 154.2°.

15.



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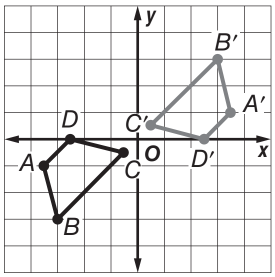
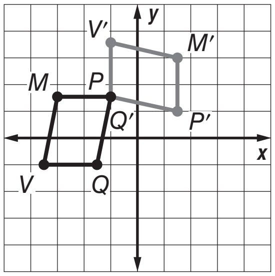
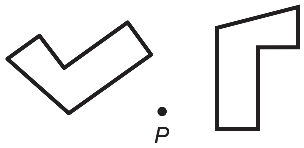


19.



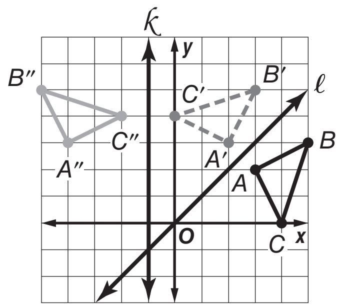
21a. 10°
21b. about 1.7 seconds
23. 125°



25. $y = -x + 2$; parallel **27.** $y = -x - 2$; collinear
29. x-intercept: $y = 2x + 4$; y-intercept: $y = 2x + 4$

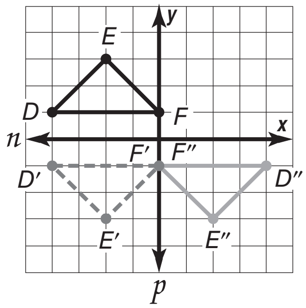
31 After 31 seconds, the ride will have rotated 31 • 0.25 or 7.75 times. So, she will be in the bottom position of the ride, with the car having the center at $(0, -4)$. After 31 seconds, the car will have rotated 31 • 0.5 or 15.5 times. So, she will be in the position directly across from her starting position. Her position after 31 seconds would be $(2, -4)$.

33 **a.**

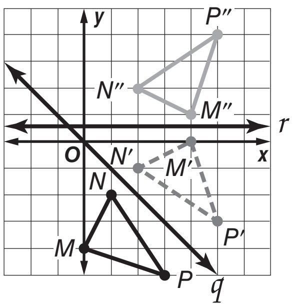


b. Sample answer: Let the x-axis be line n and the y-axis be line p. Draw $\triangle DEF$ with vertices $D(-4, 1)$, $E(-2, 3)$, and $F(0, 1)$. Reflect $\triangle DEF$ in the line n to get $\triangle D'E'F'$ with vertices $D'(-4, -1)$,

$E'(-2, -3)$, and $F'(0, -1)$. Reflect $\triangle D'E'F'$ in the line p to get $\triangle D''E''F''$ with vertices $D''(4, -1)$, $E''(2, -3)$, and $F''(0, -1)$.



Sample answer: Let the line $y = -x$ be line q and the line $y = 0.5$ be line r. Draw $\triangle MNP$ with vertices $M(0, -4)$, $N(1, 2)$, and $P(3, -5)$. Reflect $\triangle MNP$ in the line q to get $\triangle M'N'P'$ with vertices $M'(4, 0)$, $N'(2, -1)$, and $P'(5, -3)$. Reflect $\triangle M'N'P'$ in the line r to get $\triangle M''N''P''$ with vertices $M''(4, 1)$, $N''(2, 2)$, and $P''(5, 4)$.

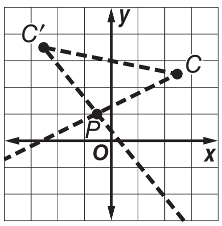


c.

Angle of Rotation Between Figures		Angle Between Intersecting Lines	
$\triangle ABC$ and $\triangle A''B''C''$	90°	ℓ and m	45°
$\triangle DEF$ and $\triangle D''E''F''$	180°	n and p	90°
$\triangle MNP$ and $\triangle M''N''P''$	90°	q and r	45°

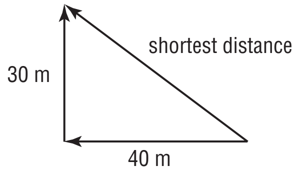
d. Sample answer: The measure of the angle of rotation about the point where the lines intersect is twice the measure of the angle between the two intersecting lines.

35. Sample answer: $(-1, 2)$; Since $\triangle CC'P$ is isosceles and the vertex angle of the triangle is formed by the angle of rotation, both $m\angle PCC'$ and $m\angle PC'C$ are 40° because the base angles of isosceles triangles are congruent. When you construct a 40° angle with a vertex at C and a 40° angle with a vertex at C', the intersection of the rays forming the two angles intersect at the point of rotation, or $(-1, 2)$.

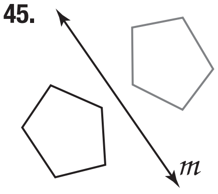


37. No; sample answer: When a figure is reflected about the x-axis, the x-coordinates of the transformed figure remain the same, and the y-coordinates are negated. When a figure is rotated 180° about the origin, both the x- and y-coordinates are negated. Therefore, the transformations are not equivalent. **39.** D **41.** J

43. 50 mi



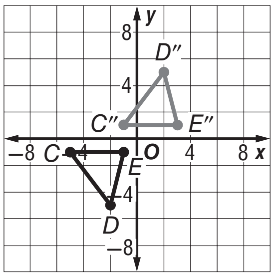
45.



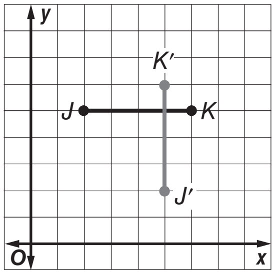
47. reflection **49.** rotation or reflection

Lesson 9-4

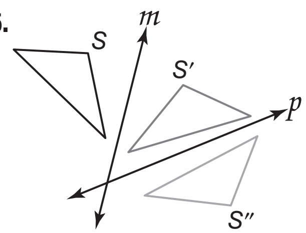
1.



3.



5.

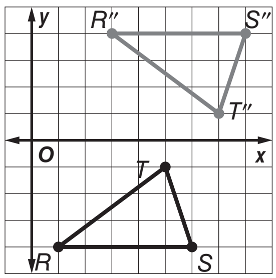


rotation clockwise 100° about the point where lines m and p intersect.

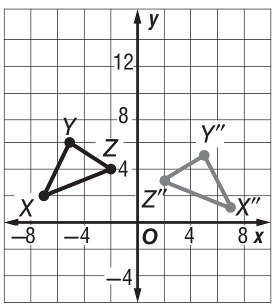
7 translation along $\langle 2, 0\rangle$ reflection in x-axis

(x, y)	$\rightarrow$	$(x + 2, y)$	(x, y)	$\rightarrow$	$(x, -y)$
$R(1, -4)$	$\rightarrow$	$R'(3, -4)$	$R'(3, -4)$	$\rightarrow$	$R''(3, 4)$
$S(6, -4)$	$\rightarrow$	$S'(8, -4)$	$S'(8, -4)$	$\rightarrow$	$S''(8, 4)$
$T(5, -1)$	$\rightarrow$	$T'(7, -1)$	$T'(7, -1)$	$\rightarrow$	$T''(7, 1)$

Graph $\triangle RST$ and its image $\triangle R''S''T''$.



9.



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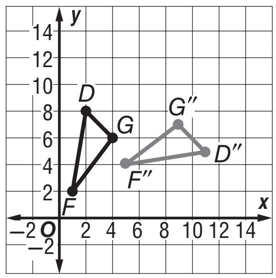
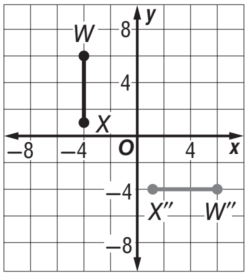
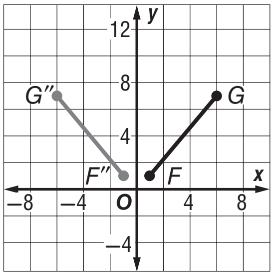


13.



15.



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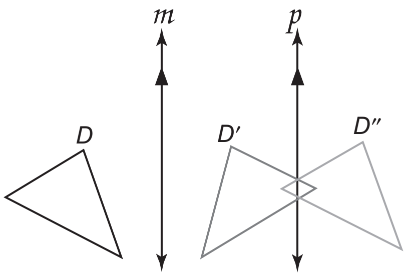
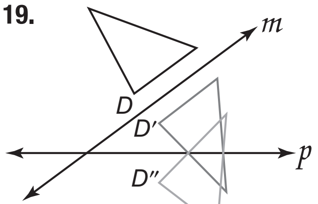


horizontal translation 4 cm to the right.

19.



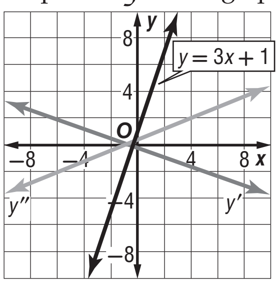
70° rotation about the point where lines m and p intersect

21. translation **23.** reflection

25 rotation 90° about origin reflection in x-axis

(x, y)	→	**$(-y, x)$**	**(x, y)**	→	**$(x, -y)$**
$(-2, -5)$	→	$(5, -2)$	$(5, -2)$	→	$(5, 2)$
$(0, 1)$	→	$(-1, 0)$	$(-1, 0)$	→	$(-1, 0)$

Graph line y' through points at $(5, -2)$ and $(-1, 0)$.
Graph line y'' through points at $(5, 2)$ and $(-1, 0)$.



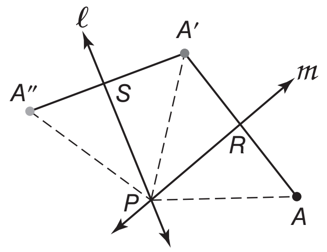
27. $A''(3, 1)$, $B''(2, 3)$, $C''(1, 0)$

29 A translation occurs from Step 1 to Step 2. A rotation 90° counterclockwise occurs from Step 2 to Step 3. So, the transformations are a translation and a 90° rotation.

31. $(x + 5.5, y)$ reflected in the line that separates the left prints from the right prints **33.** double reflection **35.** rotation 180° about the origin and reflection in the x-axis

37. Given: Lines ℓ and m intersect at point P. A is any point not on ℓ or m.
Prove: a. If you reflect point A in line m, and then reflect its image A' in line ℓ, A'' is the image of A after a rotation about point P.

b. $m\angle APA'' = 2(m\angle SPR)$



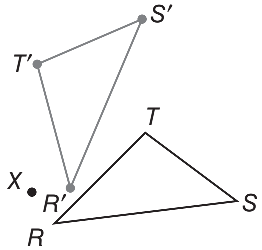
Proof: We are given that line ℓ and line m intersect at point P and that A is not on line ℓ or line m. Reflect A over line m to A' and reflect A' over line ℓ to A''. By the definition of reflection, line m is the perpendicular bisector of $\overline{AA'}$ at R, and line ℓ is the perpendicular bisector of $\overline{A'A''}$ at S. $\overline{AR} \cong \overline{A'R}$ and $\overline{A'S} \cong \overline{A''S}$ by the definition of a perpendicular bisector. Through any two points there is exactly one line, so we can draw auxiliary segments $\overline{AP}$, $\overline{A'P}$, and $\overline{A''P}$. $\angle ARP$, $\angle A'RP$, $\angle A'SP$ and $\angle A''SP$ are right angles by the definition of perpendicular bisectors. $\overline{RP} \cong \overline{RP}$ and $\overline{SP} \cong \overline{SP}$ by the Reflexive Property. $\triangle ARP \cong \triangle A'RP$ and $\triangle A'SP \cong \triangle A''SP$ by the SAS Congruence Postulate. Using CPCTC, $\overline{AP} \cong \overline{A'P}$ and $\overline{A'P} \cong \overline{A''P}$, and $\overline{AP} \cong \overline{A''P}$ by the Transitive Property. By the definition of a rotation, A'' is the image of A after a rotation about point P. Also using CPCTC, $\angle APR \cong \angle A'PR$ and $\angle A'PS \cong \angle A''PS$. By the definition of congruence, $m\angle APR = m\angle A'PR$ and $m\angle A'PS = m\angle A''PS$. $m\angle APR + m\angle A'PR + m\angle A'PS + m\angle A''PS = m\angle APA''$ and $m\angle A'PS + m\angle A'PR = m\angle SPR$ by the Angle Addition Postulate. $m\angle A'PR + m\angle A'PR + m\angle A'PS + m\angle A'PS = m\angle APA''$ by Substitution, which simplifies to $2(m\angle A'PR + m\angle A'PS) = m\angle APA''$. By Substitution, $2(m\angle SPR) = m\angle APA''$.

39. Sample answer: No; there are not invariant points in a glide reflection because all of the points are translated along a vector. Perhaps for compositions of transformations, there may be invariant points when a figure is rotated and reflected, rotated twice, or reflected twice. **41.** Yes; sample answer: If a segment with endpoints (a, b) and (c, d) is to be reflected about the x-axis, the coordinates of the endpoints of the reflected image are $(a, -b)$ and $(c, -d)$. If the segment is then reflected about the line $y = x$, the coordinates of the endpoints of the final image are $(-b, a)$ and $(-d, c)$. If the original image is first reflected about $y = x$, the coordinates of the endpoints of the reflected image are (b, a) and (d, c). If the segment is then reflected about the x-axis, the coordinates of the endpoints of the final image are $(b, -a)$ and $(d, -c)$.

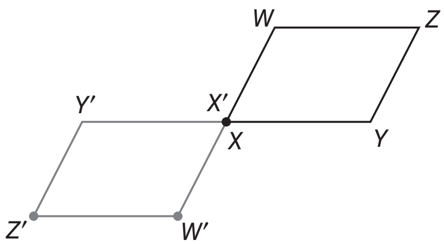
43 Sample answer: When two rotations are performed on a single image, the order of the rotations does not affect the final image when the two rotations are centered at the same point. For example, if $\triangle ABC$ is rotated 45° clockwise about the origin and then rotated 60° clockwise about the origin, $\triangle A''B''C''$ is the same as if the figure were first rotated 60° clockwise about the origin and then rotated 45° clockwise about the origin. If $\triangle ABC$ is rotated 45° clockwise about the origin and then rotated 60° clockwise about $P(2, 3)$, $\triangle A''B''C''$ is different than if the figure were first rotated 60° clockwise about $P(2, 3)$ and then rotated 45° clockwise about the origin. So, the order of the rotations sometimes affects the location of the final image.

45. A **47.** H

49.



51.



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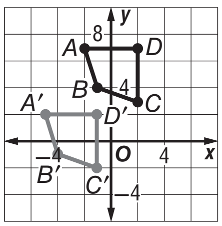
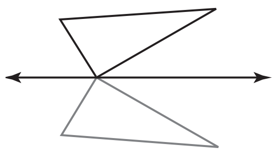


55.



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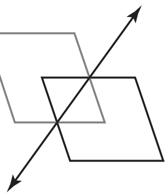
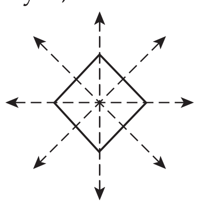
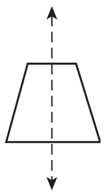


Lesson 9-5

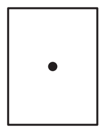
1. yes; 4



3. yes; 1

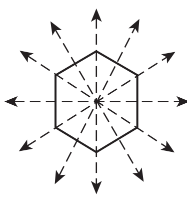


5. yes; 2; 180

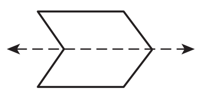


7 **a.** There is no plane in which the dome can be reflected onto itself horizontally. So, there are no horizontal planes of symmetry. The dome has 18 • 2 or 36 vertical planes of symmetry. **b.** The dome has order 36 symmetry and magnitude 360° ÷ 36 or 10°.

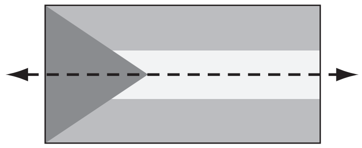
9. no **11.** yes; 6



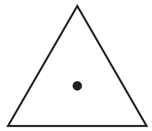
13. yes; 1



15. no **17.** yes; 1

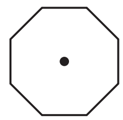


19. yes; 3; 120°



21 No; there is no rotation between 0° and 360° that maps the shape onto itself. So, the figure does not have rotational symmetry.

23. yes; 8; 45°



25. yes; 8; 45° **27.** both **29.** both

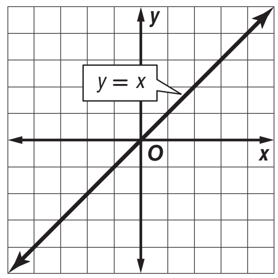
31. no horizontal, infinitely many vertical

33. 1 horizontal, infinitely many vertical

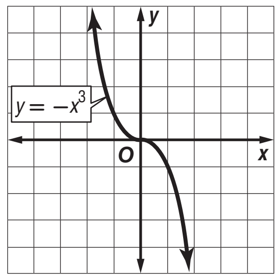
35 Figure *ABCD* is a square. It has 4 lines of symmetry and has order 4 symmetry and magnitude 360° ÷ 4 or 90°. So, it has line symmetry and rotational symmetry.

37. line and rotational

39. rotational; 2; 180°; line symmetry; $y = -x$



41. rotational; 2; 180°



43. plane and axis; 180

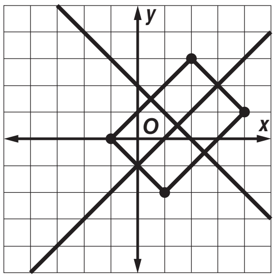
45a. 3 **45b.** 3

45c.

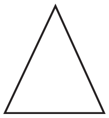
Polygon	Lines of Symmetry	Order of Symmetry
equilateral triangle	3	3
square	4	4
regular pentagon	5	5
regular hexagon	6	6

45d. Sample answer: A regular polygon with n sides has n lines of symmetry and order of symmetry n.

47. Sample answer: (−1, 0), (2, 3), (4, 1), and (1, −2)

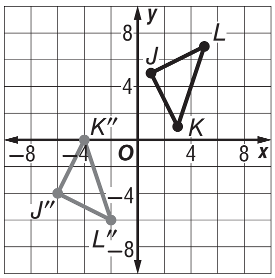


49.



Sample answer: An isosceles triangle has line symmetry from the vertex angle to the base of the triangle, but it does not have rotational symmetry because it cannot be rotated from 0° to 360° and map onto itself. **51.** B **53.** H

55.



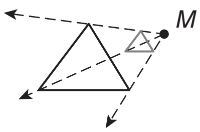
57. (7, −7)

59. reduction; $\frac{1}{2}$

61. enlargement; 3

Lesson 9-6

1.



3 The figure increases in size from B to B', so it is an enlargement.

$$\frac{\text{image length}}{\text{preimage length}} = \frac{QB'}{QB}$$

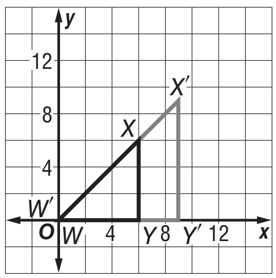
$$= \frac{8}{6} \text{ or } \frac{4}{3}$$

$$QB + BB' = QB'$$

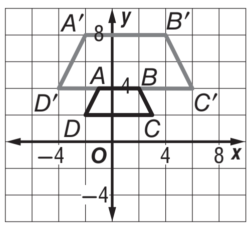
$$6 + x = 8$$

$$x = 2$$

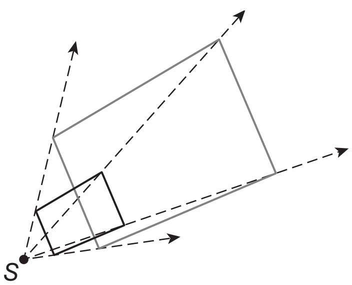
5.



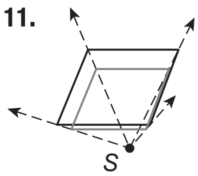
7.



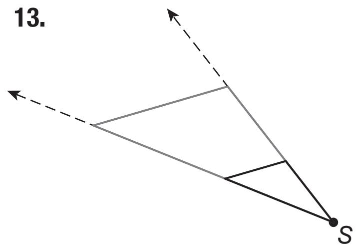
9.



11.



13.



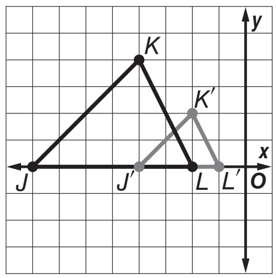
15. enlargement; 2; 4.5 **17.** reduction; $\frac{3}{4}$; 3.5

19. 15×; The insect's image length in millimeters is 3.75 • 10 or 37.5 mm. The scale factor of the dilation is $\frac{37.5}{2.5}$ or 15.

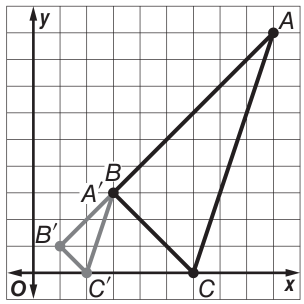
21 Multiply the x- and y-coordinates of each vertex by the scale factor, 0.5.

(x, y)	→	**$(0.5x, 0.5y)$**
$J(-8, 0)$	→	$J'(-4, 0)$
$K(-4, 4)$	→	$K'(-2, 2)$
$L(-2, 0)$	→	$L'(-1, 0)$

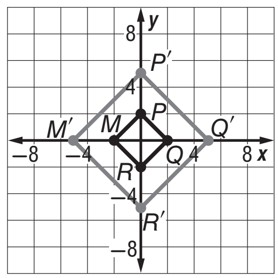
Graph JKL and its image $J'K'L'$.



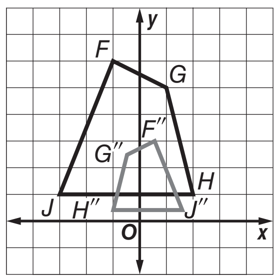
23.



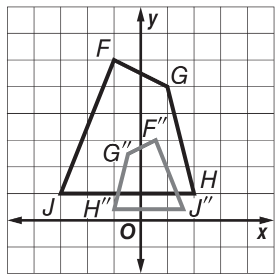
25.



27a.



27b.



27c. no **27d.** Sometimes; sample answer: For the order of a composition of a dilation centered at the origin and a reflection to be unimportant, the line of reflection must contain the origin, or must be of the form $y = mx$.

29. No; sample answer: The measures of the sides of the rectangles are not proportional, so they are not similar and cannot be a dilation.

31 **a.** $T = Ph + 2B$ Surface area of a prism
$= (16)(4) + 2(12)$ $P = 16$ cm, $h = 4$ cm, $B = 12$ cm^2
$= 88$ cm^2 Simplify.

$V = Bh$ Volume of a prism
$= (12)(4)$ $B = 12$ cm^2, $h = 4$ cm
$= 48$ cm^3 Simplify.

b. Multiply the dimensions by the scale factor 2: length $= 6 \cdot 2$ or 12 cm, width $= 2 \cdot 2$ or 4 cm, height $= 4 \cdot 2$ or 8 cm.

$T = Ph + 2B$ Surface area of a prism
$= (32)(8) + 2(48)$ $P = 32$ cm, $h = 8$ cm, $B = 48$ cm^2
$= 352$ cm^2 Simplify.

$V = Bh$ Volume of a prism
$= (48)(8)$ $B = 48$ cm^2, $h = 8$ cm
$= 384$ cm^3 Simplify.

c. Multiply the dimensions by the scale factor $\frac{1}{2}$: length $= 6 \cdot \frac{1}{2}$ or 3 cm, width $= 2 \cdot \frac{1}{2}$ or 1 cm, height $= 4 \cdot \frac{1}{2}$ or 2 cm.

$T = Ph + 2B$ Surface area of a prism
$= (8)(2) + 2(3)$ $P = 8$ cm, $h = 2$ cm, $B = 3$ cm^2
$= 22$ cm^2 Simplify.

$V = Bh$ Volume of a prism
$= (3)(2)$ $B = 3$ cm^2, $h = 2$ cm
$= 6$ cm^3 Simplify.

d. surface area of preimage: 88 cm^2
surface area of image with scale factor 2: 352 cm^2 or $(88 \cdot 4)$ cm^2
surface area of image with scale factor $\frac{1}{2}$: 22 cm^2 or $\left(88 \cdot \frac{1}{4}\right)$ cm^2
The surface area is 4 times greater after dilation with scale factor 2, $\frac{1}{4}$ as great after dilation with scale factor $\frac{1}{2}$.

volume of preimage: 48 cm^3
volume of image with scale factor 2: 384 cm^3 or $(48 \cdot 8)$ cm^3
volume of image with scale factor $\frac{1}{2}$: 6 cm^3 or $\left(48 \cdot \frac{1}{8}\right)$ cm^2
The volume is 8 times greater after dilation with scale factor 2; $\frac{1}{8}$ as great after dilation with scale factor $\frac{1}{2}$.

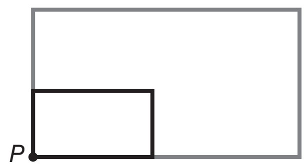
e. The surface area of the preimage would be multiplied by r^2. The volume of the preimage would be multiplied by r^3.

33 **a.** $k = \dfrac{\text{diameter of image}}{\text{diameter of preimage}}$
$= \dfrac{2 \text{ mm}}{1.5 \text{ mm}}$
$= \dfrac{2}{1\frac{1}{2}}$
$= 2 \cdot \dfrac{2}{3}$
$= \dfrac{4}{3}$ or $1\frac{1}{3}$

b. $A = \pi r^2$ Area of a circle
$= \pi(0.75)^2$ $r = 1.5 \div 2$ or 0.75
≈ 1.77 mm^2 Use a calculator.

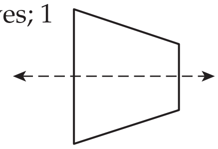
$A = \pi r^2$ Area of a circle
$= \pi(1)^2$ $r = 2 \div 2$ or 1
≈ 3.14 mm^2 Use a calculator.

35. $\frac{11}{5}$



37. $y = 4x - 3$

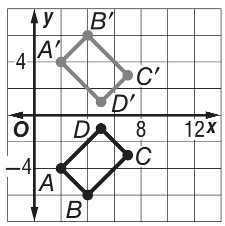
39a. Always; sample answer: Since a dilation of 1 maps an image onto itself, all four vertices will remain invariant under the dilation. **39b.** Always; sample answer: Since the rotation is centered at B, point B will always remain invariant under the rotation. **39c.** Sometimes: sample answer: If one of the vertices is on the x-axis, then that point will remain invariant under reflection. If two vertices are on the x-axis, then the two vertices located on the x-axis will remain invariant under reflection. **39d.** Never; when a figure is translated, all points move an equal distance. Therefore, no points can remain invariant under translation. **39e.** Sometimes; sample answer: If one of the vertices of the triangle is located at the origin, then that vertex would remain invariant under the dilation. If none of the points on $\triangle XYZ$ are located at the origin, then no points will remain invariant under the dilation.
41. Sample answer: Translations, reflections, and rotations produce congruent figures because the sides and angles of the preimage are congruent to the corresponding sides and angles of the image. Dilations produce similar figures, because the angles of the preimage and the image are congruent and the sides of the preimage are proportional to the corresponding sides of the image. A dilation with a scale factor of 1 produces an equal figure because the image is mapped onto its corresponding parts in the preimage.
43. A **45.** D
47. yes; 1



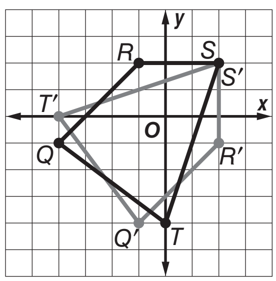
49. translation along $\langle -1, 8 \rangle$ and reflection in the y-axis
51. $2\sqrt{51}$ ft ≈ 14.3 ft
53. 34.6 **55.** 107.1

Chapter 9 Study Guide and Review

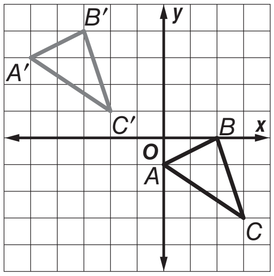
1. composition of transformations **3.** dilation
5. line of reflection **7.** translation **9.** reflection
11.



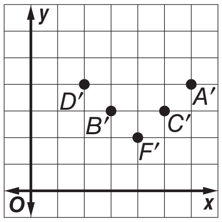
13.



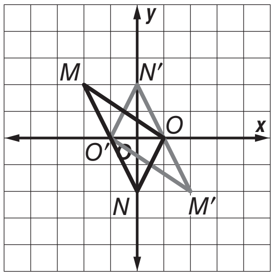
15.



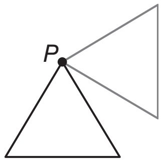
17.



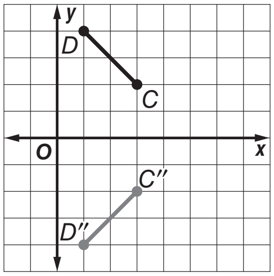
19.



21. 90°

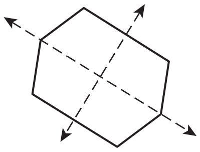


23.

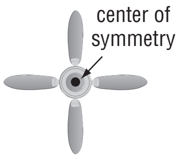


25. Sample answer: translation right and down, translation of result right and up.

27. yes; 2



29. yes; 4; 90°



31. 4 **33.** reduction; 8.25; 0.45

CHAPTER 10
Circles

Chapter 10 Get Ready

1. 130 **3.** 15.58 **5.** 82.8 **7.** \$5.85 **9.** 8.5 ft **11.** −3, 4

Lesson 10-1

1. ⊙N **3.** 8 cm **5.** 14 in. **7.** 22 ft; 138.23 ft
9. $4\pi\sqrt{13}$ cm **11.** $\overline{SU}$ **13.** 8.1 cm

15 $d = 2r$ Diameter formula
$= 2(14)$ or 28 in. Substitute and simplify.

17. 3.7 cm **19.** 14.6 **21.** 30.6 **23.** 13 in.; 81.68 in.
25. 39.47 ft; 19.74 ft **27.** 830.23 m; 415.12 m

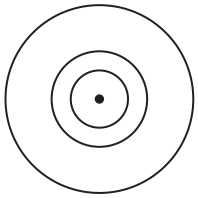
29 $a^2 + b^2 = c^2$ Pythagorean Theorem
$(6\sqrt{2})^2 + (6\sqrt{2})^2 = c^2$ Substitution
$144 = c^2$ Simplify.
$12 = c$ Take the positive square root of each side.

The diameter is 12π feet.
$C = \pi d$ Circumference formula
$= \pi(12)$ Substitution
$= 12\pi$ ft Simplify.

31. 10π in. **33.** 14π yd **35a.** 31.42 ft **35b.** 4 ft
37. 22.80 ft; 71.63 ft **39.** $0.25x$; $0.79x$ **41.** neither

43 The radius is 3 units, or 3 • 25 = 75 feet.
$C = 2\pi r$ Circumference formula
$= 2\pi(75)$ Substitution
$= 150\pi$ Simplify.
≈ 471.2 ft Use a calculator.

45a. Sample answer:



45b.

Circle Radius (cm)	Circumference (cm)
0.5	3.14
1	6.28
2	12.57

45c. They all have the same shape—circular.
45d. The ratio of their circumferences is also 2.
45e. $(C_B) = \frac{b}{a}(C_A)$ **45f.** 4 in.

47 **a.** $C = 2\pi r$ Circumference formula
$= 2\pi(30)$ Substitution
$= 60\pi$ Simplify.

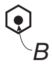
$C = 2\pi r$ Circumference formula
$= 2\pi(5)$ Substitution
$= 10\pi$ Simplify.

$60\pi - 10\pi = 50\pi \approx 157.1$ mi

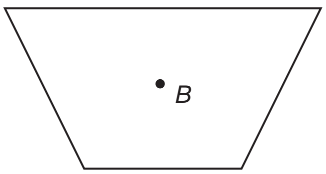
b. If $r = 5$, $C = 10\pi$; if $r = 10$, $C = 20\pi$; if $r = 15$, $C = 30\pi$, and so on. So, as r increases by 5, C increases by 10π or by about 31.4 miles.

49a. $8r$ and $6r$; Twice the radius of the circle, $2r$ is the side length of the square, so the perimeter of the square is $4(2r)$ or $8r$. The regular hexagon is made up of six equilateral triangles with side length r, so the perimeter of the hexagon is $6(r)$ or $6r$. **49b.** less; greater; $6r < C < 8r$ **49c.** $3d < C < 4d$; The circumference of the circle is between 3 and 4 times its diameter. **49d.** These limits will approach a value of πd, implying that $C = \pi d$.
51. Always; a radius is a segment drawn between the center of the circle and a point on the circle. A segment drawn from the center to a point inside the circle will always have a length less than the radius of the circle.

53. $\frac{8\pi}{\sqrt{3}}$ or $\frac{8\pi\sqrt{3}}{3}$ **55.** 40.8 **57.** J **59.**



61.



63. no **65.** no

67. True; sample answer: Since the hypothesis is true and the conclusion is true, then the statement is true for the conditions. **69.** 90 **71.** 20

Lesson 10-2

1. 170 **3.** major arc; 270 **5.** semicircle; 180 **7.** 147
9. 123 **11.** 13.74 cm

13 $65 + 70 + x = 360$ Sum of central angles
$135 + x = 360$ Simplify.
$x = 225$ Subtract 135 from each side.

15. 40 **17.** minor arc; 125 **19.** major arc; 305 **21.** semicircle; 180 **23.** major arc; 270 **25a.** 280.8; 36 **25b.** major arc; minor arc **25c.** No; no categories share the same percentage of the circle. **27.** 60 **29.** 300 **31.** 180 **33.** 220 **35.** 120

37 $\ell = \frac{x}{360} \cdot 2\pi r$ Arc length equation

$= \frac{112}{360} \cdot 2\pi(4.5)$ Substitution.

≈ 8.80 cm Use a calculator.

39. 17.02 in. **41.** 12.04 m **43.** The length of the arc would double. **45.** 40.84 in. **47.** 9.50 ft **49.** 142

51 **a.** $m\widehat{AB} = m\angle ACB$ $\widehat{AB}$ is a minor arc.

$= 180 - (22 + 22)$ Angle Addition Postulate

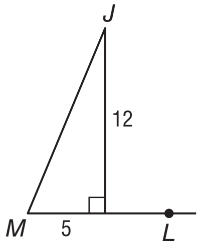
$= 180 - 44$ or 136 Simplify.

b. $\ell = \frac{x}{360} \cdot 2\pi r$ Arc length equation

$= \frac{136}{360} \cdot 2\pi(62)$ Substitution.

≈ 147.17 ft Use a calculator.

53 **a.**



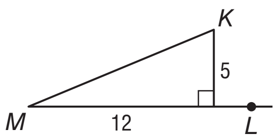
$\tan \angle JML = \frac{12}{5}$

$m\angle JML = \tan^{-1}\left(\frac{12}{5}\right)$

$\approx 67.4°$

$m\widehat{JL} = m\angle JML \approx 67.4°$

b.



$\tan \angle KML = \frac{5}{12}$

$m\angle KML = \tan^{-1}\left(\frac{5}{12}\right)$

$\approx 22.6°$

$m\widehat{KL} = m\angle KML \approx 22.6°$

c. $m\angle JMK = m\angle JML - m\angle KML$

$\approx 67.4 - 22.6$

$\approx 44.8°$

$m\widehat{JK} = m\angle JMK \approx 44.8°$

d. $r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

$= \sqrt{(5 - 0)^2 + (12 - 0)^2}$ $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (5, 12)$

$= 13$ Simplify.

$\ell = \frac{x}{360} \cdot 2\pi r$ Arc length equation

$= \frac{67.4}{360} \cdot 2\pi(13)$ Substitution

≈ 15.29 units Use a calculator.

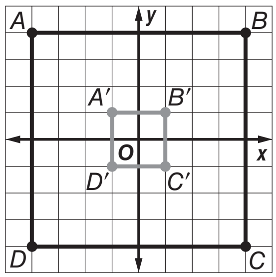
e. $\ell = \frac{x}{360} \cdot 2\pi r$ Arc length equation

$= \frac{44.8}{360} \cdot 2\pi(13)$ Substitution

≈ 10.16 units Use a calculator.

55. Selena; the circles are not congruent because they do not have congruent radii. So, the arcs are not congruent. **57.** Never; obtuse angles intersect arcs between 90° and 180° **59.** $m\widehat{LM} = 150$, $m\widehat{MN} = 90$, $m\widehat{NL} = 120$ **61.** 175 **63.** B **65.** H **67.** J **69.** 6.2

71.



73. $x = \frac{50}{3}$; $y = 10$; $z = \frac{40}{3}$ **75.** 10, −10 **77.** 46.1, −46.1

Lesson 10-3

1 $\widehat{ST}$ is a minor arc, so $m\widehat{ST} = 93$. $\overline{RS}$ and $\overline{ST}$ are congruent chords, so the corresponding arcs $\widehat{RS}$ and $\widehat{ST}$ are congruent.

$\widehat{RS} \cong \widehat{ST}$ Corresponding arcs are congruent.

$m\widehat{RS} = m\widehat{ST}$ Definition of congruent arcs

$x = 93$ Substitution

3. 3 **5.** 3.32 **7.** 21 **9.** 127 **11.** 7

13 $\overline{KL}$ and $\overline{AJ}$ are congruent chords in congruent circles, so the corresponding arcs $\widehat{KL}$ and $\widehat{AJ}$ are congruent.

$\widehat{KL} \cong \widehat{AJ}$ Corresponding arcs are congruent.

$m\widehat{KL} = m\widehat{AJ}$ Definition of congruent arcs

$5x = 3x + 54$ Substitution

$2x = 54$ Subtract $3x$ from each side.

$x = 27$ Divide each side by 2.

15. 122.5° **17.** 5.34 **19.** 6.71

21 $DE + EC = DC$ Segment Addition Postulate

$15 + EC = 88$ Substitution

$EC = 73$ Subtract 15 from each side.

$EC^2 + EB^2 = CB^2$ Pythagorean Theorem

$73^2 + EB^2 = 88^2$ Substitution

$EB^2 = 2415$ Subtract 73^2 from each side.

$EB \approx 49.14$ the positive square root of each side.

$EB = \frac{1}{2}AB$ $\overline{DC} \perp \overline{AB}$, so $\overline{DC}$ bisects $\overline{AB}$.

$2EB = AB$ Multiply each side by 2.

$2(49.14) \approx AB$ Substitution

$98.3 \approx AB$ Simplify.

23. 4

25. Proof:

Because all radii are congruent, $\overline{QP} \cong \overline{PR} \cong \overline{SP} \cong \overline{PT}$. You are given that $\overline{QR} \cong \overline{ST}$, so $\triangle PQR \cong \triangle PST$ by SSS. Thus, $\angle QPR \cong \angle SPT$ by CPCTC. Since the central angles have the same measure, their intercepted arcs have the same measure and are therefore congruent.

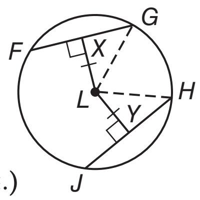
Thus, $\widehat{QR} \cong \widehat{ST}$.

27. Each arc is 90°, and each chord is 2.12 ft.

29. Given: $\odot L$, $\overline{LX} \perp \overline{FG}$, $\overline{LY} \perp \overline{JH}$, $\overline{LX} \cong \overline{LY}$

Prove: $\overline{FG} \cong \overline{JH}$

Proof:



Statements (Reasons)

1. $\overline{LG} \cong \overline{LH}$ (All radii of a $\odot$ are $\cong$.)
2. $\overline{LX} \perp \overline{FG}$, $\overline{LY} \perp \overline{JH}$, $\overline{LX} \cong \overline{LY}$ (Given)
3. $\angle LXG$ and $\angle LYH$ are right ∠s. (Def. of ⊥ lines)
4. $\triangle XGL \cong \triangle YHL$ (HL)
5. $\overline{XG} \cong \overline{YH}$ (CPCTC)

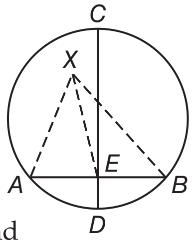
6. $XG = YH$ (Def. of $\cong$ segments)
7. $2(XG) = 2(YH)$ (Multiplication Property)
8. $\overline{LX}$ bisects $\overline{FG}$; $\overline{LY}$ bisects $\overline{JH}$. (A radius $\perp$ to a chord bisects the chord.)
9. $FG = 2(XG)$, $JH = 2(YH)$ (Def. of seg. bisector)
10. $FG = JH$ (Substitution)
11. $\overline{FG} \cong \overline{JH}$ (Def. of $\cong$ segments)

31 Since $\overline{AB} \perp \overline{CE}$ and $\overline{DF} \perp \overline{CE}$, $\overline{CE}$ bisects $\overline{AB}$ and $\overline{DF}$.
Since $\overline{AB} \cong \overline{DF}$, $AB = DF$.

$CB = \frac{1}{2}AB$	Definition of bisector
$CB = \frac{1}{2}DF$	Substitution
$CB = DE$	Definition of bisector
$9x = 2x + 14$	Substitution
$7x = 14$	Subtract $2x$ from each side.
$x = 2$	Divide each side by 7.

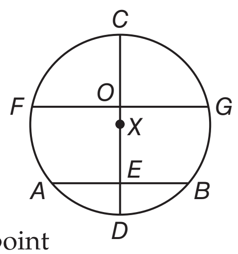
33. 5 **35.** About 17.3; P and Q are equidistant from the endpoints of $\overline{AB}$ so they both lie on the perpendicular bisector of $\overline{AB}$, so $\overline{PQ}$ is the perpendicular bisector of $\overline{AB}$. Hence, both segments of $\overline{AB}$ are 5. Since $\overline{PS}$ is perpendicular to chord $\overline{AB}$, $\angle PSA$ is a right angle. So, $\triangle PSA$ is a right triangle. By the Pythagorean Theorem, $PS = \sqrt{(PA)^2 - (AS)^2}$. By substitution, $PS = \sqrt{11^2 - 5^2}$ or $\sqrt{96}$. Similarly, $\triangle ASQ$ is a right triangle with $SQ = \sqrt{(AQ)^2 - (AS)^2} = \sqrt{9^2 - 5^2}$ or $\sqrt{56}$. Since $PQ = PS + SQ$, $PQ = \sqrt{96} + \sqrt{56}$ or about 17.3.

37a. Given: $\overline{CD}$ is the perpendicular bisector of chord $\overline{AB}$ in $\odot X$.
Prove: $\overline{CD}$ contains point X.
Proof:



Suppose X is not on $\overline{CD}$. Draw $\overline{XE}$ and radii $\overline{XA}$ and $\overline{XB}$. Since $\overline{CD}$ is the perpendicular bisector of $\overline{AB}$, E is the midpoint of $\overline{AB}$ and $\overline{AE} \cong \overline{EB}$. Also, $\overline{XA} \cong \overline{XB}$, since all radii of a $\odot$ are $\cong$. $\overline{XE} \cong \overline{XE}$ by the Reflexive Property. So, $\triangle AXE \cong \triangle BXE$ by SSS. By CPCTC, $\angle XEA \cong \angle XEB$. Since they also form a linear pair $\angle XEA$ and $\angle XEB$ are right angles. So, $\overline{XE} \perp \overline{AB}$. By definition $\overline{XE}$ is the perpendicular bisector of $\overline{AB}$. But $\overline{CD}$ is also the perpendicular bisector of $\overline{AB}$. This contradicts the uniqueness of a perpendicular bisector of a segment. Thus, the assumption is false, and center X must be on $\overline{CD}$.

37b. Given: In $\odot X$, X is on $\overline{CD}$ and $\overline{FG}$ bisects $\overline{CD}$ at O.
Prove: Point O is point X.
Proof:

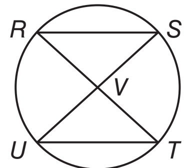


Since point X is on $\overline{CD}$ and C and D are on $\odot X$, $\overline{CD}$ is a diameter of $\odot X$. Since $\overline{FG}$ bisects $\overline{CD}$ at O, O is the midpoint of $\overline{CD}$. Since the midpoint of a diameter is the center of a circle, O, is the center of the circle. Therefore, point O is point X.

39. No; sample answer: In a circle with a radius of 12, an arc with a measure of 60 determines a chord of length 12. (the triangle related to a central angle of 60 is equilateral.) If the measure of the arc is tripled to 180, then the chord determined by the arc is a diameter and has a lenth of 2(12) or 24, which is not three times as long as the original chord.
41. F **43.** E **45.** 170 **47.** 275 in.
49. yes; obtuse
$31^2 \stackrel{?}{=} 20^2 + 21^2$
$961 > 400 + 441$
51. ± 11

Lesson 10-4

1. 30 **3.** 66 **5.** 54
7. Given: $\overline{RT}$ bisects $\overline{SU}$.
Prove: $\triangle RVS \cong \triangle UVT$
Proof:



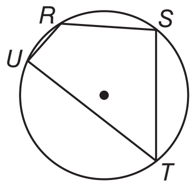
__Statements (Reasons)__
1. $\overline{RT}$ bisects $\overline{SU}$. (Given)
2. $\overline{SV} \cong \overline{VU}$ (Def. of segment bisector)
3. $\angle SRT$ intercepts $\widehat{ST}$. $\angle SUT$ intercepts $\widehat{ST}$. (Def. of intercepted arc)
4. $\angle SRT \cong \angle SUT$ (Inscribed $\measuredangle$ of same arc are $\cong$.)
5. $\angle RVS \cong \angle UVT$ (Vertical $\measuredangle$ are $\cong$.)
6. $\triangle RVS \cong \triangle UVT$ (AAS)

9. 25 **11.** 162

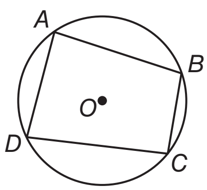
13

$m\widehat{NP} + m\widehat{PQ} + m\widehat{QN} = 360$	Addition Theorem
$120 + 100 + m\widehat{QN} = 360$	Substitution
$220 + m\widehat{QN} = 360$	Simplify.
$m\angle\widehat{QN} = 140$	Subtract 220 from each side.
$m\angle P = \frac{1}{2}m\widehat{QN}$	$\angle P$ intercepts $\widehat{QN}$.
$= \frac{1}{2}(140)$ or 70	Substitution

15. 140 **17.** 32 **19.** 20
21. Given: $m\angle T = \frac{1}{2}m\angle S$
Prove: $m\widehat{TUR} = 2m\widehat{URS}$
Proof:



$m\angle T = \frac{1}{2}m\angle S$ means that $m\angle S = 2m\angle T$. Since $m\angle S = \frac{1}{2}m\widehat{TUR}$ and $m\angle T = \frac{1}{2}m\widehat{URS}$, the equation becomes $\frac{1}{2}m\widehat{TUR} = 2\left(\frac{1}{2}m\widehat{URS}\right)$. Multiplying each side of the equation by 2 results in $m\widehat{TUR} = 2m\widehat{URS}$.
23. 30 **25.** 12.75 **27.** 135 **29.** 106
31. Given: Quadrilateral $ABCD$ is inscribed in $\odot O$.
Prove: $\angle A$ and $\angle C$ are supplementary. $\angle B$ and $\angle D$ are supplementary.



Proof: By arc addition and the definitions of arc measure and the sum of central angles, $m\widehat{DCB} + m\widehat{DAB} = 360$. Since $m\angle C = \frac{1}{2}m\widehat{DAB}$ and $m\angle A = \frac{1}{2}m\widehat{DCB}$, $m\angle C + m\angle A = \frac{1}{2}\big(m\widehat{DCB} +$

$m\widehat{DAB}\big)$, but $m\widehat{DCB} + m\widehat{DAB} = 360$, so $m\angle C + m\angle A = \frac{1}{2}(360)$ or 180. This makes $\angle C$ and $\angle A$ supplementary. Because the sum of the measures of the interior angles of a quadrilateral is 360, $m\angle A + m\angle C + m\angle B + m\angle D = 360$. But $m\angle A + m\angle C = 180$, so $m\angle B + m\angle D = 180$, making them supplementary also.

33 Since all the sides of the sign are congruent, all the corresponding arcs are congruent.

$8m\widehat{QR} = 360$, so $m\widehat{QR} = \frac{360}{8}$ or 45.

$m\angle RLQ = \frac{1}{2}m\widehat{QR}$

$= \frac{1}{2}(45)$ or 22.5

35. 135

37. Proof:

<u>Statements (Reasons)</u>

1. $m\angle ABC = m\angle ABD + m\angle DBC$ ($\angle$ Addition Postulate)

2. $m\angle ABD = \frac{1}{2}m\widehat{AD}$

$m\angle DBC = \frac{1}{2}m\widehat{DC}$ (The measure of an inscribed $\angle$ whose side is a diameter is half the measure of the intercepted arc (Case 1).)

3. $m\angle ABC = \frac{1}{2}m\widehat{AD} + \frac{1}{2}m\widehat{DC}$ (Substitution)

4. $m\angle ABC = \frac{1}{2}\big(m\widehat{AD} + m\widehat{DC}\big)$ (Factor)

5. $m\widehat{AD} + m\widehat{DC} = m\widehat{AC}$ (Arc Addition Postulate)

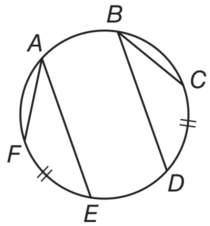
6. $m\angle ABC = \frac{1}{2}m\widehat{AC}$ (Substitution)

39 Use the Inscribed Angle Theorem to find the measures of $\angle FAE$ and $\angle CBD$. Then use the definition of congruent arcs and the Multiplication Property of Equality to help prove that the angles are congruent.

Given: $\angle FAE$ and $\angle CBD$ are inscribed; $\widehat{EF} \cong \widehat{DC}$

Prove: $\angle FAE = \angle CBD$

Proof:



<u>Statements (Reasons)</u>

1. $\angle FAE$ and $\angle CBD$ are inscribed; $\widehat{EF} \cong \widehat{DC}$ (Given)

2. $m\angle FAE = \frac{1}{2}m\widehat{EF}$; $m\angle CBD = \frac{1}{2}m\widehat{DC}$ (Measure of an inscribed $\angle$ = half measure of intercepted arc.)

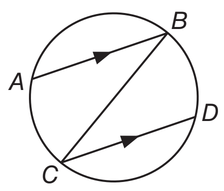
3. $m\widehat{EF} = m\widehat{DC}$ (Def. of $\cong$ arcs)

4. $\frac{1}{2}m\widehat{EF} = \frac{1}{2}m\widehat{DC}$ (Mult. Prop.)

5. $m\angle FAE = m\angle CBD$ (Substitution)

6. $\angle FAE \cong \angle CBD$ (Def. of $\cong$ ∡)

41a.



41b. Sample answer: $m\angle A = 30$, $m\angle D = 30$; $m\widehat{AC} = 60$, $m\widehat{BD} = 60$; The arcs are congruent because they have equal measures. **41c.** Sample answer: In a circle, two parallel chords cut congruent arcs.

41d. 70; 70 **43.** Always; rectangles have right angles at each vertex, therefore opposite angles will be inscribed in a semicircle. **45.** Sometimes; a rhombus can be inscribed in a circle as long as it is a square. Since the opposite angles of rhombi that are not squares are not supplementary, they can not be inscribed in a circle. **47.** $\frac{\pi}{2}$ **51.** A **53.** $d = 17$ in., $r = 8.5$ in., $C = 17\pi$ or about 53.4 in. **55.** 48 **57.** 24 **59.** 107 **61.** 144 **63.** 54 **65.** $\frac{1}{2}$

Lesson 10-5

1. no common tangent

3 $FG^2 + GE^2 \stackrel{?}{=} FE^2$

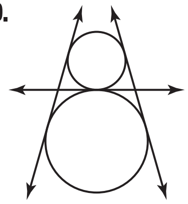
$36^2 + 15^2 \stackrel{?}{=} (24 + 15)^2$

$1521 = 1521$

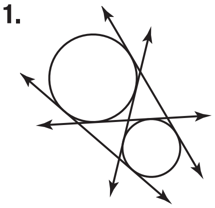
$\triangle EFG$ is a right triangle with right angle EGF. So $\overline{FG}$ is perpendicular to radius $\overline{EG}$ at point G. Therefore, by Theorem 10.10, $\overline{FG}$ is tangent to $\odot E$.

5. 16 **7.** $x = 250$; $y = 275$; 1550 ft

9.



11.



13. yes; 625 = 625

15 $XY^2 + YZ^2 \stackrel{?}{=} XZ^2$

$8^2 + 5^2 \stackrel{?}{=} (3 + 5)^2$

$89 \neq 64$

Since $\triangle XYZ$ is not a right triangle, $\overline{XY}$ is not perpendicular to radius $\overline{YZ}$. So, $\overline{XY}$ is not tangent to $\odot Z$.

17 $\overrightarrow{QP}$ is tangent to $\odot N$ at P. So, $\overrightarrow{QP} \perp \overline{PN}$ and $\triangle PQN$ is a right triangle.

$QP^2 + PN^2 = QN^2$	Pythagorean Theorem
$24^2 + 10^2 = x^2$	$QP = 24$, $PN = 10$, and $QN = x$
$576 + 100 = x^2$	Multiply.
$676 = x^2$	Simplify.
$26 = x$	Take the positive square root of each side.

19. 9 **21.** 4 **23a.** 37.95 in. **23b.** 37.95 in. **25.** 8; 52 cm **27.** 8.06

29. Given: Quadrilateral $ABCD$ is circumscribed about $\odot P$.

Prove: $AB + CD = AD + BC$

<u>Statements (Reasons)</u>

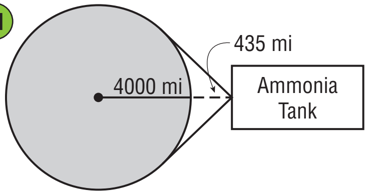
1. Quadrilateral $ABCD$ is circumscribed about $\odot P$. (Given)

2. Sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$ are tangent to $\odot P$ at points H, G, F, and E, respectively. (Def. of circumscribed)

3. $\overline{EA} \cong \overline{AH}$; $\overline{HB} \cong \overline{BG}$; $\overline{GC} \cong \overline{CF}$; $\overline{FD} \cong \overline{DE}$ (Two segments tangent to a circle from the same exterior point are $\cong$.)

4. $AB = AH + HB$, $BC = BG + GC$, $CD = CF + FD$, $DA = DE + EA$ (Segment Addition)
5. $AB + CD = AH + HB + CF + FD$; $DA + BC = DE + EA + BG + GC$ (Substitution)
6. $AB + CD = AH + BG + GC + FD$; $DA + BC = FD + AH + BG + GC$ (Substitution)
7. $AB + CD = FD + AH + BG + GC$ (Commutative Prop. of Add.)
8. $AB + CD = DA + BC$ (Substitution)

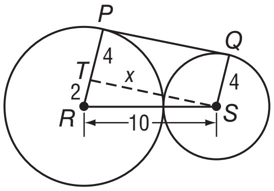
31



$4000^2 + x^2 = (4000 + 435)^2$	Pythagorean Theorem
$x^2 = 3{,}669{,}225$	Subtract 4000^2 from each side.
$x \approx 1916$ mi	Take the positive square root of each side.

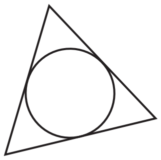
33. Proof: Assume that ℓ is not tangent to $\odot S$. Since ℓ intersects $\odot S$ at T, it must intersect the circle in another place. Call this point Q. Then $ST = SQ$. $\triangle STQ$ is isosceles, so $\angle T \cong \angle Q$. Since $\overline{ST} \perp \ell$, $\angle T$ and $\angle Q$ are right angles. This contradicts that a triangle can only have one right angle.

35. Sample answer: Using the Pythagorean Theorem, $2^2 + x^2 = 10^2$, so $x \approx 9.8$. Since $PQST$ is a rectangle, $PQ = x = 9.8$.



37. Sample answer:

circumscribed



inscribed

39. No; sample answer: From a point outside the circle, two tangents can be drawn. From a point on the circle, one tangent can be drawn. From a point inside the circle, no tangents can be drawn because a line would intersect the circle in two points.

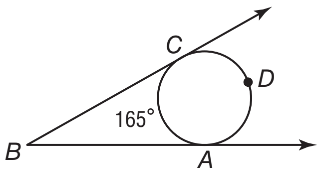
41. $6\sqrt{2}$ or about 8.5 in.

43. D **45.** 61 **47.** 71 **49.** 109 **51.** Yes; $\triangle AEC \sim \triangle BDC$ by AA Similarity. **53.** 110 **55.** 58

Lesson 10-6

1. 110 **3.** 73 **5.** 248

7 Draw and label a diagram.



$m\angle B = \frac{1}{2}(m\widehat{CDA} - m\widehat{CA})$	Theorem 10.14
$= \frac{1}{2}[(360 - 165) - 165]$	Substitution
$= \frac{1}{2}(195 - 165)$ or 15	Simplify.

9. 71.5

11

$51 = \frac{1}{2}(m\widehat{RQ} + m\widehat{NP})$	Theorem 10.12
$51 = \frac{1}{2}(m\widehat{RQ} + 74)$	Substitution
$102 = m\widehat{RQ} + 74$	Multiply each side by 2.
$28 = m\widehat{RQ}$	Subtract 74 from each side.

13. 144 **15.** 125 **17a.** 100 **17b.** 20 **19.** 74 **21.** 185 **23.** 22 **25.** 168

27

$3 = \frac{1}{2}[(5x - 6) - (4x + 8)]$	Theorem 10.14
$6 = (5x - 6) - (4x + 8)$	Multiply each side by 2.
$6 = x - 14$	Simplify.
$20 = x$	Add 14 to each side.

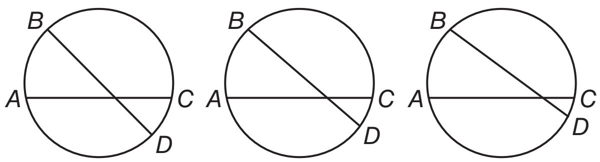
29a. 145 **29b.** 30

31. <u>Statements (Reasons)</u>

1. $\overrightarrow{FM}$ is a tangent to the circle and $\overrightarrow{FL}$ is a secant to the circle. (Given)
2. $m\angle FLH = \frac{1}{2}m\widehat{HG}$, $m\angle LHM = \frac{1}{2}m\widehat{LH}$ (The meas. of an inscribed $\angle = \frac{1}{2}$ the measure of its intercepted arc.)
3. $m\angle LHM = m\angle FLH + m\angle F$ (Exterior $\angle$s Th.)
4. $\frac{1}{2}m\widehat{LH} = \frac{1}{2}m\widehat{HG} + m\angle F$ (Substitution)
5. $\frac{1}{2}m\widehat{LH} - \frac{1}{2}m\widehat{HG} = m\angle F$ (Subtraction Prop.)
6. $\frac{1}{2}(m\widehat{LH} - m\widehat{HG}) = m\angle F$ (Distributive Prop.)

33a. Proof: By Theorem 10.10, $\overline{OA} \perp \overline{AB}$. So, $\angle FAE$ is a right $\angle$ with measure 90, and $\widehat{FCA}$ is a semicircle with measure of 180. Since $\angle CAE$ is acute, C is in the interior of $\angle FAE$. By the Angle and Arc Addition Postulates, $m\angle FAE = m\angle FAC + m\angle CAE$ and $m\widehat{FCA} = m\widehat{FC} + m\widehat{CA}$. By substitution, $90 = m\angle FAC + m\angle CAE$ and $180 = m\widehat{FC} + m\widehat{CA}$. So, $90 = \frac{1}{2}m\widehat{FC} + \frac{1}{2}m\widehat{CA}$ by Division Prop., and $m\angle FAC + m\angle CAE = \frac{1}{2}m\widehat{FC} + \frac{1}{2}m\widehat{CA}$ by substitution. $m\angle FAC = \frac{1}{2}m\widehat{FC}$ since $\angle FAC$ is inscribed, so substitution yields $\frac{1}{2}m\widehat{FC} + m\angle CAE = \frac{1}{2}m\widehat{FC} + \frac{1}{2}m\widehat{CA}$. By Subt. Prop., $m\angle CAE = \frac{1}{2}m\widehat{CA}$. **33b.** Use same reasoning to prove $m\angle CAB = \frac{1}{2}m\widehat{CDA}$

35 **a.** Sample answer:



b. Sample answer:

	Circle 1	Circle 2	Circle 3
$\widehat{CD}$	25	15	5
$\widehat{AB}$	50	50	50
x	37.5	32.5	27.5

c. As the measure of $\widehat{CD}$ gets closer to 0, the

measure of x approaches half of $m\widehat{AB}$; $\angle AEB$ becomes an inscribed angle.
d. Theorem 10.12 states that if two chords intersect in the interior of a circle, then the measure of an angle formed is one half the sum of the measure of the arcs intercepted by the angle and its vertical angle. Use this theorem to write an equation relating x, $m\widehat{AB}$, and $m\widehat{CD}$. Then let $m\widehat{CD} = 0$ and simplify. The result is Theorem 10.6, the Inscribed Angle Theorem.

$x = \frac{1}{2}(m\widehat{AB} + m\widehat{CD})$

$x = \frac{1}{2}(m\widehat{AB} + 0)$

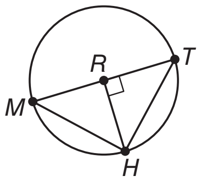
$x = \frac{1}{2}m\widehat{AB}$

37. 15

39a. $m\angle G \leq 90$; $m\angle G < 90$ for all values except when $\overrightarrow{JG} \perp \overrightarrow{GH}$ at G, then $m\angle G = 90$. **39b.** $m\widehat{KH} = 56$; $m\widehat{HJ} = 124$; Because a diameter is involved the intercepted arcs measure $(180 - x)$ and x degrees. Hence, solving $\frac{180 - x - x}{2} = 34$ leads to the answer.

41. Sample answer: Using Theorem 10.14, $60° = \frac{1}{2}[(360° - x) - x]$ or $120°$; repeat for $50°$ to get $130°$. The third arc can be found by adding $50°$ and $60°$ and subtracting from $360°$ to get $110°$. **43.** J **45.** B **47.** 8

49. Given: $\widehat{MHT}$ is a semicircle.
$\overline{RH} \perp \overline{TM}$.
Prove: $\frac{TR}{RH} = \frac{TH}{HM}$



Proof:
Statements (Reasons)
1. $\widehat{MHT}$ is a semicircle; $\overline{RH} \perp \overline{TM}$. (Given)
2. $\angle THM$ is a right angle. (If an inscribed $\angle$ intercepts a semicircle, the $\angle$ is a rt. $\angle$.)
3. $\angle TRH$ is a right angle. (Def. of $\perp$ lines)
4. $\angle THM \cong \angle TRH$ (All rt. angles are $\cong$.)
5. $\angle T \cong \angle T$ (Reflexive Prop.)
6. $\triangle TRH \sim \triangle THM$ (AA Sim.)
7. $\frac{TR}{RH} = \frac{TH}{HM}$ (Def. of $\sim \triangle$s)

51. 54.5° **53.** −4, −9 **55.** 0, −5 **57.** −6

Lesson 10-7

1. 2 **3.** 5

5 Let T be the endpoint of $\overline{QT}$, the diameter that passes through point S.

$PS \cdot SR = QS \cdot ST$	Theorem 10.15
$10 \cdot 10 = 6 \cdot ST$	Substitution
$100 = 6ST$	Simplify.
$\frac{50}{3} = ST$	Divide each side by 6.

So, the diameter of the circle is $6 + \frac{50}{3}$ or $\frac{68}{3}$ centimeters.

$C = \pi d$	Circumference formula
$= \pi\left(\frac{68}{3}\right)$	Substitution
≈ 71.21 cm	Use a calculator.

7. 5 **9.** 14 **11.** 3.1

13

$CD^2 = CB \cdot CA$	Theorem 10.17
$12^2 = x \cdot (x + 12)$	Substitution
$144 = x^2 + 12x$	Simplify.
$0 = x^2 + 12x - 144$	144 from each side.
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Quadratic Formula
$= \frac{-12 \pm \sqrt{12^2 - 4(1)(-144)}}{2(1)}$	$a = 1$, $b = 12$, $c = -144$
$= \frac{-12 \pm \sqrt{720}}{2}$	Disregard the negative solution.
≈ 7.4	Use a calculator.

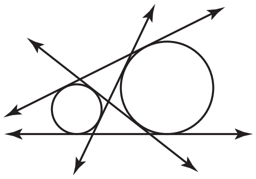
15. 13 in. **17.** 7.1 **19.** $a = 15$; $b \approx 11.3$ **21.** $c \approx 22.8$; $d \approx 16.9$

23 Inscribed angles that intercept the same arc are congruent. Use this theorem to find two pairs of congruent angles. Then use AA Similarity to show that two triangles in the figure are similar. Finally, use the definition of similar triangles to write a proportion. Find the cross products.
Proof:
Statements (Reasons)
1. $\overline{AC}$ and $\overline{DE}$ intersect at B. (Given)
2. $\angle A \cong \angle D$, $\angle E \cong \angle C$ (Inscribed ∠s that intercept the same arc are $\cong$.)
3. $\triangle ABE \sim \triangle DBC$ (AA Similarity)
4. $\frac{AB}{BD} = \frac{EB}{BC}$ (Def. of $\sim \triangle$s)
5. $AB \cdot BC = EB \cdot BD$ (Cross products)

25. Proof:
Statements (Reasons)
1. tangent $\overline{JK}$ and secant $\overline{JM}$ (Given)
2. $m\angle KML = \frac{1}{2}m\widehat{KL}$ (The measure of an inscribed $\angle$ equals half the measure of its intercept arc.)
3. $m\angle JKL = \frac{1}{2}m\widehat{KL}$ (The measure of an $\angle$ formed by a secant and a tangent = half the measure of its intercepted arc.)
4. $m\angle KML = m\angle JKL$ (Substitution)
5. $\angle KML \cong m\angle JKL$ (Definition of $\cong$ ∠s)
6. $\angle J \cong \angle J$ (Reflexive Property)
7. $\triangle JMK \sim \triangle JKL$ (AA Similarity)
8. $\frac{JK}{JL} = \frac{JM}{JK}$ (Definition of $\sim \triangle$s)
9. $JK^2 = JL \cdot JM$ (Cross products)

27. Sample answer: When two secants intersect in the exterior of a circle, the product equation equates the product of the exterior segment measure and the whole segment measure for each secant. When a secant and a tangent intersect, the product involving the tangent segment becomes (measure of tangent segments)^2 because the exterior segments and the whole segments are the same segment.
29. Sometimes; they are equal when the chords are perpendicular. **31.** Sample answer: The product of the parts on one intersecting chord equals the product of the parts of the other chord. **33.** G **35.** E

37.



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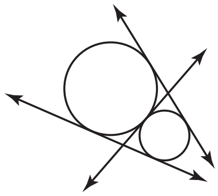

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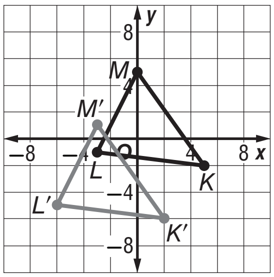

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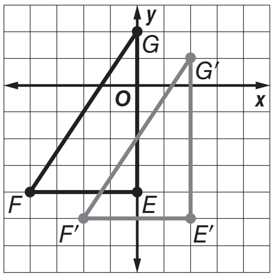


45. $y = 2x + 8$ **47.** $y = \frac{2}{9}x + \frac{1}{3}$ **49.** $y = -\frac{1}{12}x + 1$

Lesson 10-8

1. $(x - 9)^2 + y^2 = 25$

3 $r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

$= \sqrt{(2 - 0)^2 + (2 - 0)^2}$ $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (2, 2)$

$= \sqrt{8}$ Simplify.

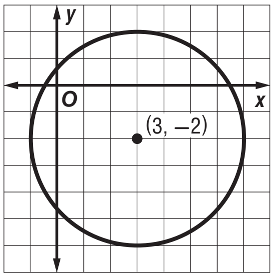
$(x - h)^2 + (y - k)^2 = r^2$ Equation of a circle

$(x - 0)^2 + (y - 0)^2 = (\sqrt{8})^2$ $h = 0, k = 0$, and $r = \sqrt{8}$

$x^2 + y^2 = 8$ Simplify.

5. $(x - 2)^2 + (y - 1)^2 = 4$

7. $(3, -2); 4$



9. $(2, -1); (x - 2)^2 + (y + 1)^2 = 40$

11. $(1, 2), (-1, 0)$

13. $x^2 + y^2 = 16$

15. $(x + 2)^2 + y^2 = 64$

17. $(x + 3)^2 + (y - 6)^2 = 9$

19. $(x + 5)^2 + (y + 1)^2 = 9$

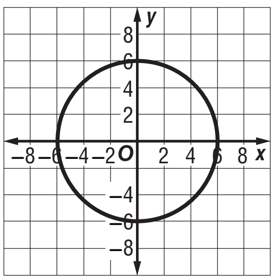
21 The third ring has a radius of 15 + 15 + 15 or 45 miles.

$(x - h)^2 + (y - k)^2 = r^2$ Equation of a circle

$(x - 0)^2 + (y - 0)^2 = 45^2$ $h = 0, k = 0$, and $r = 45$

$x^2 + y^2 = 2025$ Simplify

23. $(0, 0); 6$



25 Write equation in standard form.

$x^2 + y^2 + 8x - 4y = -4$ Original equation

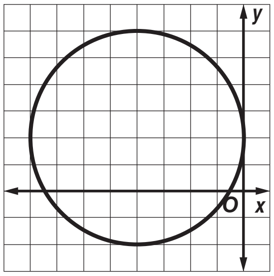
$x^2 + 8x + y^2 - 4y = -4$ Isolate and group like terms.

$x^2 + 8x + 16 + y^2 - 4y + 4 = -4 + 20$ Complete the squares.

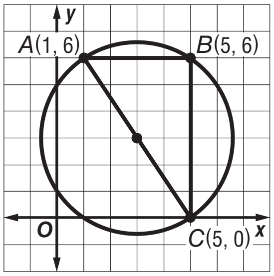
$(x + 4)^2 + (y - 2)^2 = 16$ Factor and simplify.

$[x - (-4)]^2 + (y - 2)^2 = 4^2$ Write + 4 as − (−4) and 16 as 4^2.

So $h = -4, k = 2$, and $r = 4$. The center is at $(-4, 2)$ and the radius is 4.



27. $(x - 3)^2 + (y - 3)^2 = 13$



29. $(-2, -1), (2, 1)$

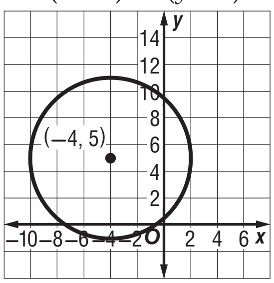
31. $(-2, -4), (2, 0)$

33. $\left(\frac{\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right), \left(-\frac{\sqrt{2}}{2}, -\frac{3\sqrt{2}}{2}\right)$

35. $(x - 3)^2 + y^2 = 25$

37a. $x^2 + y^2 = 810{,}000$ **37b.** 3000 ft

39a. $(x + 4)^2 + (y - 5)^2 = 36$



39b. The circle represents the boundary of the delivery region. All homes within the circle get free delivery. Consuela's home at (0, 0) is located outside the circle, so she cannot get free delivery.

41 The radius of a circle centered at the origin and containing the point $(0, -3)$ is 3 units. Therefore, the equation of the circle is $(x - 0)^2 + (y - 0)^2 = 3^2$ or $x^2 + y^2 = 9$. The point $(1, 2\sqrt{2})$ lies on the circle, since evaluating $x^2 + y^2 = 9$ for $x = 1$ and $y = 2\sqrt{2}$ results in a true equation.

$1^2 + (2\sqrt{2})^2 = 9$

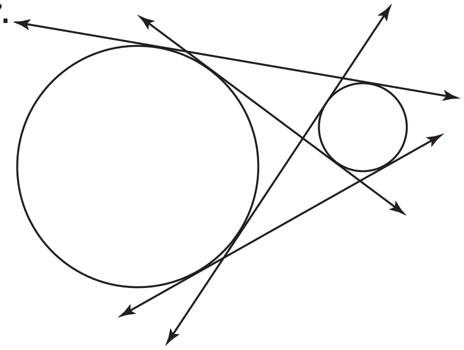
$1 + 8 = 9$

$9 = 9\checkmark$

43. $(x + 5)^2 + (y - 2)^2 = 36$ **45.** $(x - 8)^2 + (y - 2)^2 = 16$; the first circle has its center at $(5, -7)$. If the circle is shifted 3 units right and 9 units up, the new center is at $(8, 2)$, so the new equation becomes $(x - 8)^2 + (y - 2)^2 = 16$. **47a.** 4 **47b–c.** Method 1: Draw a circle of radius 200 centered on each station. Method 2: Use the Pythagorean Theorem to identify pairs of stations that are more than 200 miles apart. Using Method 2, plot the points representing the stations on a graph. Stations that are more than 4 units apart on the graph will be more than 200 miles apart and will thus be able to use the same frequency. Assign station A to the first frequency. Station B is within 4 units of station A, so it must be assigned the second frequency. Station C is within 4 units of both stations A and B, so it must be assigned a third frequency. Station D is also within 4 units of stations A, B, and C, so it must be assigned a fourth frequency. Station E is $\sqrt{29}$ or about 5.4 units away from station A, so it can share the first frequency. Station F is $\sqrt{29}$ or about 5.4 units away from station B, so it can share the second frequency. Station G is $\sqrt{32}$ or about 5.7 units away from station C, so it can share the third frequency. Therefore, the least number of frequencies that can be assigned is 4. **49.** (−6.4, 4.8) **51.** A **53.** Step 1 **55.** 3 **57.** 5.6 **59.** 53 **61.** 28.3 ft **63.** 32 cm; 64 cm^2

Chapter 10 Study Guide and Review

1. false, chord **3.** true **5.** true **7.** false, two **9.** false, congruent **11.** $\overline{DM}$ or $\overline{DP}$ **13.** 13.69 cm; 6.84 cm **15.** 34.54 ft; 17.27 ft **17.** 163 **19a.** 100.8 **19b.** 18 **19c.** minor arc **21.** 131 **23.** 50.4 **25.** 56
27.



29. 97 **31.** 214 **33.** 4 **35.** $(x + 2)^2 + (y - 4)^2 = 25$ **37.** The radius of the circle is 19 + 15 or 34 inches, and (h, k) is (0, 0). Therefore the equation is $(x - 0)^2 + (y - 0)^2 = 34^2$ or $x^2 + y^2 = 34^2$.

CHAPTER 11
Areas of Polygons and Circles

Chapter 11 Get Ready

1. 5 **3.** 20 **5.** 11 ft **7.** 123 **9.** 78 **11.** 11 **13.** $3\sqrt{2}$ cm

Lesson 11-1

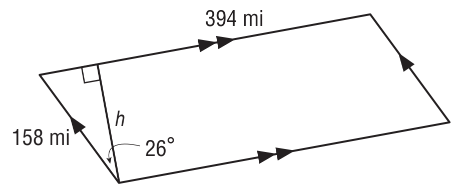
1. 56 in., 180 in^2 **3.** 64 cm, 207.8 cm^2 **5.** 43.5 in., 20 in^2 **7.** 28.5 in., 33.8 in^2 **9.** 11 cm

11 Perimeter = 21 + 17 + 21 + 17 or 76 ft
Use the Pythagorean Theorem to find the height.

$8^2 + h^2 = 17^2$	Pythagorean Theorem
$64 + h^2 = 289$	Simplify.
$h^2 = 225$	Subtract 64 from each side.
$h = 15$	Take the positive square root of each side.
$A = bh$	Area of a parallelogram
$= 21(15)$ or 315 ft^2	$b = 21$ and $h = 15$

13. 69.9 m, 129.9 m^2 **15.** 174.4 m, 1520 m^2 **17.** 727.5 ft^2 **19.** 338.4 cm^2 **21.** 480 m^2

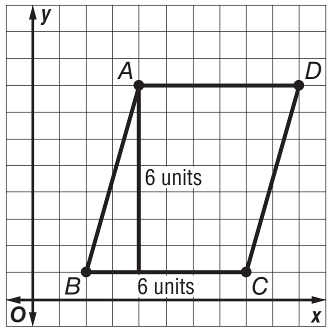
23



$\cos 26° = \frac{h}{158}$	$\cos = \frac{\text{adjacent}}{\text{hypotenuse}}$
$158 \cos 26° = h$	Multiply each side by 158.
$142 \approx h$	Use a calculator.
$A = bh$	Area of a parallelogram
$\approx 394(142)$ or 55,948 mi^2	$b = 394$ and $h = 142$

25. $b = 12$ cm; $h = 3$ cm **27.** $b = 11$ m; $h = 8$ m **29.** 1 pint yellow, 3 pints of blue **31.** 9.19 in.; 4.79 in^2

33 Graph the parallelogram then measure the length of the base and the height and calculate the area.



$A = bh$	Area of a parallelogram
$\approx 6(6)$ or 36 units^2	$b = 6$ and $h = 6$

35a. 10.9 units^2

35b.
$$\sqrt{s(s-a)(s-b)(s-c)} \stackrel{?}{=} \frac{1}{2}bh$$
$$\sqrt{15(15-5)(15-12)(15-13)} \stackrel{?}{=} \frac{1}{2}(5)(12)$$
$$\sqrt{15(10)(3)(2)} \stackrel{?}{=} 30$$
$$\sqrt{900} \stackrel{?}{=} 30$$
$$30 = 30$$

37. 15 units^2; Sample answer: I inscribed the triangle in a 6-by-6 square. I found the area of the square and subtracted the areas of the three right triangles inside the square that were positioned around the given triangle. The area of the given triangle is the difference, or 15 units^2. **39.** Sample answer: The area will not change as K moves along line p. Since lines m and p are parallel, the perpendicular distance between them is constant. That means that no matter where K is on line p, the perpendicular distance to line p, or the height of the triangle, is always the same. Since point J and L are not moving, the distance between them, or the length of the base, is constant. Since the height of the triangle and the base of the triangle are both constant, the area will always be the same. **41.** Sample answer: To find the area of the parallelogram, you can measure the height $\overline{PT}$ and then measure one of the bases $\overline{PQ}$ or $\overline{SR}$ and multiply the height by the base to get the area. You can also measure the height $\overline{SW}$ and measure one of the bases $\overline{QR}$ or $\overline{PS}$ and then multiply the height by the base to get the area. It doesn't matter which side you choose to use as the base, as long as you use the height that is perpendicular to that base to calculate the area. **43.** 6 **45.** B **47.** $x^2 + y^2 = 36$ **49.** $(x - 1)^2 + (y + 4)^2 = 17$ **51.** 5.6 **53.** Sample answer: if each pair of opposite sides are parallel, the quadrilateral is a parallelogram. **55.** 9 **57.** 12

Lesson 11-2

1. 132 ft^2 **3.** 178.5 m^2 **5.** 8 cm **7.** 6.3 ft **9.** 678.5 ft^2 **11.** 136 in^2 **13.** 137.5 ft^2

15 $A = \frac{1}{2}d_1d_2$ Area of a kite

$= \frac{1}{2}(4.8)(10.2)$ $d_1 = 4.8$ and $d_2 = 10.2$

$= 24.48$ Simplify.

The area is about 24.5 square microns.

17. 784 ft^2

19 Let x represent the length of one diagonal. Then the length of the other diagonal is $3x$.

$A = \frac{1}{2}d_1d_2$ Area of a rhombus

$168 = \frac{1}{2}(x)(3x)$ $A = 168$, $d_1 = x$, and $d_2 = 3x$

$168 = \frac{3}{2}x^2$ Simplify.

$112 = x^2$ Multiply each side by $\frac{2}{3}$.

$\sqrt{112} = x$ Take the positive square root of each side.

So the lengths of the diagonals are $\sqrt{112}$ or about 10.6 centimeters and $3(\sqrt{112})$ or about 31.7 centimeters.

21. 4 m **23.** The area of $\triangle HJF = \frac{1}{2}d_1\left(\frac{1}{2}d_2\right)$ and the area of $\triangle HGF = \frac{1}{2}d_1\left(\frac{1}{2}d_2\right)$. Therefore, the area of $\triangle HJF = \frac{1}{4}d_1d_2$, and the area of $\triangle HGF = \frac{1}{4}d_1d_2$. The area of kite $FGHJ$ is equal to the area of $\triangle HJF$ + the area of $\triangle HGF$ or $\frac{1}{4}d_1d_2 + \frac{1}{4}d_1d_2$. After simplification, the area of kite $FGHJ$ is equal to $\frac{1}{2}d_1d_2$. **25a.** 24 in^2 each of yellow, red, orange, green, and blue; 20 in^2 of purple **25b.** Yes; her kite has an area of 140 in^2, which is less than 200 in^2. **27.** 18 sq. units **29.** The area of a trapezoid is $\frac{1}{2}h(b_1 + b_2)$. So, $A = \frac{1}{2}(x + y)(x + y)$ or $\frac{1}{2}(x^2 + xy + y^2)$. The area of $\triangle 1 = \frac{1}{2}(y)(x)$, $\triangle 2 = \frac{1}{2}(z)(z)$, and $\triangle 3 = \frac{1}{2}(x)(y)$. The area of $\triangle 1 + \triangle 2 + \triangle 3 = \frac{1}{2}xy + \frac{1}{2}z^2 + \frac{1}{2}xy$. Set the area of the trapezoid equal to the combined areas of the triangles to get $\frac{1}{2}(x^2 + 2xy + y^2) = \frac{1}{2}xy + \frac{1}{2}z^2 + \frac{1}{2}xy$. Multiply by 2 on each side: $x^2 + 2xy + y^2 = 2xy + z^2$. When simplified, $x^2 + y^2 = z^2$.

31 The length of the base of the triangle is $\frac{12 - 8}{2}$ or 2. Use trigonometry to find the height of the triangle (and trapezoid).

$\tan 30 = \frac{\text{opposite}}{\text{adjacent}}$

$\frac{\sqrt{3}}{3} = \frac{2}{h}$

$\sqrt{3}h = 6$

$h = \frac{6}{\sqrt{3}}$

$h = 2\sqrt{3}$

Use the Pythagorean Theorem to find the hypotenuse of the triangle.

$a^2 + b^2 = c^2$

$2^2 + \left(2\sqrt{3}\right)^2 = c^2$

$4 + 12 = c^2$

$16 = c^2$

$4 = c$

Find the perimeter and area of the trapezoid.

perimeter $= 12 + 8 + 4 + 4$

$= 28$ in.

$= \frac{28}{12}$ ft

≈ 2.3 ft

area $= \frac{1}{2}(b_1 + b_2)h$

$= \frac{1}{2}(8 + 12)\left(2\sqrt{3}\right)$

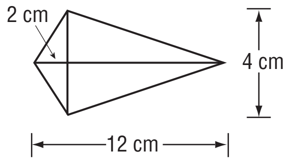
$= 20\sqrt{3}in^2$

$= 20\sqrt{3}in^2 \cdot \frac{1 \text{ ft}}{12 \text{ in.}} \cdot \frac{1 \text{ ft}}{12 \text{ in.}}$

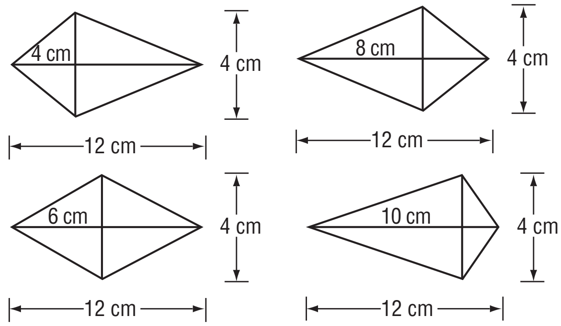
$= \frac{20\sqrt{3}}{144}$ ft^2

≈ 0.2 ft^2

33a.



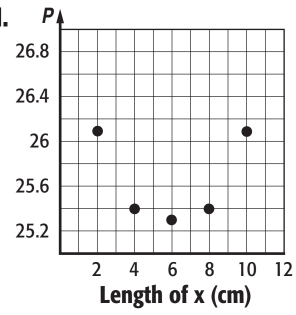
33b.



33c.

x	P
2 cm	26.1 cm
4 cm	25.4 cm
6 cm	25.3 cm
8 cm	25.4 cm
10 cm	26.1 cm

33d.



33e. Sample answer: Based on the graph, the perimeter will be minimized when $x = 6$. This value is significant because when $x = 6$, the figure is a rhombus. **35.** 7.2 **37.** Sometimes; sample answer: If the areas are equal, it means that the products of the diagonals are equal. The only time that the perimeters will be equal is when the diagonals are also equal, or when the two rhombi are congruent. **39.** A **41.** J **43.** 17.5 units^2 **45.** $x^2 + y^2 = 1600$ **47.** $x = 9$; $y = 9\sqrt{3}$ **49.** always **51.** never **53.** 18.8 in.; 28.3 in^2 **55.** 36.4 ft; 105.7 ft^2

Lesson 11-3

1. 1385.4 yd^2

3

$A = \pi r^2$	Area of a circle
$74 = \pi r^2$	$A = 74$
$23.55 \approx r^2$	Divide each side by π.
$4.85 \approx r$	Take the positive square root of each side.

So, the diameter is 2 • 4.85 or about 9.7 millimeters.

5. 4.5 in^2 **7a.** 10.6 in^2 **7b.** $48 **9.** 78.5 yd^2 **11.** 14.2 in^2 **13.** 78.5 ft^2 **15.** 10.9 mm **17.** 8.1 ft

19

$A = \frac{x}{360} \cdot \pi r^2$	Area of a sector
$= \frac{72}{360} \cdot \pi(8)^2$	$x = 72$ and $r = 8$
≈ 40.2 cm^2	Use a calculator.

21. 322 m^2 **23.** 284 in^2 **25a.** 1.7 cm^2 each **25b.** about 319.4 mg **27.** 13 **29.** 9.8

31 **a.**

$C = \pi d$	Circumference of a circle
$2.5 = \pi d$	$C = 2.5$
$0.8 \text{ ft} \approx d$	Divide each side by π.

b. age = diameter • growth factor
= 0.8 • 4.5 or 3.6 yr

33. 53.5 m^2 **35.** 10.7 cm^2 **37.** 7.9 in^2 **39.** 30 mm^2

41 The area equals the area of the large semicircle with a radius of 6 in. plus the area of a small semicircle minus 2 times the area of a small semicircle. The radius of each small semicircles is 2 in.

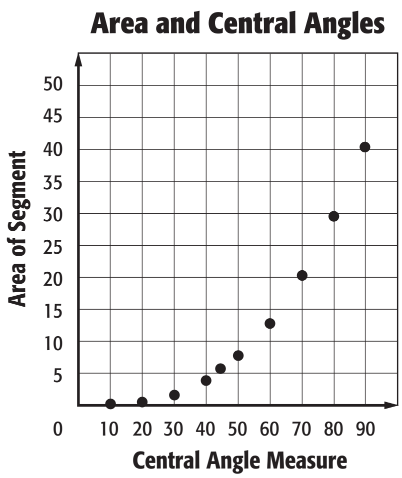
$$A = \frac{1}{2}\pi(6)^2 + \frac{1}{2}\pi(2)^2 - 2\left[\frac{1}{2}\pi(2)^2\right]$$
$$= 18\pi + 2\pi - 4\pi$$
$$= 16\pi$$
$$\approx 50.3 \text{ in}^2$$

43a. $A = \frac{x\pi r^2}{360} - r^2\left[\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)\right]$

43b.

x	A
10	0.1
20	0.5
30	1.7
40	4.0
45	5.6
50	7.7
60	13.0
70	20.3
80	29.6
90	41.1

43c.



43d. Sample answer: From the graph, it looks like the area would be about 15.5 when x is 63°. Using the formula, the area is 15.0 when x is 63°. The values are very close because I used the formula to create the graph. **45.** 449.0 cm^2 **47.** Sample answer: You can find the shaded area of the circle by subtracting x from 360° and using the resulting measure in the formula for the area of a sector. You could also find the shaded area by finding the area of the entire circle, finding the area of the unshaded sector using the formula for the area of a sector, and subtracting the area of the unshaded sector from the area of the entire circle. The method in which you find the ratio of the area of a sector to the area of the whole circle is more efficient. It requires less steps, is faster, and there is a lower probability for error. **49.** Sample answer: If the radius of the circle doubles, the area will not double. If the radius of the circle doubles, the area will be four times as great. Since the radius is squared, if you multiply the radius by 2, you multiply the area by 2^2, or 4. If the arc length of a sector is doubled, the area of the sector is doubled. Since the arc length is not raised to a power, if the arc length is doubled, the area would also be twice as large. **51.** $x = 56$; $m\angle MTQ = 117$; $m\angle PTM = 63$ **53.** E **55.** 13.2 cm, 26.4 cm **57.** 178.2 ft^2 **59.** 7 **61.** 31

Lesson 11-4

1. center: point F, radius: $\overline{FD}$, apothem: $\overline{FG}$, central angle: $\angle CFD$, 90° **3.** 162 in^2 **5.** 239 ft^2

7 **a.** The blue area equals the area of the center circle with a radius of 3 ft plus 2 times the quantity of the area a rectangle 19 ft by 12 ft minus the area of a semicircle with a radius of 6 ft.

Area
= Area of circle + 2 • (Area of rectangle − Area of semicircle)
$= \pi r^2 + 2 \cdot \left(\ell w - \frac{1}{2}\pi r^2\right)$
$= \pi(3)^2 + 2\left[(19(12) - \frac{1}{2}\pi(6)^2\right]$
$= 9\pi + 2(228 - 18\pi)$
$= 9\pi + 456 - 36\pi$
$= 456 - 27\pi$
≈ 371 ft^2

b. The red area equals the area of the center circle with a radius of 6 ft minus the center circle with a radius of 3 ft plus 2 times the area of a circle with a radius of 6 ft.

Area
= Area of large circle − Area of small circle + 2 • Area of circle
$= \pi r^2 - \pi r^2 + 2 \cdot \pi r^2$
$= \pi(6)^2 - \pi(3)^2 + 2\pi(6)^2$
$= 36\pi - 9\pi + 72\pi$
$= 99\pi$
≈ 311 ft^2

9. center: point R, radius: $\overline{RL}$, apothem: $\overline{RS}$, central angle: $\angle KRL$, 60° **11.** 59.4 cm² **13.** 584.2 in²

15 The figure can be separated into a rectangle with a length of 12 cm and a width of 10 cm and a triangle with a base of 12 cm and a height of 16 cm − 10 cm or 6 cm.

$$\text{Area of figure} = \text{Area of rectangle} + \text{Area of triangle}$$
$$= \ell w + \frac{1}{2}bh$$
$$= 12(10) + \frac{1}{2}(12)(6)$$
$$= 120 + 36 \text{ or } 156 \text{ cm}^2$$

17. 55.6 in² **19.** 42.1 yd² **21a.** 29.7 in., 52.3 in² **21b.** 16 **23.** 1.9 in² **25a.** 50.9 ft² **25b.** 4 boxes **27.** 58.1 mm; 232.4 mm²

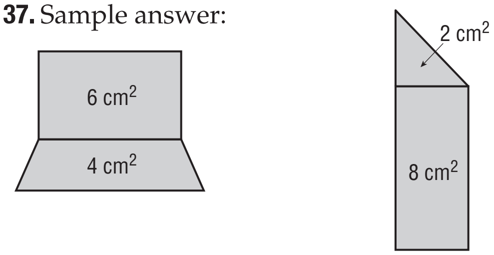
29 To find the area of the shaded region, find the area of the rectangle 8 units by 4 units minus the area of the semicircle with a radius of 2 units minus the area of the trapezoid with bases 4 units and 2 units and height 2 units.

Area of figure
$$= \text{Area of rectangle} - \text{Area of semicircle} - \text{Area of trapezoid}$$
$$= \ell w - \frac{1}{2}\pi r^2 - \frac{1}{2}h(b_1 + b_2)$$
$$= 8(4) - \frac{1}{2}\pi(2)^2 - \frac{1}{2}(2)(4+2)$$
$$= 32 - 2\pi - 6$$
$$= 26 - 2\pi$$
$$\approx 19.7 \text{ units}^2$$

31. 24 units² **33.** 0.43 in²; 0.56 in²; 0.62 in²; 0.65 in²; Sample answer: When the perimeter of a regular polygon is constant, as the number of sides increases, the area of the polygon increases.

35. Chloe; sample answer: The measure of each angle of a regular hexagon is 120°, so the segments from the center to each vertex form 60° angles. The triangles formed by the segments from the center to each vertex are equilateral, so each side of the hexagon is 11 in. The perimeter of the hexagon is 66 in. Using trigonometry, the length of the apothem is about 9.5 in. Putting the values into the formula for the area of a regular polygon and simplifying, the area is about 313.5 in².

37. Sample answer:



39. Sample answer: You can decompose the figure into shapes of which you know the area formulas. Then, you can sum all of the areas to find the total area of the figure. **41.** F **43.** D **45.** 254.5 cm² **47.** 490.9 mm² **49.** 272 in² **51.** semicircle; 180 **53.** major arc; 270 **55.** 30

Lesson 11-5

1. 9 yd² **3.** $\frac{5}{3}$; 35 **5.** 5.28 in²

7 The scale factor between the parallelograms is $\frac{7.5}{15}$ or $\frac{1}{2}$, so the ratio of their areas is $\left(\frac{1}{2}\right)^2$ or $\frac{1}{4}$.

$$\frac{\text{area of small figure}}{\text{area of large figure}} = \frac{1}{4} \quad \text{Write a proportion.}$$
$$\frac{60}{\text{area of large figure}} = \frac{1}{4} \quad \text{Substitution}$$
$$60 \cdot 4 = \text{area of large figure} \cdot 1 \quad \text{Cross multiply.}$$
$$240 = \text{area of large figure} \quad \text{Simplify.}$$

So the area of the large parallelogram is 240 ft².

9. 672 cm² **11.** $\frac{4}{5}$; 17.5 **13.** $\frac{3}{2}$; 36 **15a.** 4 in.

15b. Larger; sample answer: The area of a circular pie pan with an 8 in. diameter is about 50 in². The area of the larger pan is 52.6 in², and the area of the smaller pan is 41.6 in². The area of the larger pan is closer to the area of the circle, so Kaitlyn should choose the larger pan to make the recipe. **17a.** If the area is doubled, the radius changes from 24 in. to 33.9 in. **17b.** If the area is tripled, the radius changes from 24 in. to 41.6 in. **17c.** If the area changes by a factor of x, then the radius changes from 24 in. to $24\sqrt{x}$ in.

19 $\text{Area of } \triangle JKL = \frac{1}{2}bh$

$$= \frac{1}{2}(5)(6) \text{ or } 15 \text{ square units}$$

The scale factor between the triangles is $\frac{5}{3}$, so the ratio of their areas is $\left(\frac{5}{3}\right)^2$ or $\frac{25}{9}$.

$$\frac{\text{area of } \triangle JKL}{\text{area of } \triangle J'K'L'} = \frac{25}{9} \quad \text{Write a proportion.}$$
$$\frac{15}{\triangle J'K'L'} = \frac{25}{9} \quad \text{Area of } \triangle JKL = 15$$
$$15 \cdot 9 = \text{area of } \triangle J'K'L' \cdot 25 \quad \text{Cross multiply.}$$
$$5.4 = \text{area of } \triangle J'K'L' \quad \text{Divide each side by 25.}$$

So the area of $\triangle J'K'L'$ is 5.4 units².

21. area of $ABCD = 18$; area of $A'B'C'D' \approx 56.2$

23 **a.** Sample answer: The graph is misleading because the tennis balls used to illustrate the number of participants are similar circles. When the diameter of the tennis ball increases, the area of the tennis ball also increases. For example, the diameter of the tennis ball representing 1995 is about 2.6 and the diameter of the tennis ball representing 2000 is about 3. So, the rate of increase in the diameters is $\frac{3 - 2.6}{2000 - 1995}$ or about 8%. The area of the circle representing 1995 is $\pi(1.3)^2$ and the area of the circle representing 2000 is $\pi(1.5)^2$. So, the rate of increase in the areas is $\frac{2.25\pi - 1.69\pi}{2000 - 1995}$ or about 35%. The area of the tennis ball increases at a greater rate than the diameter of the tennis ball, so it looks like the number of participants in high school tennis is increasing more than it actually is.

b. Sample answer: If you use a figure with a constant width to represent the participation in each year and only change the height, the graph would not be misleading. For example, use rectangles of equal width and height that varies.
25. Neither; sample answer: In order to find the area of the enlarged circle, you can multiply the radius by the scale factor and substitute it into the area formula, or you can multiply the area formula by the scale factor squared. The formula for the area of the enlargment is $A = \pi(kr)^2$ or $A = k^2\pi r^2$.
27. $P_{\text{enlarged}} = Q\sqrt{R}$ **29.** Sample answer: If you know the area of the original polygon and the scale factor of the enlargement, you can find the area of the enlarged polygon by multiplying the original area by the scale factor squared. **31.** J **33.** E
35. 66.3 cm^2 **37.** 37.4 in^2 **39.** 142.5 **41.** both **43.** $\overline{LP}$
45. $\overline{BM}$, $\overline{AL}$, $\overline{EP}$, $\overline{OP}$, $\overline{PL}$, $\overline{LM}$, $\overline{MN}$

Chapter 11 Study Guide and Review

1. false; height **3.** false; radius **5.** true **7.** false; height of a parallelogram **9.** false; base **11.** $P = 50$ cm; $A = 60\ \text{cm}^2$ **13.** $P = 13.2$ mm; $A = 6\ \text{mm}^2$ **15.** 132 ft^2
17. 96 cm^2 **19.** 336 cm^2 **21.** 1.5 m^2 **23.** 59 in^2
25. 166.3 ft^2 **27.** 65.0 m^2 **29.** $\approx 695\ \text{in}^2$ **31.** $\frac{1}{2}$; 8
33. area of $\triangle RST = 18$ square units; area of $\triangle R'S'T' = 4.5$ square units **35.** 16.4 miles

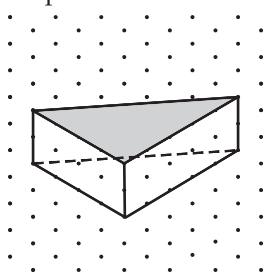
CHAPTER 12
Extending Surface Area and Volume

Chapter 12 Get Ready

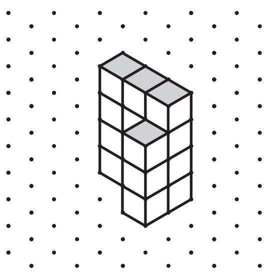
1. true **3.** true **5.** true **7.** 168 in^2 **9.** 176 in^2 **11.** ± 15

Lesson 12-1

1. Sample answer:

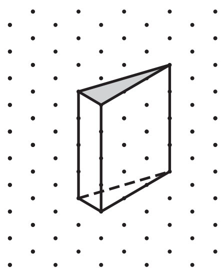


3.

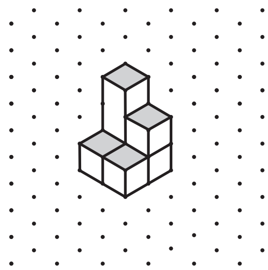


5a. slice vertically **5b.** slice horizontally **5c.** slice at an angle **7.** triangle

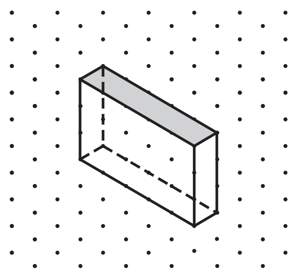
9 Sample answer: First mark the corner of the solid. Then draw 4 units down, 1 unit to the left, and 3 units to the right. Draw a triangle for the top of the solid. Draw segments 4 units down from each vertex for the vertical edges. Connect the appropriate vertices. Use a dashed line for the hidden edge.



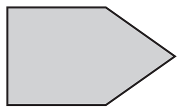
11.



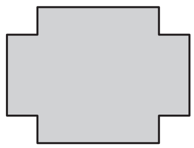
13.



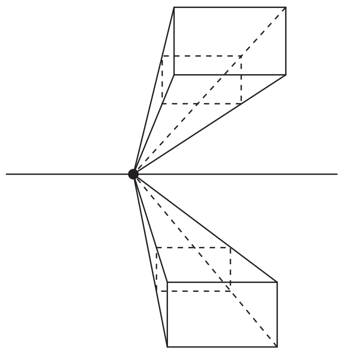
15 **a.** The cross section is a four-sided figure with opposite sides parallel and congruent and with all angles right angles. So, the cross section is a rectangle.
b. To make the cross section of a triangle, cut off the corner of the clay through one of the vertices to the opposite face of the figure.
17. hexagon **19.** trapezoid
21. Make a vertical cut. **23.** Make an angled cut.
25. Sample answer:



27. Sample answer:

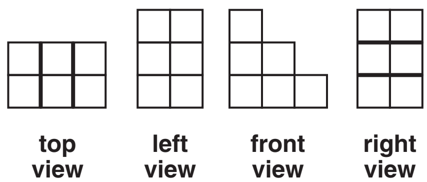


29a–b. Sample answer:

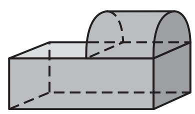


29c. Sample answer: The first drawing shows a view of the object from the bottom. The second drawing shows a view of the object from the top.

31 Top view: There are two rows and 3 columns when viewed from above. Use dark segments to indicate that there are different heights. Left view: The figure is 3 units high and two units wide. Front view: The first column is 3 units high, the second is 2 units high, and the third is 1 unit high. Right view: The figure is 3 units high and 2 units wide. Use dark segments to indicate that there are two breaks in this surface.

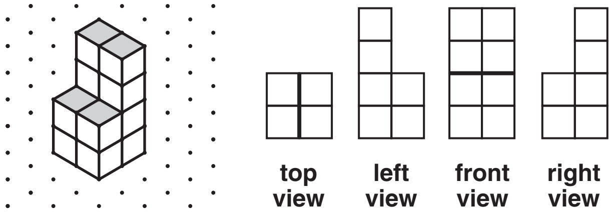


33a. Sample answer:



33b. Make a horizontal cut through the bottom part of the figure or make a vertical cut through the left side of the figure.

33c. The front view of the solid is the cross section when a vertical cut is made lengthwise. The right view of the solid is the cross section when a vertical cut is made through the right side of the figure. **35.** Sample answer: A cone is sliced at an angle through its lateral side and base.
37. Sample answer:



39. The cross section is a triangle. There are six different ways to slice the pyramid so that two equal parts are formed because the figure has six planes of symmetry. In each case, the cross section is an isosceles triangle. Only the side lengths of the triangles change. **41a.** inner side of deck = circumference of pool = $81.64 \div \pi \approx 26$ ft; outer side of deck = $26 + 3 + 3 = 32$ ft; outer perimeter of deck = $4 \times 32 = 128$ ft **41b.** area of deck = $(2 \times 3 \times 32) + (2 \times 3 \times 26) = 348$ square feet **43.** E **45.** $\frac{3}{2}$; 9
47. about 3.6 yd^2 **49.** 28.9 in.; 66.5 in^2

Lesson 12-2

1. 112.5 in^2 **3.** $L = 288\ ft^2$; $S = 336\ ft^2$ **5.** $L \approx 653.5\ yd^2$; $S \approx 1715.3\ yd^2$

7

$S = 2\pi rh + 2\pi r^2$	Surface area of a cylinder
$286.3 = 2\pi(3.4)h + 2\pi(3.4)^2$	Replace S with 286.3 and r with 3.4.
$286.3 \approx 21.4h + 72.6$	Use a calculator to simplify.
$213.7 \approx 21.4h$	Subtract 72.6 from each side.
$10.0 \approx h$	Divide each side by 21.4.

The height of the can is about 10.0 cm.

9 Find the missing side length of the base.

$c^2 = 4^2 + 3^2$	Pythagorean Theorem
$c^2 = 25$	Simplify.
$c = 5$	Take the square root of each side.

$L = Ph$	Lateral area of a prism
$= (4 + 3 + 5)2$	Substitution
$= 24$	Simplify.

The lateral area is 24 ft^2.

$S = Ph + 2B$	Surface area of a prism
$= (4 + 3 + 5)(2) + 2\left(\frac{1}{2} \cdot 4 \cdot 3\right)$	Substitution
$= 24 + 12$ or 36	Simplify.

The surface area is 36 ft^2.

11. Sample answer: $L = 64\ in^2$; $S = 88\ in^2$
13. $L = 11.2\ m^2$; $S = 13.6\ m^2$ **15.** $L = 1032\ cm^2$; $S = 1932\ cm^2$ (18×25 base); $L = 1332\ cm^2$; $S = 1932\ cm^2$ (25×12 base); $L = 1500\ cm^2$; $S = 1932\ cm^2$ (18×12 base) **17.** $L = 1484.8\ cm^2$; $S = 1745.2\ cm^2$
19. $L \approx 282.7\ mm^2$; $S \approx 339.3\ mm^2$ **21.** $L \approx 155.8\ in^2$; $S \approx 256.4\ in^2$ **23.** 42.5 m^2 **25.** $r = 9.2$ cm

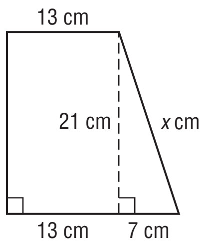
27

$S = 2\pi rh + 2\pi r^2$	Surface area of a cylinder
$256\pi = 2\pi r(8) + 2\pi r^2$	Replace S with 256π and h with 8.
$128 = 8r + r^2$	Divide each side by 2π.
$0 = r^2 + 8r - 128$	Subtract 128 from each side.
$0 = (r + 16)(r - 8)$	Factor.
$r + 16 = 0$ or $r - 8 = 0$	Zero Product Property
$r = -16$ $\quad$ $r = 8$	

Since the radius cannot be negative, $r = 8$. So, the diameter is $8 \cdot 2$ or 16 millimeters.

29a. First, find the area of the sector and double it. Then find 73% of the the lateral area of the cylinder. Next, find the areas of the two rectangles formed by the radius and height when a portion is cut. Last, find the sum of all the areas. **29b.** 283.7 in^2
31. $L = 1392.0\ cm^2$; $S = 2032\ cm^2$ **33.** about 299.1 cm^2

35 The composite figure has trapezoid bases. The trapezoids have bases 20 cm and 13 cm and a height of 21 cm. To find the length of the fourth side of the trapezoid x, use the Pythagorean Theorem.

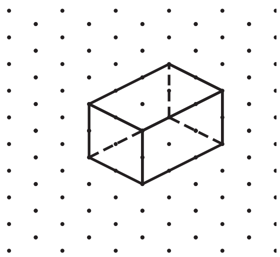


$x^2 = 21^2 + 7^2$	Pythagorean Theorem
$x^2 = 490$	Simplify.
$x \approx 22.136$	Take the square root of each side.

$S = Ph + 2B$	Surface area of a prism
$\approx (21 + 13 + 22.136 + 20)(28) + 2\left[\frac{1}{2}(21)(20 + 13)\right]$	Substitution
$\approx 2131.8 + 693$ or $2824.8\ cm^2$	Simplify.

37. 1059.3 cm^2 **39.** Derek; sample answer: $S = 2\pi r^2 + 2\pi rh$, so the surface area of the cylinder is $2\pi(6)^2 + 2\pi(6)(5)$ or $132\pi\ cm^2$ **41.** To find the surface area of any solid figure, find the area of the base (or bases) and add to the area of the lateral faces of the figure. The lateral faces and bases of a rectangular prism are rectangles. Since the bases of a cylinder are circles, the lateral faces of a cylinder is a rectangle. **43.** $\frac{\sqrt{3}}{2}\ell^2 + 3\ell h$; the area of an equilateral triangle of side ℓ is $\frac{\sqrt{3}}{4}\ell^2$ and the perimeter of the triangle is 3ℓ. So, the total surface area is $\frac{\sqrt{3}}{2}\ell^2 + 3\ell h$. **45.** A **47.** H

49.



51. One 9-inch cake; Nine minicakes have the same top area as one 9-inch cake, but nine minicakes cost 9($4) or $36 while the 9-inch cake is only $15, so the 9-inch cake is a better buy.
53. 2.5 in. **55.** 20.5

Lesson 12-3

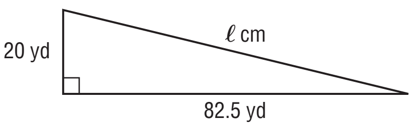
1. $L = 384$ cm^2; $S = 640$ cm^2 **3.** $L \approx 207.8$ m^2; $S \approx 332.6$ m^2 **5.** $L \approx 188.5$ m^2; $S \approx 267.0$ m^2

7

$L = \frac{1}{2}P\ell$	Lateral area of a regular pyramid
$= \frac{1}{2}(8)(5)$	$P = 2 \cdot 4$ or 8, $\ell = 5$
$= 20$ m^2	Simplify.
$S = \frac{1}{2}P\ell + B$	Surface area of a regular pyramid
$= 20 + 4$	$\frac{1}{2}P\ell = 20$, $B = 4$
$= 24$ m^2	Simplify.

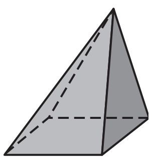
9. $L \approx 178.2$ cm^2; $S \approx 302.9$ cm^2 **11.** $L \approx 966.0$ in^2; $S \approx 1686.0$ in^2 **13.** 139,440 ft^2 **15.** $L \approx 357.6$ cm^2; $S \approx 470.7$ cm^2 **17.** $L \approx 241.1$ ft^2; $S \approx 446.1$ ft^2

19 Use the Pythagorean Theorem to find the slant height ℓ.

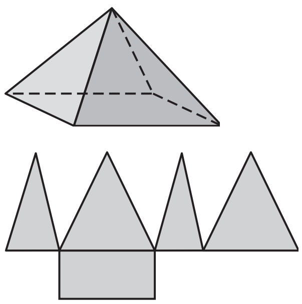


$c^2 = a^2 + b^2$	Pythagorean Theorem
$\ell^2 = 20^2 + 82.5^2$	$a = 20$, $b = 82.5$, $c = \ell$
$\ell^2 = 7206.25$	Simplify.
$\ell = \sqrt{7206.25}$	Take the square root of each side.
$L = \frac{1}{2}P\ell$	Lateral area of a regular pyramid
$= \frac{1}{2}(660)(\sqrt{7206.25})$	$P = 154 \cdot 4$ or 660, $\ell = 6$
$\approx 28{,}013.6$ yd^2	Use a calculator.

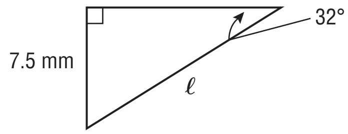
21. 34 **23.** 5 mm **25.** 16 cm **27.** 266π ft^2
29a. nonregular pyramid with a square base
29b. Sample answer:



31. Sample answer:



33 The lateral area of a cone is $L = \pi r\ell$. The radius is half the diameter, or 7.5 mm. Find the slant height ℓ.



$\sin 32° = \frac{\text{opposite}}{\text{hypotenuse}}$

$\sin 32° = \frac{7.5}{\ell}$

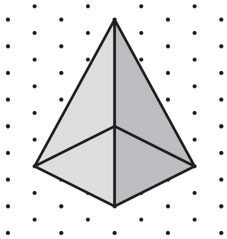
$\ell = \frac{7.5}{\sin 32°}$ or about 14.15 mm

$L = \pi(7.5)(14.15)$ or about 333.5 mm^2.

The surface area of a cone is $S = L + \pi r^2$.

$S = 333.5 + \pi(7.5)^2$ or about 510.2 mm^2.

35a. Sample answer:

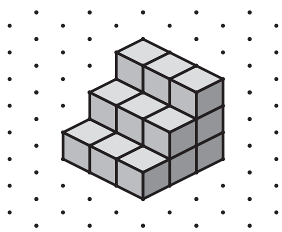


35b.

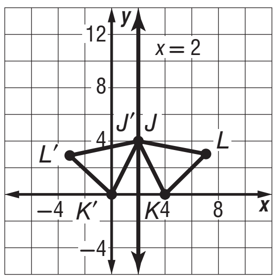
Slant Height (units)	Lateral Area (units)^2
1	6
3	18
9	54

35c. The lateral area is tripled. **35d.** The lateral area is multiplied by 3^2 or 9. **37.** Always; if the heights and radii are the same, the surface area of the cylinder will be greater since it has two circular bases and additional lateral area. **39.** Sample answer: a square pyramid with a base edge of 5 units and a slant height of 7.5 units
41. Use the apothem, the height, and the Pythagorean Theorem to find the slant height ℓ of the pyramid. Then use the central angle of the n-gon and the apothem to find the length of one side of the n-gon. Then find the perimeter. Finally, use $S = \frac{1}{2}P\ell + B$ to find the surface area. The area of the base B is $\frac{1}{2}Pa$. **43.** 3299 mm^2 **45.** D

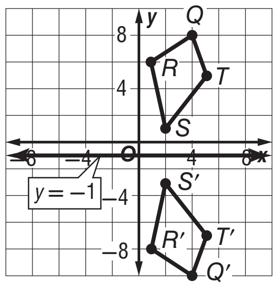
47.



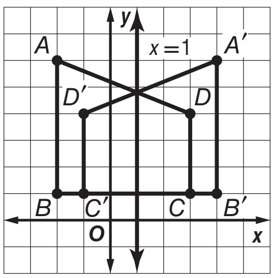
49.



51.



53.



55. 57 m, 120 m^2
57. 183.1 in., 1887 in^2

Lesson 12-4

1. 108 cm^3 **3.** 26.95 m^3 **5.** 206.4 ft^3 **7.** 1025.4 cm^3 **9.** D

11 $V = Bh$ Volume of a prism
$= 38.5(14)$ $B = \frac{1}{2}(11)(7)$ or 38.5, $h = 14$
$= 539\ m^3$ Simplify.

13. 58.14 ft^3 **15.** 1534.25 in^3

17 $V = \pi r^2 h$ Volume of a cylinder
$= \pi(6)^2(3.6)$ Replace r with 12 ÷ 2 or 6 and h with 3.6.
$\approx 407.2\ cm^3$ Use a calculator.

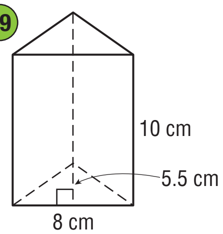
19. 2686.1 mm^3 **21.** 521.5 cm^3 **23.** 3934.9 cm^3
25. 35.1 cm **27a.** 0.0019 lb/in^3 **27b.** The plant should grow well in this soil since the bulk density of 0.0019 lb/in^3 is close to the desired bulk density of 0.0018 lb/in^3. **27c.** 8.3 lb **29.** 120 m^3

31 Find the volume of the cylinder with a height of 11.5 cm and a radius of 8.5 ÷ 2 or 4.25 cm. Subtract from that the volume of the cylinder with a height of 11.5 cm and a radius of 6.5 ÷ 2 or 3.25 cm. Add the volume of the bottom cylinder that has a height of 1 cm and a radius of 6.5 ÷ 2 or 3.25 cm.

Volume = Volume of large cylinder − Volume of small cylinder + Volume of bottom
$= \pi r_1^{\,2}h_1 - \pi r_2^{\,2}h_1 + \pi r_2^{\,2}h_2$
$= \pi(4.25)^2(11.5) - \pi(3.25)^2(11.5) + \pi(3.25)^2(1)$ $r_1 = 4.25, r_2 = 3.25, h_1 = 10.5, h_2 = 1$
$\approx 304.1\ cm^3$ Use a calculator.

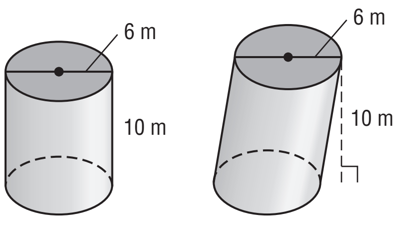
33. 678.6 in^3 **35.** 3,190,680.0 cm^3 **37.** $11\frac{1}{4}$ in.

39

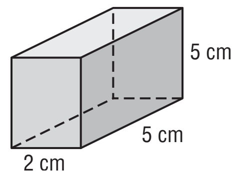


Each triangular prism has a base area of $\frac{1}{2}(8)(5.5)$ or 22 cm^2 and a height of 10 cm. The volume of each triangular prism is 22 • 10 or 220 cm^3. So, the volume of five triangular prisms is 220 • 5 or 1100 cm^3.

41a.



41b. Greater than; a square with a side length of 6 m has an area of 36 m^2. A circle with a diameter of 6 m has an area of 9π or 28.3 m^2. Since the heights are the same, the volume of the square prism is greater assuming $x > 1$. **41c.** Multiplying the radius by x; since the volume is represented by $\pi r^2 h$, multiplying the height by x makes the volume x times greater. Multiplying the radius by x makes the volume x^2 times greater. **43.** Sample answer: The can holds $\pi(2)^2(5)$ or 20π in^3 of liquid. Therefore, the container holds 60π in^3. **43a.** base 3 in. by 5 in., height 4π in. **43b.** base 5 in. per side, height $\frac{12}{5}\pi$ in. **43c.** base with legs measuring 3 in. and 4 in., height 10π in.
45. Sample answer:



47. Sample answer: Both formulas involve multiplying the area of the base by the height. The base of a prism is a polygon, so the expression representing the area varies, depending on the type of polygon it is. The base of a cylinder is a circle, so its area is πr^2.
49. F **51.** C **53.** 126 cm^2; 175 cm^2 **55.** 205 in^2
57. 11.4 cm **59.** 9.3 in. **61.** 378 m^2

Lesson 12-5

1. 75 in^3 **3.** 62.4 m^3 **5.** 51.3 in^3 **7.** 28.1 mm^3
9. 513,333.3 ft^3

11 $V = \frac{1}{3}Bh$ Volume of a pyramid
$= \frac{1}{3}(36.9)(8.6)$ $B = \frac{1}{2} \cdot 9 \cdot 8.2$ or 36.9, $h = 8.6$
$\approx 105.8\ mm^3$ Simplify.

13. 233.8 cm^3 **15.** 35.6 cm^3 **17.** 235.6 in^3
19. 1473.1 cm^3 **21.** 1072.3 in^3

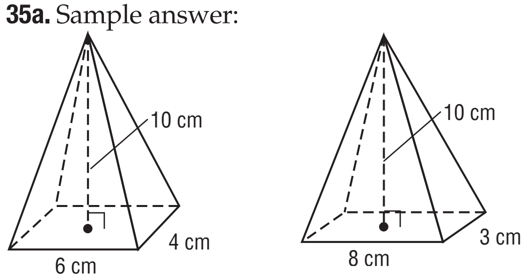
23 $V = \frac{1}{3}\pi r^2 h$ Volume of a cone
$= \frac{1}{3}\pi(4)^2(14)$ Replace r with $\frac{8}{2}$ or 4 and h with 14.
$\approx 234.6\ cm^3$ Use a calculator.

25. 32.2 ft^3 **27.** 3190.6 m^3 **29.** about 13,333 BTUs
31a. The volume is doubled. **31b.** The volume is multiplied by 2^2 or 4. **31c.** The volume is multiplied by 2^3 or 8.

33 $V = \frac{1}{3}\pi r^2 h$ Volume of a cone
$196\pi = \frac{1}{3}\pi r^2(12)$ Replace V with 196π and h with 12.
$196\pi = 4\pi r^2$ Simplify.
$49 = r^2$ Divide each side by 4π.
$7 = r$ Take the square root of each side.

The radius of the cone is 7 inches, so the diameter is 7 • 2 or 14 inches.

35a. Sample answer:



35b. The volumes are the same. The volume of a pyramid equals one third times the base area times the height. So, if the base areas of two pyramids are equal and their heights are equal, then their volumes are equal. **35c.** If the base area is multiplied by 5, the volume is multiplied by 5. If the height is multiplied by 5, the volume is multiplied by 5. If both the base area and the height are multiplied by 5, the volume is multiplied by $5 \cdot 5$ or 25. **37.** Cornelio; Alexandra incorrectly used the slant height. **39.** Sample answer: A square pyramid with a base area of 16 and a height of 12, a prism with a square base of area 16 and height of 4; if a pyramid and prism have the same base, then in order to have the same volume, the height of the pyramid must be 3 times as great as the height of the prism. **41.** A **43.** F **45.** 1008.0 in^3 **47.** 426,437.6 m^3 **49.** 30.4 cm^2 **51.** 26.6 ft^2

Lesson 12-6

1. 1017.9 m^2 **3.** 452.4 yd^2 **5.** 4188.8 ft^3 **7.** 3619.1 m^3 **9.** 277.0 in^2 **11.** 113.1 cm^2 **13.** 680.9 in^2 **15.** 128 ft^2 **17.** 530.1 mm^2

19
$V = \frac{4}{3}\pi r^3$ Volume of a sphere

$= \frac{4}{3}\pi(1)^3$ $r = \frac{2}{2}$ or 1

$\approx 4.2\ cm^3$ Use a calculator.

21. 2712.3 cm^3 **23.** 179.8 in^3 **25.** 77.9 m^3 **27.** 860,289.5 ft^3

29
Surface area $= \frac{1}{2} \cdot$ Area of sphere + Lateral area of cylinder + Area of circle

$= \frac{1}{2}(4\pi r^2) + 2\pi rh + \pi r^2$

$= \frac{1}{2}(4\pi)(4)^2 + 2\pi(4)(5) + \pi(4)^2$

$\approx 276.5\ in^2$

Volume = Volume of hemisphere + Volume of cylinder

$= \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) + \pi r^2 h$

$= \frac{1}{2}\left(\frac{4}{3}\pi \cdot 4^3\right) + \pi(4)^2(5)$

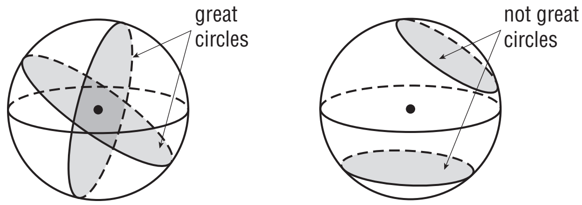
$\approx 385.4\ in^3$

31a. 594.6 cm^2; 1282.8 cm^3 **31b.** 148.7 cm^2; 160.4 cm^3 **33.** $\overline{DC}$ **35.** $\overline{AB}$ **37.** $\odot S$
39a. $\sqrt{r^2 - x^2}$ **39b.** $\pi\left(\sqrt{r^2 - x^2}\right)^2 \cdot y$ or $\pi yr^2 - \pi yx^2$
39c. The volume of the disc from the cylinder is $\pi r^2 y$ or πyr^2. The volume of the disc from the two cones is $\pi x^2 y$ or πyx^2. Subtract the volumes of the discs from the cylinder and cone to get $\pi yr^2 - \pi yx^2$, which is the expression for the volume of the disc from the sphere at height x. **39d.** Cavalieri's Principle **39e.** The volume of the cylinder is $\pi r^2(2r)$ or $2\pi r^3$. The volume of one cone is $\frac{1}{3}\pi r^2(r)$ or $\frac{1}{3}\pi r^3$, so the volume of the double napped cone is $2 \cdot \frac{1}{3}\pi r^3$ or $\frac{2}{3}\pi r^3$. Therefore, the volume of the hollowed out cylinder, and thus the sphere, is $2\pi r^3 - \frac{2}{3}\pi r^3$ or $\frac{4}{3}\pi r^3$.

41 Vertical: There is an infinite number of vertical planes that produce reflection symmetry. When any vertical plane intersects the hemisphere through a diameter, both sides of the hemisphere are mirror images. Horizontal: There are no horizontal planes that produce reflection symmetry. When any horizontal plane intersects the hemisphere, the part on top will always be slightly smaller than the bottom. Rotation: There is an infinite number of angles of rotation. When the axis of rotation passes through the center of the sphere perpendicular to its base, the hemisphere can be mapped onto itself by a rotation of any angle between 0° and 360° in the axis.

43 The surface area is divided by 3^2 or 9. The volume is divided by 3^3 or 27. **45.** 587.7 in^3

47.



49. 68.6 **51.** H **53.** 58.9 ft^3 **55.** 232.4 m^3 **57.** 154.2 $units^2$ **59.** 80.2 $units^2$ **61.** 21 $units^2$ **63.** D, B, and G **65.** $\overleftrightarrow{EF}$ and $\overleftrightarrow{AB}$ do not intersect. $\overleftrightarrow{AB}$ lies in plane $\mathcal{P}$, but only E lies in plane $\mathcal{P}$.

Lesson 12-7

1. $\overleftrightarrow{DH}$, $\overleftrightarrow{FJ}$ **3.** $\triangle JKQ$, $\triangle LMP$

5 Figure x does not go through the poles of the sphere. So, figure x is not a great circle and so not a line in spherical geometry.

7. The points on any great circle or arc of a great circle can be put into one-to-one correspondence with real numbers. **9.** Sample answers: $\overleftrightarrow{WZ}$ and $\overleftrightarrow{XY}$, $\overline{RY}$ or $\overline{TZ}$, $\triangle RST$ or $\triangle MPL$ **11a.** $\overleftrightarrow{AD}$ and $\overleftrightarrow{FC}$ **11b.** Sample answers: $\overline{BG}$ and $\overline{AH}$ **11c.** Sample answers: $\triangle BCD$ and $\triangle ABF$ **11d.** $\overline{QD}$ and $\overline{BL}$ **11e.** $\overline{MJ}$ **11f.** $\overleftrightarrow{MB}$ and $\overleftrightarrow{KF}$ **13.** no

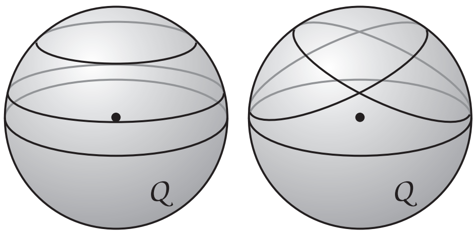
15 Every great circle (line) is finite and returns to its original starting point. Thus, there exists no great circle that goes on infinitely in two directions.

17. Yes; If three points are collinear, any one of the three points is between the other two. **19.** 14.0 in.; since 100 degrees is $\frac{5}{18}$ of 360 degrees, $\frac{5}{18} \times$ circumference of the great circle ≈ 14.0.
21a. about 913 mi; the cities are 13.2° apart on the same great circle, so $\frac{13.2}{360} \times 2\pi \times 3963$ give the distance between them. **21b.** Yes; sample answer: Since the cities lie on a great circle, the distance between the cities can be expressed as the major arc or the minor arc. The sum of the two values is the circumference of Earth. **21c.** No; sample answer:

Since lines of latitude do not go through opposite poles of the sphere, they are not great circles. Therefore, the distance cannot be calculated in the same way. **21d.** Sample answer: Infinite locations. If Phoenix were a point on the sphere, then there are infinite points that are equidistant from that point.

23 **a.** No; if $\overline{CD}$ were perpendicular to $\overline{DA}$, then $\overline{DA}$ would be parallel to $\overline{CB}$. This is not possible, since there are no parallel lines in spherical geometry. **b.** $DA < CB$ because $\overline{CB}$ appears to lie on a great circle. **c.** No; since there are no parallel lines in spherical geometry, the sides of a figure cannot be parallel. So, a rectangle, as defined in Euclidean geometry, cannot exist in non-Euclidean geometry.

25. Sample answer: In plane geometry, the sum of the measures of the angles of a triangle is 180. In spherical geometry, the sum of the measures of the angles of a triangle is greater than 180. In hyperbolic geometry, the sum of the measures of the angles of a triangle is less than 180. **27.** Sometimes; sample answer: Since small circles cannot go through opposite poles, it is possible for them to be parallel, such as lines of latitude. It is also possible for them to intersect when two small circles can be drawn through three points, where they have one point in common and two points that occur on one small circle and not the other.



29. False; sample answer; Spherical geometry is non-Euclidean, so it cannot be a subset of Euclidean geometry. **31.** C **33.** Sample answer: $\overline{BC}$ **35.** 735.4 m^3 **37.** 1074.5 cm^3 **39.** 78.5 m^3 **41.** 0.1 m^3 **43.** 2.7 cm^2 **45.** 322.3 m^2

Lesson 12-8

1. similar; 4:3 **3.** 4:25 **5.** 220,893.2 cm^3 **7.** neither **9.** similar; 6:5

11 $\frac{\text{height of large cylinder}}{\text{height of small cylinder}} = \frac{35}{25}$ or $\frac{7}{5}$

The scale factor is $\frac{7}{5}$. If the scale factor is $\frac{a}{b}$, then the ratio of volumes is $\frac{a^3}{b^3} = \frac{7^3}{5^3}$ or $\frac{343}{125}$. So, the ratio of the volumes is 343:125.

13. 5:1 **15a.** 10:13 **15b.** 419.6 cm^3

17 scale factor $= \frac{26 \text{ ft}}{14 \text{ in.}}$ Write a ratio comparing the lengths.

$= \frac{312 \text{ in.}}{14 \text{ in.}}$ 26 ft = 26 • 12 or 312 in.

$= \frac{156}{7}$ Simplify.

The scale factor is 156:7.

19. 4.1 in. **21.** 2439.6 cm^3 **23.** about 5.08 to 1

25 $\frac{\text{area of smaller tent}}{\text{area of larger tent}} = \frac{9}{12.25}$ Write a ratio comparing the floor areas.

$= \frac{3^2}{3.5^2}$ Write as $\frac{a^2}{b^2}$.

The scale factor is 3:3.5.

ratio of diameters $\rightarrow \frac{6}{d} = \frac{3}{3.5} \leftarrow$ scale factor

$6 \cdot 3.5 = d \cdot 3$ Find the cross products.

$7 = d$ Solve for d.

So, the diameter of the larger tent is 7 feet.

$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$ Volume of a hemisphere

$= \frac{1}{2}\left(\frac{4}{3}\pi \cdot 3.5^3\right)$ Radius $= \frac{7}{2}$ or 3.5

≈ 89.8 Use a calculator.

The volume of the larger tent is about 89.8 ft^3.

27. Laura; because she compared corresponding parts of the similar figures. Paloma incorrectly compared the diameter of X to the radius of Y. **29.** Since the scale factor is 15:9 or 5:3, the ratio of the surface areas is 25:9 and the ratio of the volumes is 125:27. So, the surface area of the larger prism is $\frac{25}{9}$ or about 2.8 times the surface area of the smaller prism. The volume of the larger prism is $\frac{125}{27}$ or about 4.6 times the volume the smaller prism. **31.** 14 cm **33.** B **35.** $\sqrt{85} \approx 9.2$ km **37.** yes **39.** yes **41.** 5 **43.** 6 **45.** 0.31 **47.** 0.93

Chapter 12 Study Guide and Review

1. false, Spherical geometry **3.** false, right cone **5.** true **7.** true **9.** true **11.** triangle **13.** rectangle **15.** Sample answer: 160 ft^2; 202 ft^2 **17.** 113.1 cm^2; 169.6 cm^2 **19.** 354.4 cm^2; 432.9 cm^2 **21.** 972 cm^3 **23.** 3.6 cm^3 **25.** 91,636,272 ft^3 **27.** 1017.9 m^2 **29.** 1708.6 in^3 **31.** $\overleftrightarrow{FG}$, $\overleftrightarrow{DJ}$ **33.** $\triangle CBD$ **35.** $\overline{KC}$ **37.** no **39.** congruent **41.** neither

CHAPTER 13
Probability and Measurement

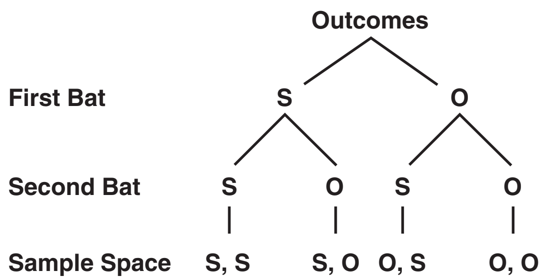
Chapter 13 Get Ready

1. $\frac{7}{8}$ **3.** $1\frac{11}{40}$ **5.** $\frac{3}{8}$ **7.** 144 **9.** $\frac{1}{2}$ or 50% **11.** $\frac{1}{3}$ or 33% **13.** $\frac{1}{5}$ or 20% **15.** $\frac{11}{20}$ or 55%

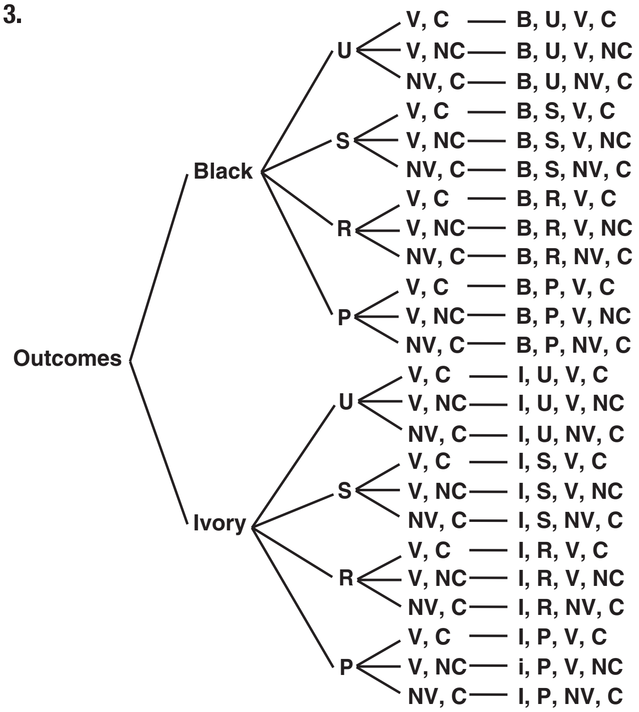
Lesson 13-1

1. S, S O, O
S, O O, S

Outcomes	Safe	Out
Safe	S, S	S, 0
Out	0, S	0, 0



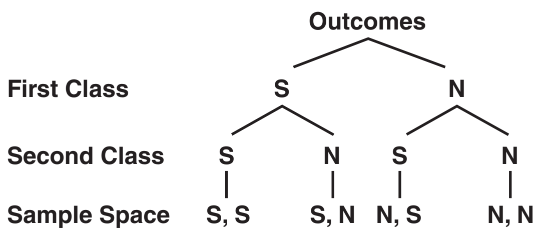
3.



5 Possible Outcomes
= Appetizers × Soups × Salads × Entrees × Desserts
= 8 × 4 × 6 × 12 × 9 or 20,736

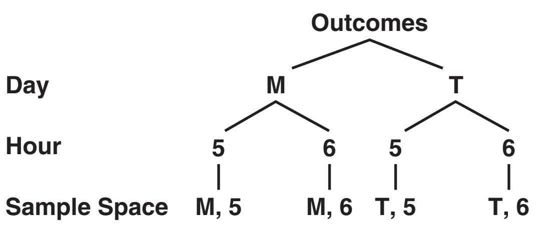
7. S, S N, N
S, N N, S

Outcomes	Smithsonian	Natural
Smithsonian	S, S	S, N
Natural	N, S	N, N



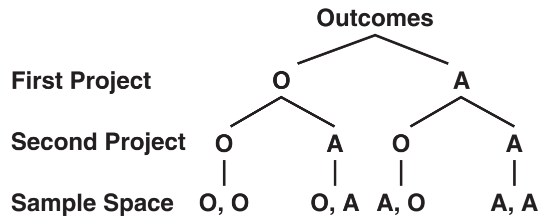
9. M, 5 T, 5
M, 6 T, 6

Outcomes	5	6
Monday	M, 5	M, 6
Thursday	T, 5	T, 6

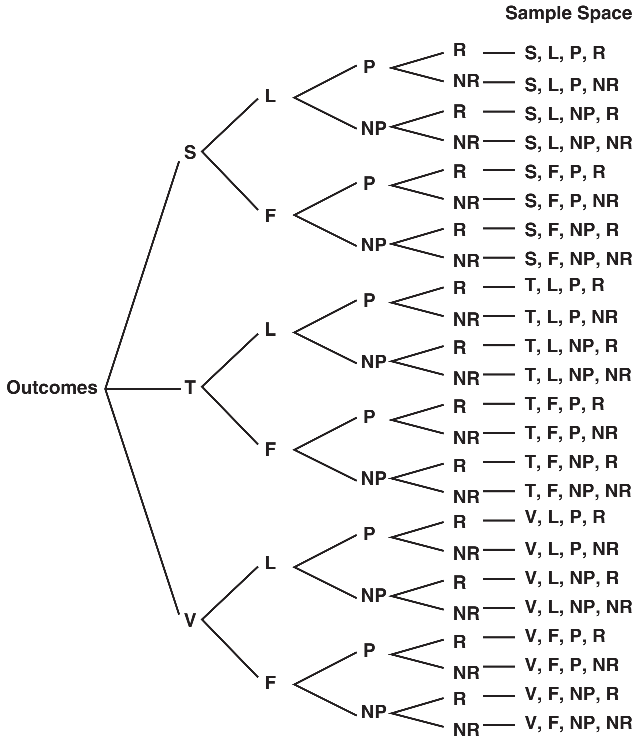


11. O, O A, A
O, A A, O

Outcomes	Oil	Acrylic
Oil	0, 0	0, A
Acrylic	A, 0	A, A



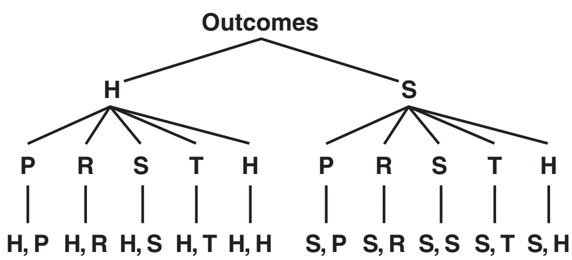
13. S = sedan, T = truck, V = van, L = leather, F = fabric, P = CD player, NP = no CD player, R = sunroof, NR = no sunroof



15 Possible Outcomes = Secretary × Treasurer × Vice President × President
= 3 × 4 × 5 × 2 or 120

17. 240 **19.** H = rhombus, P = parallelogram, R = rectangle, S = square, T = trapezoid; H, P; H, R; H, S; H, T; H, H; S, P; S, R; S, S; S, T; S, H

Outcomes	Rhombus	Square
Parallelogram	H, P	S, P
Rectangle	H, R	S, R
Square	H, S	S, S
Trapezoid	H, T	S, T
Rhombus	H, H	S, H



21. Sample answer: 6 different ways:
$4(x + 6) + 2(3) + 2(x + 4)$;

$2(x + 11) + 2(x + 8) + 2(x)$;
$2(x + 4) + 2(x + 9) + 2(x + 6)$;
$2(x) + 2(3) + 4(x + 8)$;
$2(x) + 2(x + 8) + 2(3) + 2(x + 8)$;
$2(x) + 2(3) + 2(4) + 2(x + 6) + 2(x + 6)$

23 **a.** The rolls that result in a sum of 8 are 2 and 6, 3 and 5, 4 and 4, 5 and 3, 6 and 2. So, there are 5 outcomes.
b. The rolls that result in an odd sum are shown.

1, 2	1, 4	1, 6
2, 1	2, 3	2, 5
3, 2	3, 4	3, 6
4, 1	4, 3	4, 5
5, 2	5, 4	5, 6
6, 1	6, 3	6, 5

So, there are 18 outcomes.

25. $n^3 - 3n^2 + 2n$; Sample answer: There are n objects in the box when you remove the first object, so after you remove one object, there are $n - 1$ possible outcomes. After you remove the second object, there are $n - 2$ possible outcomes. The number of possible outcomes is the product of the number of outcomes of each experiment or $n(n - 1)(n - 2)$. **27.** Sample answer: You can list the possible outcomes for one stage of an experiment in the columns and the possible outcomes for the other stage of the experiment in the rows. Since a table is two dimensional, it would be impossible to list the possible outcomes for three or more stages of an experiment. Therefore, tables can only be used to represent the sample space for a two-stage experiment. **29.** $P = n^k$; Sample answer: The total number of possible outcomes is the product of the number of outcomes for each of the stages 1 through k. Since there are k stages, you are multiplying n by itself k times which is n^k. **31.** B **33.** G **35.** 130 m high, 245 m wide, and 465 m long **37.** Sample answer: $\overline{FC}$ **39.** 1429.4 ft^2, 1737.3 ft^2 **41.** 1710.6 m^2, 3421.2 m^2 **43.** line **45.** 12.5 **47.** 12 **49.** 32

Lesson 13-2

1. $\frac{1}{20}$ **3.** $\frac{1}{420}$ **5.** $\frac{1}{124,750}$

7 The number of possible outcomes is 50!. The number of favorable outcomes is (50 − 2)! or 48!.
P(Alfonso 14, Colin 23)

$= \frac{48!}{50!}$ — $\frac{\text{Number of favorable outcomes}}{\text{Number of possible outcomes}}$

$= \frac{\overset{1}{\cancel{48!}}}{50 \cdot 49 \cdot \underset{1}{\cancel{48!}}}$ — Expand 48! and divide out common factors.

$= \frac{1}{2450}$ — Simplify.

9. $\frac{1}{15,120}$

11 There is a total of 10 letters. Of these letters, B occurs 2 times, A occurs 2 times, and L occurs 2 times. So, the number of distinguishable permutations of these letters is

$\frac{10!}{2! \cdot 2! \cdot 2!} = \frac{3,628,800}{8}$ or 453,600 — Use a calculator.

There is only 1 favorable arrangement—BASKETBALL. So, the probability that a permutation of these letters selected at random spells basketball is $\frac{1}{453,600}$.

13. $\frac{1}{7}$ **15.** $\frac{1}{10,626}$ **17a.** $\frac{1}{56}$ **17b.** $\frac{1}{40,320}$ **17c.** $\frac{1}{140}$ **17d.** $\frac{2}{7}$ **19a.** 720 **19b.** 5040

21 Find the number of ways to choose the second letter times the number of ways to choose the third letter times the number of ways to choose the last two numbers.

$$\text{possible license plates} = {}_2C_1 \cdot {}_3C_1 \cdot {}_{10}C_1 \cdot {}_{10}C_1 = 2 \cdot 3 \cdot 10 \cdot 10 \text{ or } 600$$

23. $\frac{13}{261}$ **25.** Sample answer: A bag contains seven marbles that are red, orange, yellow, green, blue, purple, and black. The probability that the orange, blue, and black marbles will be chosen if three marbles are drawn at random can be calculated using a combination.

27.
$$C(n, n - r) \stackrel{?}{=} C(n, r)$$
$$\frac{n!}{[n - (n - r)]!(n - r)!} \stackrel{?}{=} \frac{n!}{(n - r)!r!}$$
$$\frac{n!}{r!(n - r)!} \stackrel{?}{=} \frac{n!}{(n - r)!r!}$$
$$\frac{n!}{(n - r)!r!} = \frac{n!}{(n - r)!r!} \checkmark$$

29. C **31.** J **33.** 16 **35.** 2 **37.** 4.5 **39.** 3 **41.** 1 **43.** 5

Lesson 13-3

1. $\frac{1}{2}$, 0.5, or 50% **3.** $\frac{13}{33}$, 0.39, or about 39% **5.** $\frac{1}{8}$, 0.125, or 12.5% **7.** $\frac{13}{18}$, 0.72, or 72% **9.** $\frac{1}{9}$, 0.11, or 11% **11.** $\frac{1}{6}$, 0.17, or about 17%

13 You need to find the ratio of the area of the shaded region to the area of the entire region. The area of shaded region equals the area of the large semicircle minus the area of the small semicircle plus the area of the small semicircle. So, the area of the shaded region equals the area of the large semicircle. Since the area of the large semicircle equals half the total area, P(landing in shaded region) $= \frac{1}{2}$, 0.5, or 50%.

15 P(pointer landing on yellow) $= \frac{44}{360}$ or about 12.2%

17. 69.4% **19.** 62.2% **21.** Sample answer: a point between 10 and 20 **23.** $\frac{1}{2}$, 0.5, or 50% **25.** 53.5% **27.** Sample answer: The probability that a randomly chosen point will lie in the shaded region is ratio of the area of the sector to the area of the circle.

$P(\text{point lies in sector}) = \dfrac{\text{area of sector}}{\text{area of circle}}$

$\dfrac{x}{360} \stackrel{?}{=} \dfrac{\frac{x}{360} \cdot \pi r^2}{\pi r^2}$

$\dfrac{x}{360} = \dfrac{x}{360}$ ✓

29. 0.24 or 24% **31.** 0.33 or 33%

33 volume of shallow region $= Bh = (7 \cdot 20) \cdot 20$ or 2800 ft^3

volume of incline region $= Bh = \frac{1}{2}(25)(7 + 20) \cdot 20$ or 6750 ft^3

volume of deep region $= Bh = (20 \cdot 30) \cdot 20$ or 12,000 ft^3

P(bear swims in the incline region)

$= \dfrac{\text{volume of incline region}}{\text{volume of pool}}$

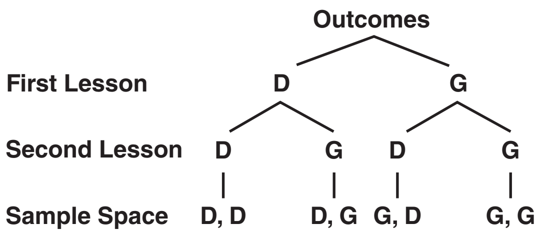
$= \dfrac{6750}{2800 + 6750 + 12{,}000}$

≈ 0.31 or 31%

35. 14.3% **37.** No; sample answer: Athletic events should not be considered random because there are other factors involved, such as pressure and ability, that have an impact on the success of the event.
39. Sample answer: The probability of a randomly chosen point lying in the shaded region of the square on the left is found by subtracting the area of the unshaded square from the area of the larger square and finding the ratio of the difference of the areas to the area of the larger square. The probability is $\dfrac{1^2 - 0.75^2}{1^2}$ or 43.75%. The probability of a randomly chosen point lying in the shaded region of the square on the right is the ratio of the area of the shaded square to the area of the larger square, which is $\dfrac{0.4375}{1}$ or 43.75%. Therefore, the probability of a randomly chosen point lying in the shaded area of either square is the same. **41.** F **43.** C

45. D, D G, G
D, G G, D

Outcomes	Drums	Guitar
Drums	D, D	D, G
Guitar	G, D	G, G

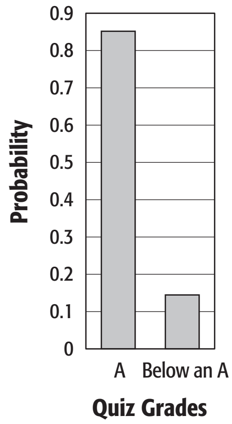


47. 45 **49.** Square; each angle intercepts a semicircle, making them 90° angles. Each side is a chord of congruent arcs, so the chords are congruent.
51. 57.1 m^2 **53.** 66.3 cm^2

Lesson 13-4

1. Sample answer: Use a spinner that is divided into two sectors, one containing 80% or 288° and the other containing 20% or 72°. Do 20 trials and record the results in a frequency table.

Outcome	Frequency
A	17
below an A	3
Total	20



The probability of Clara getting an A on her next quiz is 0.85. The probability of earning any other grade is 1 − 0.85 or 0.15. **3a.** 36 **3b.** Sample answer: Use a random number generator to generate integers 1 through 25 where 1–16 represents 25 points, 17–24 represents 50 points, and 25 represents 100 points. Do 50 trials and record the results in a frequency table.

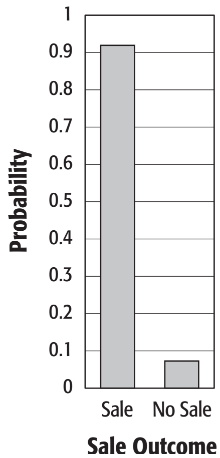
Outcome	Frequency
25	29
50	21
100	0

The average value is 35.5. **3c.** Sample answer: The expected value and average value are very close.

5 Sample answer: Use a spinner that is divided into two sectors, one containing 95% or 342° and the other containing 5% or 18°.
Sale: 95% of 360 = 342°
No Sale: 5% of 360 = 18°

Do 50 trials and record the results in a frequency table.

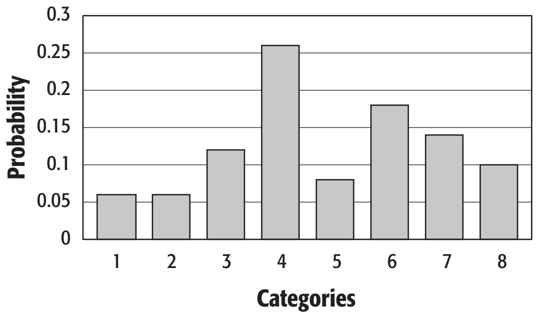
Outcome	Frequency
sale	46
no sale	4
Total	50



Based on the simulation, the experimental probability of Ian selling a game is 0.92. The experimental probability of not selling a game is 1 − 0.92 or 0.08.

7. Sample answer: Use a spinner that is divided into 8 equal sectors, each 45°. Do 50 trials and record the results in a frequency table.

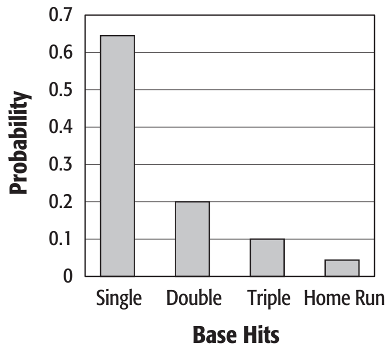
Outcome	Frequency
Category 1	3
Category 2	3
Category 3	6
Category 4	13
Category 5	4
Category 6	9
Category 7	7
Category 8	5
Total	50



The probability of landing on Categories 1 and 2 is 0.06, Category 3 is 0.12, Category 4 is 0.26, Category 5 is 0.08, Category 6 is 0.18, Category 7 is 0.14, and Category 8 is 0.1.

9. Sample answer: Use a random number generator to generate integers 1 through 20, where 1–12 represents a single, 13–17 represents a double, 18–19 represents a triple, and 20 represents a home run. Do 20 trials and record the results in a frequency table.

Outcome	Frequency
single	13
double	4
triple	2
home run	1
Total	20

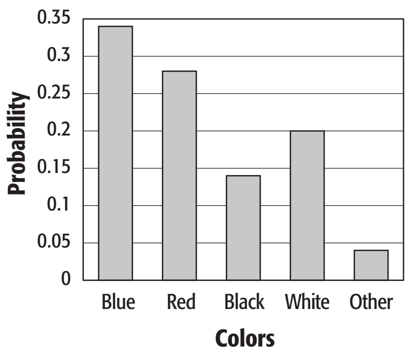


The probability of the baseball player hitting a single is 0.65, a double is 0.2, a triple is 0.1, and a home run is 0.05.

11. Sample answer: Use a random number generator to generate integers 1 through 20, where 1–7 represents blue, 8–13 represents red, 14–16 represents white, 17–19 represents black, and 20 represents all other colors. Do 50 trials and record the results in a frequency table.

Outcome	Frequency
blue	17
red	14
black	7
white	10
other	2
Total	50

The probability of a customer buying a blue car is 0.34, buying a red car is 0.28, buying a black car is 0.14, buying a white car is 0.2, and any other color is 0.04.



13 Calculate the geometric probability of landing on each color.

$P(\text{red}) = \dfrac{\text{area of red}}{\text{area of circle}}$

$= \dfrac{\pi(0.5)^2}{\pi(5)^2}$

$= 0.01$

The blue and white areas are equal. Their total area is the area of the large circle minus the area of the red circle or $\pi(5)^2 - \pi(0.5)^2$ or about 77.8 square units. So, the blue area and the white area are each about 38.9 square units.

$P(\text{blue}) = \dfrac{\text{area of blue}}{\text{area of circle}}$

$= \dfrac{38.9}{\pi(5)^2}$

≈ 0.495

$P(\text{white}) = P(\text{blue}) \approx 0.495$

$E(Y) = 25 \cdot 0.495 + 50 \cdot 0.495 + 0.01 \cdot 100$

$E(Y) = 38.125$

Sample answer: Assign the integers 1–1000 to accurately represent the probability data. Blue = integers 1–495, White = integers 496–990, Red = integers 991–1000. Use the calculator to generate 500 trials of random integers from 1 to 1000. Record the results in a frequency table.

Outcome	Frequency
red	0
blue	29
white	21
total	50

Then calculate the average value of the outcomes.

$\frac{0}{50} \cdot 100 + \frac{29}{50} \cdot 25 + \frac{21}{50} \cdot 50 = 35.5$

Average value = 35.5; the expected value is greater than the average value.

15a. 0.75 **15b.** Sample answer: Use a random number generator to generate integers 1 through 20, where 1–7 represents 0 points, 8–19 represents 1 point, and 20 represents 3 points. Do 50 trials and record the results in a frequency table; average value = 0.76.

Outcome	Frequency
0	16
1	32
3	2

15c. Sample answer: the two values are almost equal.

17a. There is a $\frac{1}{6}$ or 16.7% probability of throwing a strike in each box.

17b. Sample answer:

Strike Area	Accuracy (%)
1	15
2	17
3	19
4	22
5	19
6	8
Total	100

17c. Sample answer: Some of the values are higher or lower, but most are very close to 16.7%.

19a. Sample answer:

Sum of Die Roll
9
10
6
6
7
9
5
9
5
7
6
5
7
3
9
7
6
7
8
7

19b. Sample answer:

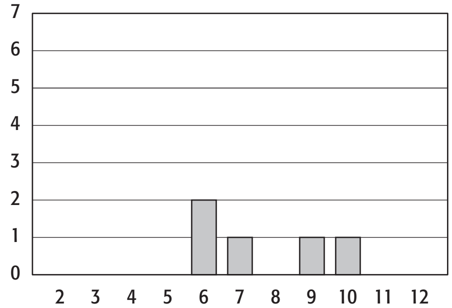
Sum of Output from Random Number Generator
4
10
5
10
6
7
12
3
7
4
7
9
3
6
4
11
5
7
5
3

19c. Sample answer:

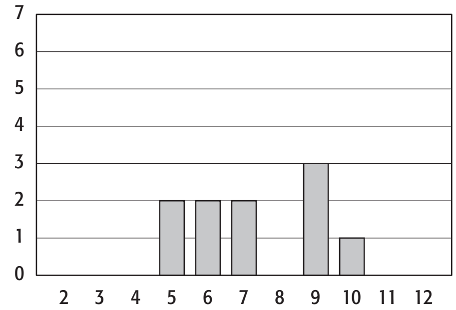
Trial	Sum of Die Roll	Sum of Output from Random Number Generator
1	9	4
2	10	10
3	6	5
4	6	10
5	7	6
6	9	7
7	5	12
8	9	3
9	5	7
10	7	4
11	6	7
12	5	9
13	7	3
14	3	6
15	9	4
16	7	11
17	6	5
18	7	7
19	8	5
20	7	3

19d. Sample answer:

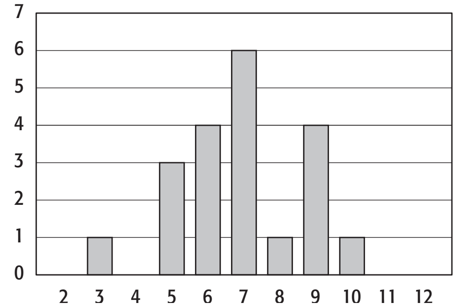
Dice–5 Rolls



Dice–10 Rolls

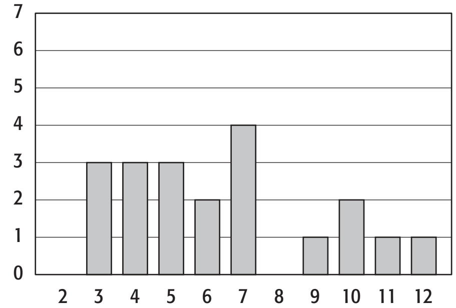


Dice – 20 Rolls



19e. Sample answer: The bar graph has more data points at the middle sums as more trials are added. **19f.** Sample answer:

Random Number Generator



19g. Sample answer: They both have the most data points at the middle sums. **19h.** Sample answer: The expected value in both experiments is 7 because it is the sum that occurs most frequently.

21 Sometimes; sample answer: Flipping a coin can be used to simulate and experiment with two possible outcomes when both of the outcomes are equally likely. For example, if there is an equal number of boys and girls in a class, then flipping a coin can be used to simulate choosing one person from the class. If the probabilities of the occurrence of the two outcomes are different, flipping a coin is not an appropriate simulation. For example, if 55% of the class is girls and 45% is boys, then flipping a coin cannot be used to simulate choosing one person from the class.

23. Sample answer: We assume that the object lands within the target area, and that it is equally likely that the object will land anywhere in the region. These are needed because in the real world it will not be

equally likely to land anywhere in the region.
25. Sample answer: Expectation, or expected value, deals with all possible events, while probability typically deals with only one event. Probability is the likelihood that an event will happen, is between 0 and 1 inclusive, and is typically expressed as a decimal, fraction, or percent. Expectation is the weighted average of all possible events and can take on any value, even a value that is not a possible outcome. For example, the likelihood of rolling a "1" with a six-sided die is 1 out of every 6 rolls. So, the probability is $\frac{1}{6}$ or about 17%. When the same die is rolled, the expectation is 3.5, which isn't even a possible outcome. **27.** H **29.** B **31.** $\frac{4}{7}$, 0.57, or 57%
33. 50.3 ft^2 **35.** 1017.9 in^2

Lesson 13-5

1. The outcome of Jeremy taking the SAT in no way changes the probability of the outcome of his ACT test. Therefore, these two events are *independent*.

3. $\frac{1}{2704}$ or 3.7×10^{-4} **5.** $\frac{1}{5}$ or 0.20

7 These events are dependent since the card is not replaced.

$P(A \text{ and } A) = P(A) \cdot P(A|A)$ — Probability of dependent events

$= \frac{4}{52} \cdot \frac{3}{51}$ — After first ace is drawn, 51 cards remain, and 3 of those are aces.

$= \frac{12}{2652}$ or $\frac{1}{221}$ — Multiply.

The probability is $\frac{1}{221}$ or about 0.5%.

9. independent; $\frac{1}{36}$ or about 3% **11.** $\frac{1}{306}$ or about 0.3%

13. $\frac{20}{161}$ or about 12% **15.** $\frac{1}{4}$ or 25% **17.** $\frac{1}{6}$ or 17%

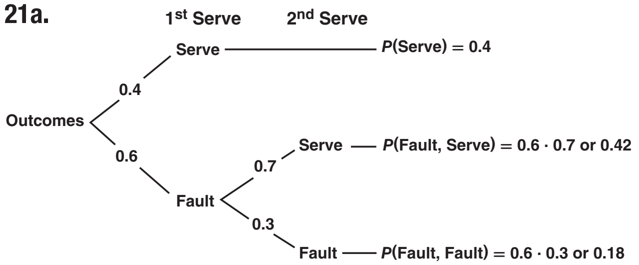
19 $P(\text{own MP3 player} \mid \text{own CD player})$

$= \frac{P(\text{own MP3 player and CD player})}{P(\text{own CD player})}$

$= \frac{0.28}{0.43}$

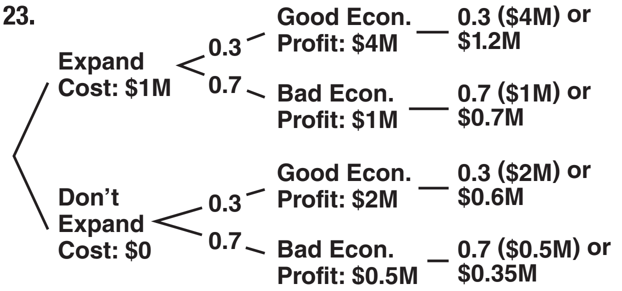
≈ 0.65

21a.



21b. 0.18 or 18% **21c.** Sample answer: I would use a random number generator to generate integers 1 through 50. The integers 1–9 will represent a double fault, and the integers 10–50 will represent the other possible outcomes. The simulation will consist of 50 trials.

23.



Sample answer: The expected value of choosing to expand is \$1.2M + \$0.7M or \$1.9M, and the expected value of choosing not to expand is \$0.95M. When we subtract the costs of expanding and of not expanding, we find the net expected value of expanding is \$1.9M − \$1M or \$0.9M and the net expected value of not expanding is \$0.95M − \$0 or \$0.95M. Since \$0.9M < \$0.95M, you should not expand the business.
25. *A* and *B* are independent events.
27. In order for the events to be independent, two things must be true: 1) the chance that a person smokes is the same as the chance that a person smokes given that the person's parent smokes, and 2) the chance that a person's parent smokes is the same as the chance that a person's parent smokes given that the person smokes. **29.** J **31.** B **33.** 0.25 **35.** 0.07
37. neither **39a.** 5026.5 ft **39b.** 500–600 ft
39c. 3142 ft; 3770 ft **41.** 12 **43.** 216

Lesson 13-6

1. not mutually exclusive **3.** $\frac{2}{3}$ or about 67%

5. The probability of missing the spare is $\frac{8}{10}$ or 80%.

7. 17.3%

9 Since rolling two fours is both getting doubles and getting a sum of 8, the events are not mutually exclusive.

$P(\text{doubles or a sum of 8})$

$= P(\text{doubles}) + P(\text{a sum of 8}) - P(\text{doubles and a sum of 8})$

$= \frac{6}{36} + \frac{5}{36} - \frac{1}{36}$

$= \frac{10}{36}$ or about 27.8%

11. mutually exclusive; 100% **13.** mutually exclusive; $\frac{2}{9}$ or about 22.2% **15.** $\frac{7}{16}$ or about 43.8%

17. $\frac{3}{4}$ or about 75% **19.** $\frac{7}{8}$ or about 87.5%

21 Find the probability that the first worker is paid by the hour plus the probability that that the second worker is paid by the hour. The probability that a worker is not paid by the hour is 1 − 0.71 or 0.29.

$P(\text{first paid}) + P(\text{second paid})$

$= P(\text{first paid}) \cdot P(\text{second not paid}) + P(\text{second paid}) \cdot P(\text{first not paid})$

$= 0.71(0.29) + 0.71(0.29)$

$\approx 0.21 + 0.21$ or 0.42

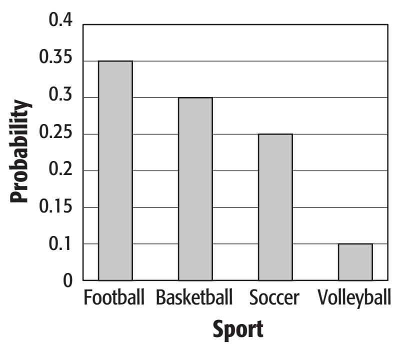
The probability is about 0.42 or 42%.

23. $\frac{1}{13}$ or 7.7% **25.** $\frac{3}{13}$ or 23.1%

27a. 71.3% **27b.** 11.3% **27c.** 36.2% **27d.** 3.8%
29. 0.74; sample answer: There are three outcomes in which the values of two or more of the dice are less than or equal to 4 and one outcome where the values of all three of the dice are less than or equal to 4. You have to find the probability of each of the four scenarios and add them together. **31.** Not mutually exclusive; sample answer: If a triangle is equilateral, it is also equiangular. The two can never be mutually exclusive. **33.** Sample answer: If you pull a card from a deck, it can be either a 3 or a 5. The two events are mutually exclusive. If you pull a card from a deck, it can be a 3 and it can be red. The two events are not mutually exclusive. **35.** D **37.** J
39. dependent; $\frac{1}{221}$ or 0.5%

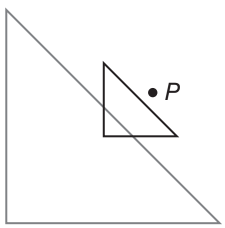
41. Sample answer: Use a random number generator to generate integers 1 through 20 in which 1–7 represent football, 8–13 represent basketball, 14–17 represent soccer, and 18–20 represent volleyball. Do 20 trials, and record the results in a frequency table.

Outcome	Frequency
football	7
basketball	6
soccer	5
volleyball	2
Total	20



The probability that an athlete plays only football is 0.35, only basketball is 0.30, only soccer is 0.25, and only volleyball is 0.1.

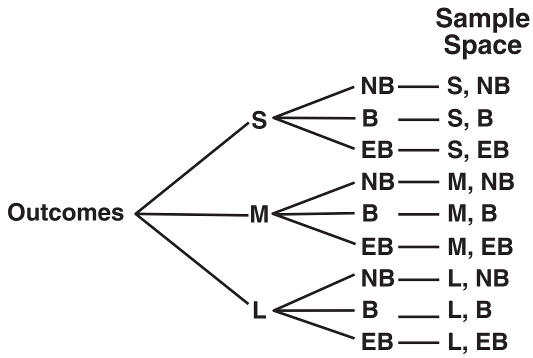
43.



Chapter 13 Study Guide and Review

1. true **3.** true **5.** true **7.** true **9.** false, simulation
11. S, NB; S, B; S, EB; M, NB; M, B; M, EB; L, NB; L, LB; L, EB

Outcomes	No Butter	Butter	Extra Butter
Small	S, NB	S, B	S, EB
Medium	N, NB	M, B	M, EB
Large	L, NB	L, B	L, EB



13. 4 **15.** 35,960 **17a.** $\frac{2}{9}$ **17b.** $\frac{7}{9}$ **19.** Sample answer: Use a spinner that is divided into 4 sectors, 108°, 79.2°, 82.8°, and 90°. Perform 50 trials and record the results in a frequency table. The results can be used to determine the probability of when a particular book will be purchased. **21.** $\frac{6}{35}$ **23.** 37%
25. $\frac{4}{13}$

Glossary/Glosario

MultilingualeGlossary

Go to connectED.mcgraw-hill.com for a glossary of terms in these additional languages:

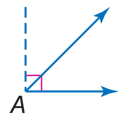
Arabic	Chinese	Hmong	Spanish	Vietnamese
Bengali	English	Korean	Tagalog	
Brazilian Portuguese	Haitian Creole	Russian	Urdu	

English

Español

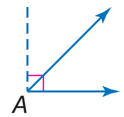
A

acute angle (p. 38) An angle with a degree measure less than 90.



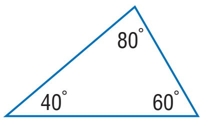
$0 < m\angle A < 90$

ángulo agudo Ángulo cuya medida en grados es menos de 90.



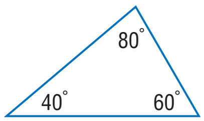
$0 < m\angle A < 90$

acute triangle (p. 237) A triangle in which all of the angles are acute angles.



three acute angles

triángulo acutángulo Triángulo cuyos ángulos son todos agudos.



tres ángulos agudos

adjacent angles (p. 46) Two angles that lie in the same plane, have a common vertex and a common side, but no common interior points.

ángulos adyacentes Dos ángulos que yacen sobre el mismo plano, tienen el mismo vértice y un lado en común, pero ningún punto interior en común.

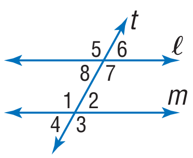
adjacent arcs (p. 708) Arcs in a circle that have exactly one point in common.

arcos adyacentes Arcos en un círculo que tienen un solo punto en común.

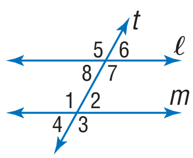
algebraic proof (p. 136) A proof that is made up of a series of algebraic statements. The properties of equality provide justification for many statements in algebraic proofs.

demostración algebraica Demostración que se realiza con una serie de enunciados algebraicos. Las propiedades de la igualdad proveen justificación para muchas enunciados en demostraciones algebraicas.

alternate exterior angles (p. 174) In the figure, transversal t intersects lines ℓ and m. $\angle 5$ and $\angle 3$, and $\angle 6$ and $\angle 4$ are alternate exterior angles.



ángulos alternos externos En la figura, la transversal t interseca las rectas ℓ y m. $\angle 5$ y $\angle 3$, y $\angle 6$ y $\angle 4$ son ángulos alternos externos.



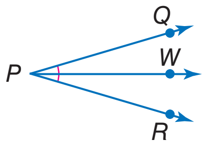
alternate interior angles (p. 174) In the figure at the bottom of page R115, transversal t intersects lines ℓ and m. $\angle 1$ and $\angle 7$, and $\angle 2$ and $\angle 8$ are alternate interior angles.

altitude **1.** (p. 337) In a triangle, a segment from a vertex of the triangle to the line containing the opposite side and perpendicular to that side. **2.** (p. 846) In a prism or cylinder, a segment perpendicular to the bases with an endpoint in each plane. **3.** (p. 854) In a pyramid or cone, the segment that has the vertex as one endpoint and is perpendicular to the base.

ambiguous case of the Law of Sines (p. 598) Given the measures of two sides and a nonincluded angle, there exist two possible triangles.

angle (p. 36) The intersection of two noncollinear rays at a common endpoint. The rays are called *sides* and the common endpoint is called the *vertex*.

angle bisector (p. 39) A ray that divides an angle into two congruent angles.



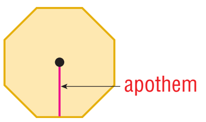
$\overrightarrow{PW}$ is the bisector of $\angle P$.

angle of depression (p. 580) The angle between the line of sight and the horizontal when an observer looks downward.

angle of elevation (p. 580) The angle between the line of sight and the horizontal when an observer looks upward.

angle of rotation (p. 640) The angle through which a preimage is rotated to form the image.

apothem (p. 807) A segment that is drawn from the center of a regular polygon perpendicular to a side of the polygon.



arc (p. 706) A part of a circle that is defined by two endpoints.

area (p. 58) The number of square units needed to cover a surface.

auxiliary line (p. 246) An extra line or segment drawn in a figure to help complete a proof.

axiom (p. 127) A statement that is accepted as true.

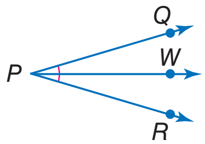
ángulos alternos internos En la figura anterior, la transversal t interseca las rectas ℓ y m. $\angle 1$ y $\angle 7$, y $\angle 2$ y $\angle 8$ son ángulos alternos internos.

altura **1.** En un triángulo, segmento trazado desde uno de los vértices del triángulo hasta el lado opuesto y que es perpendicular a dicho lado. **2.** En un prisma o un cilindro, segmento perpendicular a las bases con un extremo en cada plano. **3.** En una pirámide o un cono, segmento que tiene un extremo en el vértice y que es perpendicular a la base.

caso ambiguo de la ley de los senos Dadas las medidas de dos lados y de un ángulo no incluido, existen dos triángulos posibles.

ángulo La intersección de dos rayos no colineales en un extremo común. Las rayos se llaman *lados* y el punto común se llama *vértice*.

bisectriz de un ángulo Rayo que divide un ángulo en dos ángulos congruentes.



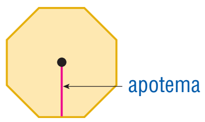
$\overrightarrow{PW}$ es la bisectriz del $\angle P$.

ángulo de depresión Ángulo formado por la horizontal y la línea de visión de un observador que mira hacia abajo.

ángulo de elevación Ángulo formado por la horizontal y la línea de visión de un observador que mira hacia arriba.

ángulo de rotación Ángulo a través del cual se rota una preimagen para formar la imagen.

apotema Segmento trazado desde el centro de un polígono regular hasta uno de sus lados y que es perpendicular a dicho lado.



arco Parte d e un círculo definida por dos extremos.

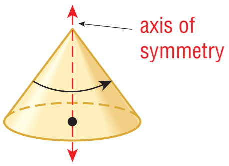
área Número de unidades cuadradas para cubrir una superficie.

línea auxiliar Recta o segmento de recta adicional que es traza en una figura para ayudar a completar una demostración.

axioma Enunciado que se acepta como verdadero.

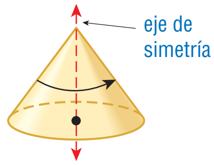
axis **1.** (p. 848) In a cylinder, the segment with endpoints that are the centers of the bases. **2.** (p. 856) In a cone, the segment with endpoints that are the vertex and the center of the base.

axis symmetry (p. 665) Symmetry in a three-dimensional figure that occurs if the figure can be mapped onto itself by a rotation between 0° and 360° in a line.



eje **1.** En un cilindro, el segmento cuyos extremos son el centro de las bases. **2.** En un cono, el segmento cuyos extremos son el vértice y el centro de la base.

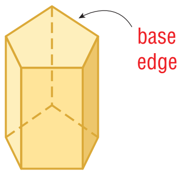
eje simetría Simetriá que ocurre en una figura tridimensional si la figura se puede aplicar sobre sí misma, mientras se gira entre 0° y 360° sobre una recta.



B

base angle of an isosceles triangle (p. 285) See *isosceles triangle* and isosceles trapezoid.

base edges (p. 846) The intersection of the lateral faces and bases in a solid figure.



base of parallelogram (p. 779) Any side of a parallelogram.

base of a polyhedron (p. 67) The two parallel congruent faces of a polyhedron.

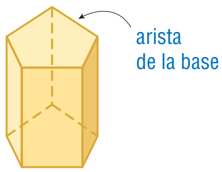
between (p. 15) For any two points *A* and *B* on a line, there is another point *C* between *A* and *B* if and only if *A*, *B*, and *C* are collinear and $AC + CB = AB$.

betweenness of points (p. 15) See *between*.

biconditional (p. 116) The conjunction of a conditional statement and its converse.

ángulo de la base de un triángulo isósceles Ver *triángulo isósceles* y trapecio isósceles.

aristas de las bases Intersección de las base con las caras laterales en una figura sólida.



base de un paralelogramo Cualquier lado de un paralelogramo.

base de poliedro Las dos caras paralelas y congruentes de un poliedro.

entre Para cualquier par de puntos *A* y *B* de una recta, existe un punto *C* ubicado entre *A* y *B* si y sólo si *A*, *B* y *C* son colineales y $AC + CB = AB$.

intermediación de puntos Ver *entre*.

bicondicional Conjunción entre un enunciado condicional y su recíproco.

C

center of circle (p. 697) The central point where radii form a locus of points called a circle.

center of dilation (p. 511) The center point from which dilations are performed.

center of rotation (p. 640) A fixed point around which shapes move in a circular motion to a new position.

center of symmetry (p. 664) See *point of symmetry*.

central angle (p. 706) An angle that intersects a circle in two points and has its vertex at the center of the circle.

centro de un círculo Punto central desde el cual los radios forman un lugar geométrico de puntos llamado círculo.

centro de la homotecia Punto fijo en torno al cual se realizan las homotecias.

centro de rotación Punto fijo alrededor del cual gira una figura hasta alcanzar una posición dada.

centro de la simetría Vea *el punto de simetría*.

ángulo central Ángulo que interseca un círculo en dos puntos y cuyo vértice está en el centro del círculo.

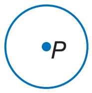
central angle of a regular polygon (p. 807) An angle that has its vertex at the center of a polygon and with sides that pass through consecutive vertices of the polygon.

centroid (p. 335) The point of concurrency of the medians of a triangle.

chord **1.** (p. 697) For a given circle, a segment with endpoints that are on the circle. **2.** (p. 880) For a given sphere, a segment with endpoints that are on the sphere.

chord segments (p. 750) Segments that form when two chords intersect inside a circle.

circle (p. 697) The locus of all points in a plane equidistant from a given point called the *center* of the circle.



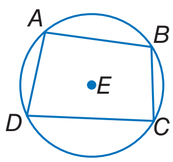
P is the center of the circle.

circular permutation (p. 925) A permutation of objects that are arranged in a circle or loop.

circumcenter (p. 325) The point of concurrency of the perpendicular bisectors of a triangle.

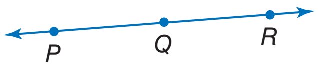
circumference (pp. 58, 699) The distance around a circle.

circumscribed (p. 700) A circle is circumscribed about a polygon if the circle contains all the vertices of the polygon.



⊙*E* is circumscribed about quadrilateral *ABCD*.

collinear (p. 5) Points that lie on the same line.



P, *Q*, and *R* are collinear.

combination (p. 926) An arrangement or listing in which order is not important.

common tangent (p. 732) A line or segment that is tangent to two circles in the same plane.

complement (p. 959) The complement of an event *A* consists of all the outcomes in the sample space that are not included as outcomes of event *A*.

complementary angles (p. 47) Two angles with measures that have a sum of 90.

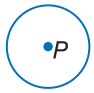
ángulo central de un polígono regular Ángulo cuyo vértice esta en el centro del polígono y cuyos lados pasan por vértices consecutivas del polígono.

baricentro Punto de intersección de las medianas de un triángulo.

cuerda **1.** Para cualquier círculo, segmento cuyos extremos están en el círculo. **2.** Para cualquier esfera, segmento cuyos extremos están en la esfera.

segmentos de cuerda Segmentos que se forman cuando dos cuerdas se intersecan dentro de un círculo.

círculo Lugar geométrico formado por todos los puntos en un plano, equidistantes de un punto dado llamado *centro* del círculo.



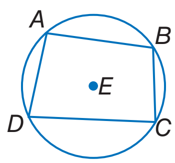
P es el centro del círculo.

permutación circular Permutación de objetos que se arreglan en un círculo o un bucle.

circuncentro Punto de intersección de las mediatrices de un triángulo.

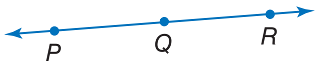
circunferencia Distancia alrededor de un círculo.

circunscrito Un polígono está circunscrito a un círculo si todos sus vértices están contenidos en el círculo.



⊙*E* está circunscrito al cuadrilátero *ABCD*.

colineal Puntos que yacen sobre la misma recta.



P, *Q* y *R* son colineales.

combinación Arreglo o lista en que el orden no es importante.

tangente común Recta o segmento de recta tangente a dos círculos en el mismo plano.

complemento El complemento de un evento *A* consiste en todos los resultados en el espacio muestral que no se incluyen como resultados del evento *A*.

ángulos complementarios Dos ángulos cuyas medidas suman 90.

component form (p. 602) A vector expressed as an ordered pair, ⟨change in x, change in y⟩.

composite figure (p. 809) A figure that can be separated into regions that are basic figures.

composite solid (p. 852) A three-dimensional figure that is composed of simpler figures.

composition of reflections (p. 652) Successive reflections in parallel lines.

composition of transformations (p. 651) The resulting transformation when a transformation is applied to a figure and then another transformation is applied to its image.

compound event (p. 947) An event that consists of two or more simple events.

compound statement (p. 99) A statement formed by joining two or more statements.

concave polygon (p. 56) A polygon for which there is a line containing a side of the polygon that also contains a point in the interior of the polygon.

concentric circles (p. 698) Coplanar circles with the same center.

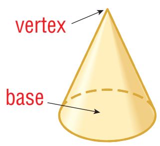
conclusion (p. 107) In a conditional statement, the statement that immediately follows the word *then*.

concurrent lines (p. 325) Three or more lines that intersect at a common point.

conditional probability (p. 949) The probability of an event under the condition that some preceding event has occurred.

conditional statement (p. 107) A statement that can be written in *if-then* form.

cone (p. 67) A solid with a circular base, a vertex not contained in the same plane as the base, and a lateral surface area composed of all points in the segments connecting the vertex to the edge of the base.



congruence transformations (p. 296) A mapping for which a geometric figure and its image are congruent.

congruent (pp. 16, 255) Having the same measure.

congruent arcs (p. 707) Arcs in the same circle or in congruent circles that have the same measure.

componente Vector expresado en forma de par ordenado, ⟨cambio en x, cambio en y⟩.

figura compuesta Figura que se puede separar en regiones formas de figuras básicas.

solido compuesto Figura tridimensional formada por figuras más simples.

composición de reflexiones Reflexiones sucesivas en rectas paralelas.

composición de transformaciones Transformación que resulta cuando se aplica una transformación a una figura y luego se le aplica otra transformación a su imagen.

evento compuesto Evento que consiste de dos o más eventos simples.

enunciado compuesto Enunciado formado por la unión de dos o más enunciados.

polígono cóncavo Polígono para el cual existe una recta que contiene un lado del polígono y un punto en el interior del polígono.

círculos concéntricos Círculos coplanarios con el mismo centro.

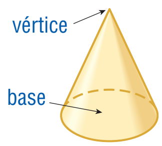
conclusión Parte de un enunciado condicional que está escrito justo después de la palabra *entonces*.

rectas concurrentes Tres o más rectas que se intersecan en un punto común.

probabilidad condicional La probabilidad de un acontecimiento bajo condición que ha ocurrido un cierto acontecimiento precedente.

enunciado condicional Enunciado escrito en la forma *si-entonces*.

cono Sólido de base circular cuyo vértice no yace en el mismo plano que la base y cuya área de superficie lateral está formada por todos los puntos en los segmentos que conectan el vértice con el bonde de la base.



transformaciones de congruencia Aplicación en la cual una figura geométrica y su imagen son congruentes.

congruente Que tienen la misma medida.

arcos congruentes Arcos que tienen la misma medida y que pertenecen al mismo círculo o a círculos congruentes.

congruent polygons (p. 255) Polygons in which all matching parts are congruent.

polígonos congruentes Polígonos cuyas partes correspondientes son todas congruentes.

congruent solids (p. 896) Two solids with the same shape, size and scale factor of 1:1.

sólidos congruentes Dos sólidos con la misma forma, tamaño y factor de escala de 1:1.

conic section (p. 764) Any figure that can be obtained by slicing a cone.

sección cónica Cualquier figura obtenida mediante el corte de un cono doble.

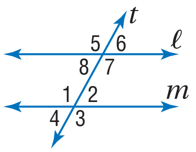
conjecture (p. 91) An educated guess based on known information.

conjetura Juicio basado en información conocida.

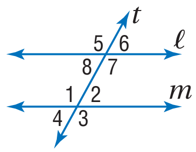
conjunction (p. 99) A compound statement formed by joining two or more statements with the word *and*.

conjunción Enunciado compuesto que se obtiene al unir dos o más enunciados con la palabra *y*.

consecutive interior angles (p. 174) In the figure, transversal t intersects lines ℓ and m. There are two pairs of consecutive interior angles: $\angle 8$ and $\angle 1$, and $\angle 7$ and $\angle 2$.



ángulos internos consecutivos En la figura, la transversal t interseca las rectas ℓ y m. La figura presenta dos pares de ángulos internos consecutivos; $\angle 8$ y $\angle 1$; y $\angle 7$ y $\angle 2$.



construction (p. 17) A method of creating geometric figures without the benefit of measuring tools. Generally, only a pencil, straightedge, and compass are used.

construcción Método para dibujar figuras geométricas sin el uso de instrumentos de medición. En general, sólo requiere de un lápiz, una regla y un compás.

contrapositive (p. 109) The statement formed by negating both the hypothesis and conclusion of the converse of a conditional statement.

antítesis Enunciado formado por la negación tanto de la hipótesis como de la conclusión del recíproco de un enunciado condicional.

converse (p. 109) The statement formed by exchanging the hypothesis and conclusion of a conditional statement.

recíproco Enunciado que se obtiene al intercambiar la hipótesis y la conclusión de un enunciado condicional dado.

convex polygon (p. 56) A polygon for which there is no line that contains both a side of the polygon and a point in the interior of the polygon.

polígono convexo Polígono para el cual no existe recta alguna que contenga un lado del polígono y un punto en el interior del polígono.

coordinate proofs (p. 303) Proofs that use figures in the coordinate plane and algebra to prove geometric concepts.

demostraciones en coordinadas Demostraciones que usan figuras en el plano de coordinados y álgebra para demostrar conceptos geométricos.

coplanar (p. 5) Points that lie in the same plane.

coplanar Puntos que yacen en el mismo plano.

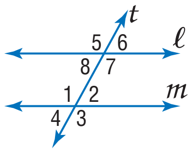
corner view (p. 839) The view from a corner of a three-dimensional figure, also called the *isometric view*.

vista de esquina Vista desde una de las esquinas de una figura tridimensional. También se conoce como *vista en perspectiva*.

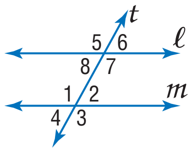
corollary (p. 249) A statement that can be easily proved using a theorem is called a corollary of that theorem.

corolario Un enunciado que se puede demostrar fácilmente usando un teorema se conoce como corolario de dicho teorema.

corresponding angles (p. 174) In the figure, transversal t intersects lines ℓ and m. There are four pairs of corresponding angles: $\angle 5$ and $\angle 1$, $\angle 8$ and $\angle 4$, $\angle 6$ and $\angle 2$, and $\angle 7$ and $\angle 3$.



ángulos correspondientes En la figura, la transversal t interseca las rectas ℓ y m. La figura muestra cuatro pares de ángulos correspondientes: $\angle 5$ y $\angle 1$, $\angle 8$ y $\angle 4$, $\angle 6$ y $\angle 2$; y $\angle 7$ y $\angle 3$.



corresponding parts (p. 255) Matching parts of congruent polygons.

partes correspondientes Partes que coinciden de polígonos congruentes.

cosecant (p. 578) The reciprocal of the sine of an angle in a right triangle.

cosecante Recíproco del seno de un ángulo en un triángulo rectangulo.

cosine (p. 568) For an acute angle of a right triangle, the ratio of the measure of the leg adjacent to the acute angle to the measure of the hypotenuse.

coseno Para cualquier ángulo agudo de un triángulo rectángulo, razón de la medida del cateto adyacente al ángulo agudo a la medida de la hipotenusa.

cotangent (p. 578) The ratio of the adjacent to the opposite side of a right triangle.

cotangente Razón de la medida del cateto adyacente a la medida de cateto opuesto de un triángulo rectángulo.

counterexample (p. 94) An example used to show that a given statement is not always true.

contraejemplo Ejemplo que se usa para demostrar que un enunciado dado no siempre es verdadero.

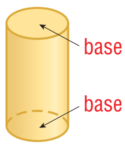
cross products (p. 462) In the proportion $\frac{a}{b} = \frac{c}{d}$, where $b \neq 0$ and $d \neq 0$, the cross products are ad and bc. The proportion is true if and only if the cross products are equal.

productos cruzados En la proporción $\frac{a}{b} = \frac{c}{d}$, donde $b \neq 0$ y $d \neq 0$, los productos cruzados son ad y bc. La proporción es verdadera si y sólo si los productos cruzados son iguales.

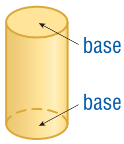
cross section (p. 840) The intersection of a solid and a plane.

sección transversal Intersección de un sólido con un plano.

cylinder (p. 67) A figure with bases that are formed by congruent circles in parallel planes.



cilindro Figura cuyas bases son círculos congruentes ubicados en planos paralelos.



D

deductive argument (p. 129) A proof formed by a group of algebraic steps used to solve a problem.

argumento deductivo Demostración que consta de un conjunto de pasos algebraicos que se usan para resolver un problema.

deductive reasoning (p. 117) A system of reasoning that uses facts, rules, definitions, or properties to reach logical conclusions.

razonamiento deductivo Sistema de razonamiento que emplea hechos, reglas, definiciones o propiedades para obtener conclusiones lógicas.

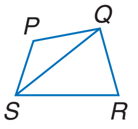
degree (p. 37) A unit of measure used in measuring angles and arcs. An arc of a circle with a measure of 1° is $\frac{1}{360}$ of the entire circle.

grado Unidad de medida que se usa para medir ángulos y arcos. El arco de un círculo que mide 1° equivale a $\frac{1}{360}$ del círculo completo.

dependent events (p. 947) Two or more events in which the outcome of one event affects the outcome of the other events.

eventos dependientes Dos o más eventos en que el resultado de un evento afecta el resultado de los otros eventos.

diagonal **(p. 393)** In a polygon, a segment that connects nonconsecutive vertices of the polygon.



$\overline{SQ}$ is a diagonal.

diameter **1. (p. 697)** In a circle, a chord that passes through the center of the circle. **2. (p. 880)** In a sphere, a segment that contains the center of the sphere, and has endpoints that are on the sphere.

dilation **(pp. 511, 674)** A transformation that enlarges or reduces the original figure proportionally. A dilation with center C and positive scale factor k, $k \neq 1$, is a function that maps a point P in a figure to its image such that

- if point P and C coincide, then the image and preimage are the same point, or
- if point P is not the center of dilation, then P' lies on $\overrightarrow{CP}$ and $CP' = k(CP)$.

If $k < 0$, P' is the point on the ray opposite $\overrightarrow{CP}$ such that $CP' = |k|(CP)$.

direct isometry **(p. 298)** An isometry in which the image of a figure is found by moving the figure intact within the plane.

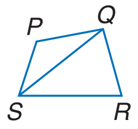
direction **(p. 600)** The measure of the angle that a vector forms with the positive x-axis or any other horizontal line.

directrix **(p. 765)** The fixed line in a parabola that is equidistant from the locus of all points in a plane.

disjunction **(p. 100)** A compound statement formed by joining two or more statements with the word *or*.

distance between two points **(p. 25)** The length of the segment between two points.

diagonal Recta que conecta vértices no consecutivos de un polígono.



$\overline{SQ}$ es una diagonal.

diámetro **1.** En un círculo cuerda que pasa por el centro. **2.** En una estera segmento que incluye el centro de la esfera y cuyos extremos están ubicados en la esfera.

homotecia Transformación que amplía o disminuye proporcionalmente el tamaño de una figura. Una homotecia con centro C y factor de escala positivo k, $k \neq 1$, es una función que aplica un punto P a su imagen, de modo que si el punto P coincide con el punto C, entonces la imagen y la preimagen son el mismo punto, o si el punto P no es el centro de la homotecia, entonces P' yace sobre $\overrightarrow{CP}$ y $CP' = k(CP)$. Si $k < 0$, P' es el punto sobre el rayo opuesto a $\overrightarrow{CP}$, tal que $CP' = |k|(CP)$.

isometría directa Isometría en la cual se obtiene la imagen de una figura, al mover la figura intacta dentro del plano.

dirección Medida del ángulo que forma un vector con el eje x positivo o con cualquier otra recta horizontal.

directriz Línea fija en una parábola que está equidistante del lugar geométrico de todos los puntos en un plano.

disyunción Enunciado compuesto que se forma al unir dos o más enunciados con la palabra *o*.

distancia entre dos puntos Longitud del segmento entre dos puntos.

E

edge **(p. 964)** A line that connects two nodes in a network.

edge of a polyhedron **(p. 67)** A line segment where the faces of a polyhedron intersect.

efficient route **(p. 965)** The path in a network with the least weight.

enlargement **(p. 511)** An image that is larger that the original figure.

equiangular polygon **(p. 57)** A polygon with all congruent angles.

arista Recta que conecta dos nodos en una red.

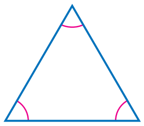
arista de un poliedro Segmento de recta donde se intersecan las caras de un poliedro.

ruta eficiente Ruta en una red con el menor peso.

ampliación Imagen que es más grande que la figura original.

polígono equiangular Polígono cuyos ángulos son todos congruentes.

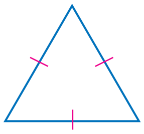
equiangular triangle (p. 237) A triangle with all angles congruent.



equidistant (p. 218) The distance between two lines measured along a perpendicular line is always the same.

equilateral polygon (p. 57) A polygon with all congruent sides.

equilateral triangle (p. 238) A triangle with all sides congruent.



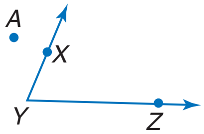
equivalent vectors (p. 601) Vectors that have the same magnitude and direction.

Euclidean geometry (p. 889) A geometrical system in which a plane is a flat surface made up of points that extend infinitely in all directions.

expected value (p. 941) Also *mathematical expectation*, is the average value of a random variable that one expects after repeating an experiment or simulation an infinite number of times.

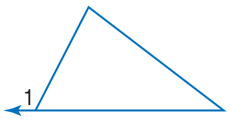
extended ratios (p. 461) Ratios that are used to compare three or more quantities.

exterior (p. 36) A point is in the exterior of an angle if it is neither on the angle nor in the interior of the angle.



A is in the exterior of $\angle XYZ$.

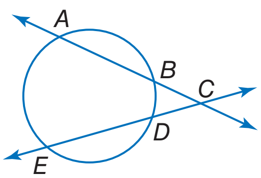
exterior angle (p. 248) An angle formed by one side of a triangle and the extension of another side.



$\angle 1$ is an exterior angle.

exterior angles (p. 174) An angle that lies in the region that is not between two transversals that intersect the same line.

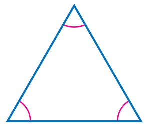
external secant segment (p. 752) A secant segment that lies in the exterior of the circle.



$\overline{BC}$ and $\overline{CD}$ are external secant segments.

extremes (p. 462) In $\frac{a}{b} = \frac{c}{d}$, the numbers *a* and *d*.

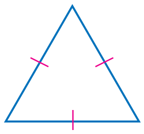
triángulo equiangular Triángulo cuyos ángulos son todos congruentes.



equidistante La distancia entre dos rectas que siempre permanece constante cuando se mide a lo largo de una perpendicular.

polígono equilátero Polígono cuyos lados son todos congruentes.

triángulo equilátero Triángulo cuyos lados son todos congruentes.



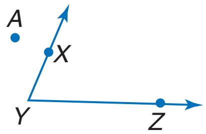
vectores iguales Vectores con la misma magnitud y dirección.

geometría euclidiana Sistema en el cual un plano es una superficie plana formada por puntos que se extienden infinitamente en todas las direcciones.

valor esperado También *expectativa matemática*, el valor promedio de una variable aleatoria que uno *espera* después de repetir un experimento o un simulacro un número infinito de veces.

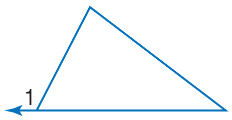
razones extendmientes Razones que se utilizan para comparar tres o más cantidades.

exterior Un punto yace en el exterior de un ángulo si no se ubica ni en el ángulo ni en el interior del ángulo.



A está en el exterior del $\angle XYZ$.

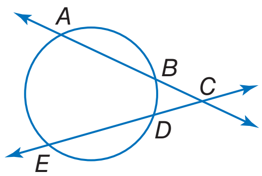
ángulo externo Ángulo formado por un lado de un triángulo y la prolongación de otro de sus lados.



$\angle 1$ es un ángulo externo.

ángulos externos Un ángulo que está en la región que no está entre dos transversales que cruzan la misma línea.

segmento secante externo Segmento secante que yace en el exterior del círculo.



$\overline{BC}$ y $\overline{CD}$ son segmentos secantes externos.

extremos Los números *a* y *d* en $\frac{a}{b} = \frac{c}{d}$.

F

face of a polyhedron (p. 67) A flat surface of a polyhedron.

cara de un poliedro Superficie plana de un poliedro.

factorial (p. 922) The product of the integers less than or equal to a positive integer *n*, written as *n*!

factorial Producto de los enteros menores o iguales a un número positivo *n*, escrito como *n*!

finite plane (p. 10) A plane that has boundaries or does not extend indefinitely.

plano finito Plano que tiene límites o que no se extiende indefinidamente.

flow proof (p. 248) A proof that organizes statements in logical order, starting with the given statements. Each statement is written in a box with the reason verifying the statement written below the box. Arrows are used to indicate the order of the statements.

demostración de flujo Demostración que organiza los enunciados en orden lógico, comenzando con los enunciados dados. Cada enunciado se escribe en una casilla y debajo de cada casilla se escribe el argumento que verifica dicho enunciado. El orden de los enunciados se indica con flechas.

focus (p. 765) The fixed point in a parabola that is equidistant from the locus of all points in a plane.

foco Punto fijo en una parábola que está equidistante del lugar geométrico de todos los puntos en un plano.

formal proof (p. 137) A two-column proof containing statements and reasons.

demonstración formal Demonstración en dos columnas que contiene enunciados y razonamientos.

fractal (p. 509) A figure generated by repeating a special sequence of steps infinitely often. Fractals often exhibit self-similarity.

fractal Figura que se obtiene mediante la repetición infinita de una sucesión particular de pasos. Los fractales a menudo exhiben autosemejanza.

frustum (p. 861) The part of a solid that remains after the top portion has been cut by a plane parallel to the base.

tronco Parte de un sólido que queda después de que la parte superior ha sido cortada por un plano paralelo a la base.

Fundamental Counting Principle (p. 917) A method used to determine the number of possible outcomes in a sample space by multiplying the number of possible outcomes from each stage or event.

principio fundamental de contar Método para determinar el número de resultados posibles en un espacio muestral multiplicando el número de resultados posibles de cada etapa o evento.

G

geometric mean (p. 537) For any positive numbers *a* and *b*, the positive number *x* such that $\frac{a}{x} = \frac{x}{b}$.

media geométrica Para todo número positivo *a* y *b*, existe un número positivo *x* tal que $\frac{a}{x} = \frac{x}{b}$.

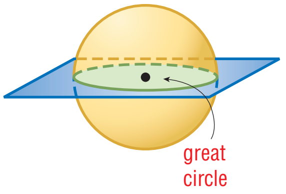
geometric probability (p. 931) Using the principles of length and area to find the probability of an event.

probabilidad geométrica Uso de los principios de longitud y área para calcular la probabilidad de un evento.

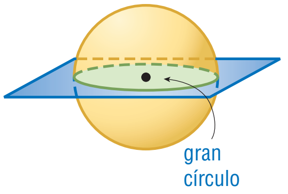
glide reflection (p. 651) The composition of a translation followed by a reflection in a line parallel to the translation vector.

reflexión del deslizamiento Composición de una traslación seguida por una reflexión en una recta paralela al vector de la traslación.

great circle (p. 881) A circle formed when a plane intersects a sphere with its center at the center of the sphere.



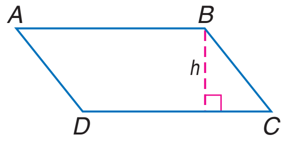
círculo mayor Círculo que se forma cuando un plano interseca una esfera y cuyo centro es el mismo que el centro de la esfera.



Glossary/Glosario

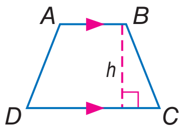
H

height of a parallelogram (p. 779) The length of an altitude of a parallelogram.



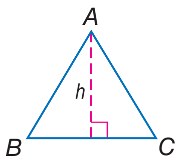
h is the height of parallelogram *ABCD*.

height of a trapezoid (p. 789) The perpendicular distance between the bases of a trapezoid.



h is the height of trapezoid *ABCD*.

height of a triangle (p. 781) The length of an altitude drawn to a given base of a triangle.

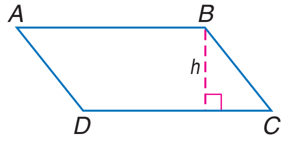


h is the height of triangle *ABC*.

hemisphere (p. 881) One of the two congruent parts into which a great circle separates a sphere.

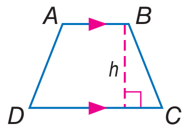
hypothesis (p. 107) In a conditional statement, the statement that immediately follows the word *if*.

altura de un paralelogramo Longitud del segmento perpendicular que va desde la base hasta el vértice opuesto a ella.



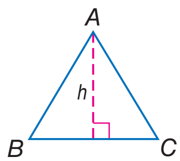
h es la altura del paralelogramo *ABCD*.

altura de un trapecio Distancia perpendicular entre las bases de un trapecio.



h es la altura del trapecio *ABCD*.

altura de un triángulo Longitud de una altura trazada a una base dada de un triángulo.



h es la altura de triángulo *ABC*.

hemisferio Una de las dos partes congruentes en las cuales un círculo mayor divide una esfera.

hipótesis Enunciado escrito inmediatamente después de la palabra *si* en un enunciado condicional.

I

if-then statement (p. 107) A compound statement of the form "if *p*, then *q*," where *p* and *q* are statements.

image (p. 296) A figure that results from the transformation of a geometric figure.

incenter (p. 328) The point of concurrency of the angle bisectors of a triangle.

included angle (p. 266) In a triangle, the angle formed by two sides is the included angle for those two sides.

included side (p. 275) The side of a polygon that is a side of each of two angles.

enunciado si-entonces Enunciado compuesto de la forma "si *p*, entonces *q*," donde *p* y *q* son enunciados.

imagen Figura que resulta de la transformación de una figura geométrica.

incentro Punto de intersección de las bisectrices interiores de un triángulo.

ángulo incluido En un triángulo, el ángulo formado por dos lados es el ángulo incluido de esos dos lados.

lado incluido Lado de un polígono común a dos de sus ángulos.

independent events (p. 947) Two or more events in which the outcome of one event does not affect the outcome of the other events.

eventos independientes El resultado de un evento no afecta el resultado del otro evento.

indirect isometry (p. 298) An isometry that cannot be performed by maintaining the orientation of the points, as in a direct isometry.

isometría indirecta Tipo de isometría que no se puede obtener manteniendo la orientación de los puntos, como ocurre con la isometría directa.

indirect proof (p. 355) In an indirect proof, one assumes that the statement to be proved is false. One then uses logical reasoning to deduce that a statement contradicts a postulate, theorem, or one of the assumptions. Once a contradiction is obtained, one concludes that the statement assumed false must in fact be true.

demostración indirecta En una demostración indirecta, se supone que el enunciado a demostrar es falso. Después, se deduce lógicamente que existe un enunciado que contradice un postulado, un teorema o una de las conjeturas. Una vez hallada una contradicción, se concluye que el enunciado que se suponía falso debe ser, en realidad, verdadero.

indirect reasoning (p. 355) Reasoning that assumes that the conclusion is false and then shows that this assumption leads to a contradiction of the hypothesis like a postulate, theorem, or corollary. Then, since the assumption has been proved false, the conclusion must be true.

razonamiento indirecto Razonamiento en que primero se supone que la conclusión es falsa y luego se demuestra que esta conjetura lleva a una contradicción de la hipótesis como un postulado, un teorema o un corolario. Finalmente, como se ha demostrado que la conjetura es falsa, la conclusión debe ser verdadera.

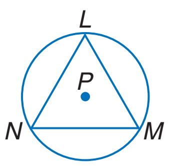
inductive reasoning (p. 91) Reasoning that uses a number of specific examples to arrive at a plausible generalization or prediction. Conclusions arrived at by inductive reasoning lack the logical certainty of those arrived at by deductive reasoning.

razonamiento inductivo Razonamiento que usa varios ejemplos específicos para lograr una generalización o una predicción pausible. Las conclusiones obtenidas por razonamiento inductivo carecen de la certeza lógica de aquellas obtenidas por razonamiento deductivo.

informal proof (p. 129) A paragraph proof.

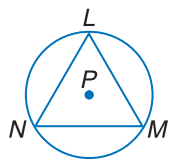
demostración informal Demostración en forma de párrafo.

inscribed (p. 700) A polygon is inscribed in a circle if each of its vertices lie on the circle.



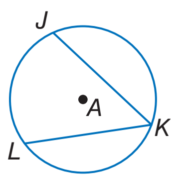
$\triangle LMN$ is inscribed in $\odot P$.

inscrito Un polígono está inscrito en un círculo si todos sus vértices yacen en el círculo.



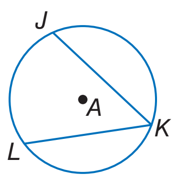
$\triangle LMN$ está inscrito en $\odot P$.

inscribed angle (p. 723) An angle that has a vertex on a circle and sides that contain chords of the circle.



In $\odot A$, $\angle JKL$ is an inscribed angle.

ángulo inscrito Ángulo cuyo vértice esté en un círculo y cuyos lados contienen cuerdas del círculo.



En $\odot A$, $\angle JKL$ es un ángulo inscrito.

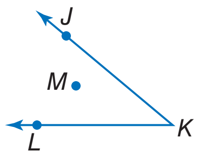
intercepted arc (p. 723) An angle intercepts an arc if and only if each of the following conditions are met.

1. The endpoints of the arc lie on the angle.
2. All points of the arc except the endpoints are in the interior of the circle.
3. Each side of the angle has an endpoint of the arc.

arco intersecado Un ángulo interseca un arco si y sólo si se cumple cada una de las siguientes condiciones.

1. Los extremos del arco yacen en el ángulo.
2. Todos los puntos del arco, excepto los extremos, yacen en el interior del círculo.
3. Cada lado del ángulo tiene un extremo del arco.

interior (p. 36) A point is in the interior of an angle if it does not lie on the angle itself and it lies on a segment with endpoints that are on the sides of the angle.



M is in the interior of $\angle JKL$.

interior angles (p. 174) Angles that lie between two transversals that intersect the same line.

intersection (p. 6) A set of points common to two or more geometric figures.

inverse (p. 109) The statement formed by negating both the hypothesis and conclusion of a conditional statement.

inverse cosine (p. 571) The inverse function of cosine, or $\cos^{-1}$. If the cosine of an acute $\angle A$ is equal to x, then $\cos^{-1} x$ is equal to the measure of $\angle A$.

inverse sine (p. 571) The inverse function of sine, or $\sin^{-1}$. If the sine of an acute $\angle A$ is equal to x, then $\sin^{-1} x$ is equal to the measure of $\angle A$.

inverse tangent (p. 571) The inverse function of tangent, or $\tan^{-1}$. If the tangent of an acute $\angle A$ is equal to x, then $\tan^{-1} x$ is equal to the measure of $\angle A$.

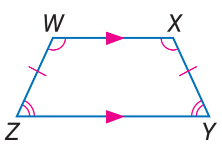
irrational number (p. 26) A number that cannot be expressed as a terminating or repeating decimal.

irregular figure (p. 57) A polygon with sides and angles that are not all congruent.

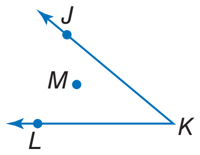
isometric view (p. 839) Corner views of three-dimensional objects on two-dimensional paper.

isometry (p. 296) A mapping for which the original figure and its image are congruent.

isosceles trapezoid (p. 439) A trapezoid in which the legs are congruent, both pairs of base angles are congruent, and the diagonals are congruent.



interior Un punto se encuenta en el interior de un ángulo si no yace en el ángulo como tal y si está en un segmento cuyos extremos están en los lados del ángulo.



M está en el interior del $\angle JKL$.

ángulos interiores Ángulos que yacen entre dos transversales que intersecan la misma recta.

intersección Conjunto de puntos comunes a dos o más figuras geométricas.

inverso Enunciado que se obtiene al negar tanto la hipótesis como la conclusión de un enunciado condicional.

inverso del coseno Función inversa del coseno, o $\cos^{-1}$. Si el coseno de un $\angle A$ agudo es igual a x, entonces $\cos^{-1} x$ es igual a la medida del $\angle A$.

inverso del seno Función inversa del seno, o $\sin^{-1}$. Si el seno de un $\angle A$ agudo es igual a x, entonces $\sin^{-1} x$ es igual a la medida del A.

inverse del tangente Función inversa de la tangente, o $\tan^{-1}$. Si la tangente de un $\angle A$ agudo es igual a x, entonces $\tan^{-1} x$ es igual a la medida del $\angle A$.

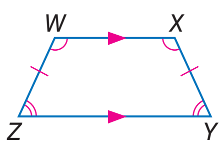
número irracional Número que no se puede expresar como un decimal terminal o periódico.

figura irregular Polígono cuyos lados y ángulos no son todo congruentes.

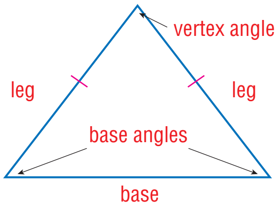
vista isométrica Vistas de las esquinas de sólidos geométricos tridimensionales sobre un papel bidimensional.

isometría Aplicación en la cual la figura original y su imagen son congruentes.

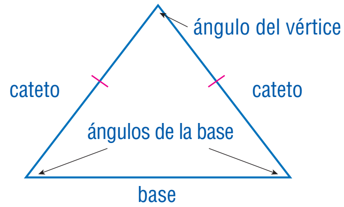
trapecio isósceles Trapecio cuyos catetos son congruentes, ambos pares de ángulos de las bases son congruentes y las diagonales son congruentes.



isosceles triangle (pp. 238, 285) A triangle with at least two sides congruent. The congruent sides are called *legs*. The angles opposite the legs are *base angles*. The angle formed by the two legs is the *vertex angle*. The side opposite the vertex angle is the *base*.



triángulo isósceles Triángulo que tiene por lo menos dos lados congruentes. Los lados congruentes se llaman *catetos*. Los ángulos opuestos a los catetos son los *ángulos de la base*. El ángulo formado por los dos catetos es el *ángulo del vértice*. El lado opuesto al ángulo del vértice es la *base*.



iteration (p. 509) A process of repeating the same procedure over and over again.

iteración Proceso de repetir el mismo procedimiento una y otra vez.

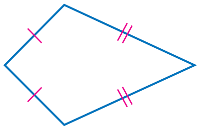
J

joint frequencies (p. 954) In a two-way frequency table, the frequencies reported in the cells in the interior of the table.

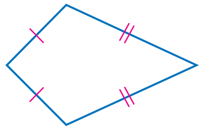
frecuencias conjuntas En una tabla de double entrada o de frecuencias, las frecuencias reportadas en las celdas en el interior de la tabla.

K

kite (p. 442) A quadrilateral with exactly two distinct pairs of adjacent congruent sides.



cometa Cuadrilátero que tiene exactamente dos pares differentes de lados congruentes y adyacentes.



L

lateral area (p. 846) For prisms, pyramids, cylinders, and cones, the area of the faces of the figure not including the bases.

área lateral En prismas, pirámides, cilindros y conos, es el área de la caras de la figura sin incluir el área de las bases.

lateral edges **1.** (p. 846) In a prism, the intersection of two adjacent lateral faces. **2.** (p. 854) In a pyramid, lateral edges are the edges of the lateral faces that join the vertex to vertices of the base.

aristas laterales **1.** En un prisma, la intersección de dos caras laterales adyacentes. **2.** En una pirámide, las aristas de las caras laterales que unen el vértice de la pirámide con los vértices de la base.

lateral faces **1.** (p. 846) In a prism, the faces that are not bases. **2.** (p. 854) In a pyramid, faces that intersect at the vertex.

caras laterales **1.** En un prisma, las caras que no forman las bases. **2.** En una pirámide, las caras que se intersecan en el vértice.

latitude (p. 895) A measure of distance north or south of the equator.

latitud Medida de la distancia al norte o al sur del ecuador.

Law of Cosines (p. 589) Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of sides opposite the angles with measures A, B, and C respectively. Then the following equations are true.

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

ley de los cosenos Sea $\triangle ABC$ cualquier triángulo donde a, b y c son las medidas de los lados opuestos a los ángulos que miden A, B y C respectivamente. Entonces las siguientes ecuaciones son verdaderas.

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

Law of Detachment (p. 117) If $p \rightarrow q$ is a true conditional and p is true, then q is also true.

Law of Large Numbers (p. 942) Law that states that as the number of trials of a random process increases, the average value will approach the expected value.

Law of Sines (p. 588) Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of sides opposite the angles with measures A, B, and C respectively.

Then, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

Law of Syllogism (p. 119) If $p \rightarrow q$ and $q \rightarrow r$ are true conditionals, then $p \rightarrow r$ is also true.

legs of a right triangle (p. 26) The shorter sides of a right triangle.

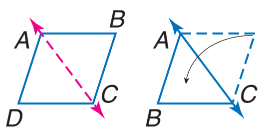
legs of a trapezoid (p. 439) The nonparallel sides of a trapezoid.

legs of an isosceles triangle (p. 285) The two congruent sides of an isosceles triangle.

line (p. 5) A basic undefined term of geometry. A line is made up of points and has no thickness or width. In a figure, a line is shown with an arrowhead at each end. Lines are usually named by lowercase script letters or by writing capital letters for two points on the line, with a double arrow over the pair of letters.

line of reflection (p. 623) A line in which each point on the preimage and its corresponding point on the image are the same distance from this line.

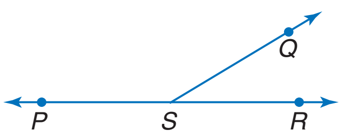
line of symmetry (p. 663) A line that can be drawn through a plane figure so that the figure on one side is the reflection image of the figure on the opposite side.



$\overleftrightarrow{AC}$ is a line of symmetry.

line segment (p. 14) A measurable part of a line that consists of two points, called endpoints, and all of the points between them.

linear pair (p. 46) A pair of adjacent angles whose non-common sides are opposite rays.



$\angle PSQ$ and $\angle QSR$ are a linear pair.

ley de indiferencia Si $p \rightarrow q$ es un enunciado condicional verdadero y p es verdadero, entonces q también es verdadero.

ley de los grandes números Ley que establece que a medida que aumenta el número de ensayos de un proceso aleatorio, el valor promedio se aproximará al valor esperado.

ley de los senos Sea $\triangle ABC$ cualquier triángulo donde a, b y c representan las medidas de los lados opuestos a los ángulos que miden A, B y C respectivamente.

Entonces, $\frac{\text{scn } A}{a} = \frac{\text{scn } B}{b} = \frac{\text{scn } C}{c}$.

ley del silogismo Si $p \rightarrow q$ y $q \rightarrow r$ son enunciados condicionales verdaderos, entonces $p \rightarrow r$ también es verdadero.

catetos de un triángulo rectángulo Lados más cortos de un triángulo rectángulo.

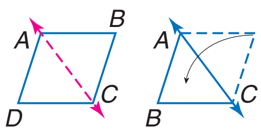
catetos de un trapecio Los lados no paralelos de un trapecio.

catetos de un triángulo isósceles Las dos lados congruentes de un triángulo isósceles.

recta Término geometrico basico no definido. Una recta está formada por puntos y carece de grosor o ancho. En una figura, una recta se representa con una flecha en cada extremo. Generalmente se designan con letras minúsculas o con las dos letras mayúsculas de dos puntos sobre la recta y una flecha doble sobre el par de letras.

línea de reflexión Una línea en la cual cada punto en el preimage y el su corresponder senalañ en la imagen es la misma distancia de esta línea.

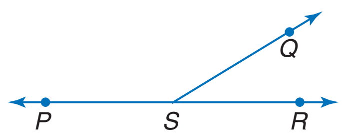
eje de simetría Recta que se traza a través de una figura plana, de modo que un lado de la figura es la imagen reflejada del lado opuesto.



$\overleftrightarrow{AC}$ es un eje de simetría.

segmento de recta Sección medible de una recta que consta de dos puntos, llamados extremos, y todos los puntos entre ellos.

par lineal Par de ángulos adyacentes cuyos lados no comunes forman rayos opuestos.



$\angle PSQ$ y $\angle QSR$ forman un par lineal.

locus (p. 11) The set of points that satisfy a given condition.

lugar geométrico Conjunto de puntos que satisfacen una condición dada.

logically equivalent (p. 110) Statements that have the same truth values.

lógicamente equivalentes Enunciados que poseen los mismos valores verdaderos.

longitude (p. 895) A measure of distance east or west of the Prime Meridian.

longitud Medida de la distancia del este o al oeste del Primer Meridiano.

M

magnitude (p. 600) The length of a vector.

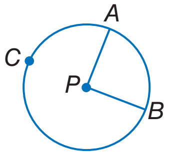
magnitud Longitud de un vector.

magnitude of symmetry (p. 664) The smallest angle through which a figure can be rotated so that it maps onto itself.

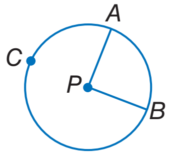
magnitud de la simetria El angulo mas pequeno con el cual una figura puede serrotada de modo que traz sobre si mismo.

major arc (p. 707) An arc with a measure greater than 180.

$\overset{\frown}{ACB}$ is a major arc.



arco mayor Arco que mide más de 180. $\overset{\frown}{ACB}$ es un arco mayor.



marginal frequencies (p. 954) In a two-way frequency table, the accumulated frequencies reported in the Totals row and Totals column.

frecuencias marginales En una tabla de double entrada o de frecuencias, las frecuencias acumuladas que se reportan en la hilera de los totales y en la columna de los totales.

matrix logic (p. 353) A rectangular array in which learned clues are recorded in order to solve a logic or reasoning problem.

lógica matricial Arreglo rectangular en que las claves aprendidas se escriben en orden para resolver un problema de lógica o razonamiento.

means (p. 462) In $\frac{a}{b} = \frac{c}{d}$, the numbers b and c.

medias Los números b y c en la proporción $\frac{a}{b} = \frac{c}{d}$.

median (p. 335) In a triangle, a line segment with endpoints that are a vertex of a triangle and the midpoint of the side opposite the vertex.

mediana En un triángulo, Segmento de recta de cuyos extremos son un vértice del triángulo y el punto medio del lado opuesto a dicho vértice.

meridians (p. 895) Imaginary vertical lines drawn around the Earth through the North and South Poles.

meridianos Líneas verticales imaginarias dibujadas alrededor de la Tierra que von del polo norte al polo sur.

midpoint (p. 27) The point on a segment exactly halfway between the endpoints of the segment.

punto medio Punto en un segmento que yace exactamente en la mitad, entre los extremos del segmento.

midsegment of trapezoid (p. 441) A segment that connects the midpoints of the legs of a trapezoid.

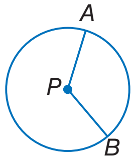
segmento medio de un trapecio Segmento que conecta los puntos medios de los catetos de un trapecio.

midsegment of triangle (p. 491) A segment with endpoints that are the midpoints of two sides of a triangle.

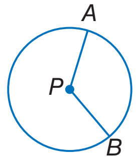
segmento medio de un triángulo Segmento cuyas extremos son los puntos medianos de dos lados de un triángulo.

minor arc (p. 707) An arc with a measure less than 180.

$\overset{\frown}{AB}$ is a minor arc.



arco menor Arco que mide menos de 180. $\overset{\frown}{AB}$ es un arco menor.



multi-stage experiments (p. 916) Experiments with more than two stages.

experimentos multietápicos Experimentos con más de dos etapas.

mutually exclusive (p. 956) Two events that have no outcomes in common.

mutuamente exclusivos Eventos que no tienen resultados en común.

N

negation (p. 99) If a statement is represented by p, then *not* p is the negation of the statement.

negación Si p representa un enunciado, entonces *no* p es la negación del enunciado.

net (p. 76) A two-dimensional figure that when folded forms the surfaces of a three-dimensional object.

red Figura bidimensional que al ser plegada forma las superficies de un objeto tridimensional.

network (p. 964) A graph of interconnected vertices.

red Gráfico de vértices interconectados.

***n*-gon** (p. 57) A polygon with n sides.

***en*ágono** Polígono con n lados.

node (p. 964) A collection of vertices.

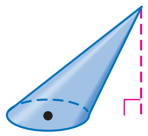
nodo Colección de vértices.

non-Euclidean geometry (p. 890) The study of geometrical systems that are not in accordance with the Parallel Postulate of Euclidean geometry.

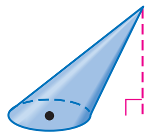
geometría no euclidiana El estudio de sistemas geométricos que no satisfacen el postulado de las paralelas de la geometría euclidiana.

O

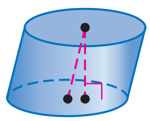
oblique cone (p. 856) A cone that is not a right cone.



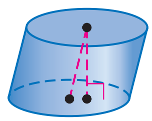
cono oblicuo Cono que no es un cono recto.



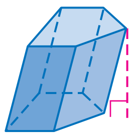
oblique cylinder (p. 848) A cylinder that is not a right cylinder.



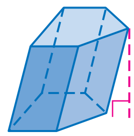
cilindro oblicuo Cilindro que no es un cilindro recto.



oblique prism (p. 846) A prism in which the lateral edges are not perpendicular to the bases.



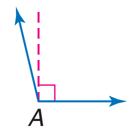
prisma oblicuo Prisma cuyas aristas laterales no son perpendiculares a las bases.



oblique solid (p. 838) A solid with base(s) that are not perpendicular to the edges connecting the two bases or vertex.

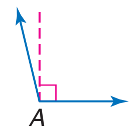
sólido oblicuo Sólido con base o bases que no son perpendiculares a las aristas, las cuales conectan las dos bases o vértice.

obtuse angle (p. 38) An angle with degree measure greater than 90 and less than 180.



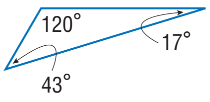
$90 < m\angle A < 180$

ángulo obtuso Ángulo que mide más de 90 y menos de 180.



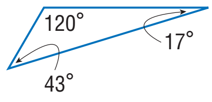
$90 < m\angle A < 180$

obtuse triangle (p. 237) A triangle with an obtuse angle.



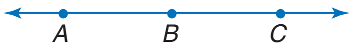
one obtuse angle

triángulo obtusángulo Triángulo con un ángulo obtuso.

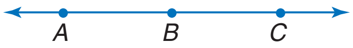


un ángulo obtuso

opposite rays (p. 36) Two rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$ such that B is between A and C.



rayos opuestos Dos rayos $\overrightarrow{BA}$ y $\overrightarrow{BC}$ donde B esta entre A y C.



opposite vectors (p. 601) Vectors that have the same magnitude but opposite direction.

vectores opuestos Vectores que tienen la misma magnitud pero enfrente de la dirección.

order of symmetry (p. 664) The number of times a figure can map onto itself as it rotates from 0° to 360°.

orden de la simetría Número de veces que una figura se puede aplicar sobre sí misma mientras gira de 0° a 360°.

ordered triple (p. 556) Three numbers given in a specific order used to locate points in space.

triple ordenado Tres números dados en un orden específico que sirven para ubicar puntos en el espacio.

orthocenter (p. 337) The point of concurrency of the altitudes of a triangle.

ortocentro Punto de intersección de las alturas de un triángulo.

orthographic drawing (p. 75) The two-dimensional top view, left view, front view, and right view of a three-dimensional object.

proyección ortogonal Vista bidimensional superior, del lado izquierda, frontal y del lado derecho de un objeto tridimensional.

P

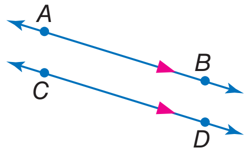
parabola (p. 764) The graph of a quadratic function. The set of all points in a plane that are the same distance from a given point, called the focus, and a given line, called the directrix.

parábola La grafica de una funcion cuadratica. Conjunto de todos los puntos de un plano que estan a la misma distancia de un punto dado, llamado foco, y de una recta dada, llamada directriz.

paragraph proof (p. 129) An informal proof written in the form of a paragraph that explains why a conjecture for a given situation is true.

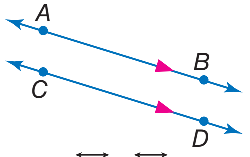
demostración de párrafo Demostración informal escrita en párrafo que explica por qué una conjetura para una situación dada es verdadera.

parallel lines (p. 173) Coplanar lines that do not intersect.



$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

rectas paralelas Rectas coplanares que no se intersecan.



$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

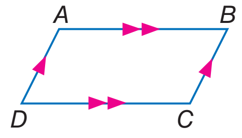
parallel planes (p. 173) Planes that do not intersect.

planos paralelos Planos que no se intersecan.

parallel vectors (p. 601) Vectors that have the same or opposite direction.

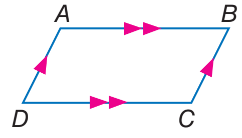
vectores paralelos Vectores con la misma dirección o dirección opuesta.

parallelogram (p. 403) A quadrilateral with parallel opposite sides. Any side of a parallelogram may be called a *base*.



$\overline{AB} \parallel \overline{DC}$; $\overline{AD} \parallel \overline{BC}$

paralelogramo Cuadrilátero cuyos lados opuestos son paralelos y cuya *base* puede ser cualquier de sus lados.



$\overline{AB} \parallel \overline{DC}$; $\overline{AD} \parallel \overline{BC}$

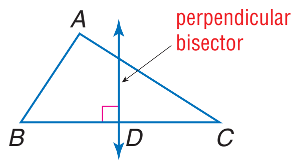
parallelogram method (p. 601) A method used to find the resultant of two vectors in which you place the vectors at the same initial point, complete a parallelogram, and draw the diagonal.

parallels (p. 895) Imaginary horizontal lines parallel to the equator.

perimeter (p. 58) The sum of the lengths of the sides of a polygon.

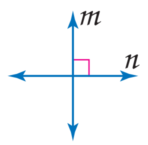
permutation (p. 922) An arrangement of objects in which order is important.

perpendicular bisector (p. 324) In a triangle, a line, segment, or ray that passes through the midpoint of a side and is perpendicular to that side.



D is the midpoint of $\overline{BC}$.

perpendicular lines (p. 48) Lines that form right angles.



line $m \perp$ line n

pi (π) (p. 699) An irrational number represented by the ratio of the circumference of a circle to the diameter of the circle.

plane (p. 5) A basic undefined term of geometry. A plane is a flat surface made up of points that has no depth and extends indefinitely in all directions. In a figure, a plane is often represented by a shaded, slanted four-sided figure. Planes are usually named by a capital script letter or by three noncollinear points on the plane.

plane Euclidean geometry (p. 889) Geometry based on Euclid's axioms dealing with a system of points, lines, and planes.

plane symmetry (p. 665) Symmetry in a three- dimensional figure that occurs if the figure can be mapped onto itself by a reflection in a plane.

Platonic solids (p. 68) The five regular polyhedra: tetrahedron, hexahedron, octahedron, dodecahedron, or icosahedron.

point (p. 5) A basic undefined term of geometry. A point is a location. In a figure, points are represented by a dot. Points are named by capital letters.

point of concurrency (p. 325) The point of intersection of concurrent lines.

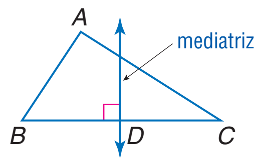
método del paralelogramo Método que se usa para hallar la resultante de dos vectores en que se dibujan los vectores con el mismo punto de origen, se completa un paralelogramo y se traza la diagonal.

paralelos Rectas horizontales imaginarias paralelas al ecuador.

perímetro Suma de la longitud de los lados de un polígono.

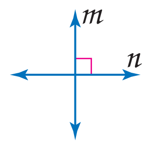
permutación Disposicion de objetos en la cual el orden es importante.

mediatriz Recta, segmento de recta o rayo perpendicular que corta un lado del triángulo en su punto medio.



D es el punto medio de $\overline{BC}$.

rectas perpendiculares Rectas que forman ángulos rectos.



recta $m \perp$ recta n

pi (π) Número irracional representado por la razón de la circunferencia de un círculo al diámetro del mismo.

plano Término geométrico básico no definido. Superficie plana sin espesor formada por puntos y que se extiende hasta el infinito en todas direcciones. En una figura, los planos a menudo se representan con una figura inclinada y sombreada y se designan con una letra mayúscula o con tres puntos no colineales del plano.

geometría del plano euclidiano Geometría basada en los axiomas de Euclides, los cuales abarcan un sistema de puntos, rectas y planos.

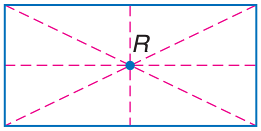
simetría plana Simetría en una figura tridimensional que ocurre si la figura se puede aplicar sobre sí misma mediante una reflexión en el plano.

sólidos platónicos Los cinco poliedros regulares siguientes: tetraedro, hexaedro, octaedro, dodecaedro e icosaedro.

punto Término geométrico básico no definido. Un punto representa un lugar o ubicación. En una figura, se representa con una marca puntual y se designan con letras mayúsculas.

punto de concurrencia Punto de intersección de rectas concurrentes.

point of symmetry (p. 664) A figure that can be mapped onto itself by a rotation of 180°.



R is a point of symmetry.

point of tangency (p. 732) For a line that intersects a circle in only one point, the point at which they intersect.

point-slope form (p. 198) An equation of the form $y - y_1 = m(x - x_1)$, where (x_1, y_1) are the coordinates of any point on the line and *m* is the slope of the line.

poles (p. 881) The endpoints of the diameter of a great circle.

polygon (p. 56) A closed figure formed by a finite number of coplanar segments called *sides* such that the following conditions are met:

1. The sides that have a common endpoint are noncollinear.
2. Each side intersects exactly two other sides, but only at their endpoints, called the *vertices*.

polyhedrons (p. 67) Closed three-dimensional figures made up of flat polygonal regions. The flat regions formed by the polygons and their interiors are called *faces*. Pairs of faces intersect in segments called *edges*. Points where three or more edges intersect are called *vertices*.

population density (p. 797) A measurement of population per unit of area.

postulate (p. 127) A statement that describes a fundamental relationship between the basic terms of geometry. Postulates are accepted as true without proof.

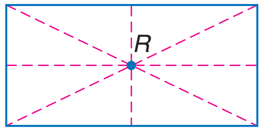
preimage (p. 296) The graph of an object before a transformation.

principle of superposition (p. 682) Two figures are congruent if and only if there is a rigid motion or a series of rigid motions that maps one figure exactly onto the other.

prism (p. 67) A solid with the following characteristics:

1. Two faces, called *bases*, are formed by congruent polygons that lie in parallel planes.
2. The faces that are not bases, called *lateral faces*, are formed by parallelograms.

punto de simetría Una figura que se puede traz sobre sí mismo por una rotación de 180°.



R es un punto de simetría.

punto de tangencia Punto de intersección de una recta en un círculo en un solo punto.

forma punto-pendiente Ecuación de la forma $y - y_1 = m(x - x_1)$, donde (x_1, y_1) representan las coordenadas de un punto cualquiera sobre la recta y *m* representa la pendiente de la recta.

postes Las extremos del diámetro de un círculo mayor.

polígono Figura cerrada formada por un número finito de segmentos coplanares llamados *lados*, tal que satisface las siguientes condiciones:

1. Los lados que tienen un extremo común son no colineales.
2. Cada lado interseca exactamente dos lados mas, pero sólo en sus extremos, llamados *vértices*.

poliedros Figuras tridimensionales cerrada formadas por regiones poligonales planas. Las regiones planas definidas por un polígono y sus interiores se llaman *caras*. Cada intersección entre dos caras se llama *arista*. Los puntos donde se intersecan tres o más aristas se llaman *vértices*.

densidad demográfica Medida de la población por unidad de área.

postulado Enunciado que describe una relación fundamental entre los términos geométricos básicos. Los postulados se aceptan como verdaderos sin necesidad de demostración.

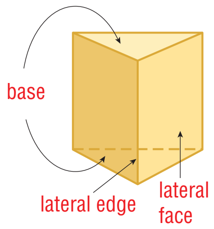
preimagen Gráfica de una figura antes de una transformación.

principio de superposición Dos figuras son congruentes si y sólo si existe un movimiento rígido o una serie de movimientos rígidos que aplican una de las figuras exactamente sobre la otra.

prisma Sólido con las siguientes características:

1. Dos caras llamadas *bases*, formadas por polígonos congruentes que yacen en planos paralelos.
2. Las caras que no son las bases, llamadas *caras laterales*, son paralelogramos.

3. The intersections of two adjacent lateral faces are called *lateral edges* and are parallel segments.



triangular prism

probability model (p. 939) A mathematical model used to match a random phenomenon.

probability tree (p. 949) An organized table of line segments (branches) that shows the probability of each outcome.

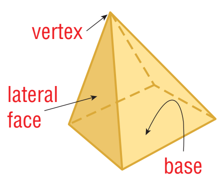
proof (p. 128) A logical argument in which each statement you make is supported by a statement that is accepted as true.

proof by contradiction (p. 355) An indirect proof in which one assumes that the statement to be proved is false. One then uses logical reasoning to deduce a statement that contradicts a postulate, theorem, or one of the assumptions. Once a contradiction is obtained, one concludes that the statement assumed false must in fact be true.

proportion (p. 462) An equation of the form $\frac{a}{b} = \frac{c}{d}$ that states that two ratios are equal.

pyramid (p. 67) A solid with the following characteristics:

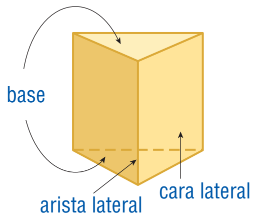
1. All of the faces, except one face, intersect at a point called the *vertex*.
2. The face that does not contain the vertex is called the *base* and is a polygonal region.
3. The faces meeting at the vertex are called *lateral faces* and are triangular regions.



rectangular pyramid

Pythagorean triple (p. 548) A group of three whole numbers that satisfies the equation $a^2 + b^2 = c^2$, where c is the greatest number.

3. Las intersecciones de dos caras laterales adyacentes se llaman *aristas laterales* y son segmentos paralelos.



prisma triangular

modelo de probabilidad Modelo matemático que se usa para relacionar un fenómeno aleatorio.

árbol de la probabilidad Tabla organizada de segmentos de recta (ramas) que muestra la probabilidad de cada resultado.

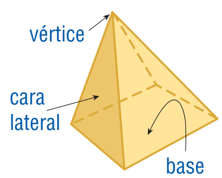
demostración Argumento lógico en el cual cada enunciado que se hace está respaldado por un enunciado que se acepta como verdadero.

demostración por contradicción Demostración indirecta en la cual se supone que el enunciado a demostrarse es falso. Luego, se usa el razonamiento lógico para inferir un enunciado que contradiga el postulado, teorema o una de las conjeturas. Una vez que se obtiene una contradicción, se concluye que el enunciado que se supuso falso es, en realidad, verdadero.

proporción Ecuación de la forma $\frac{a}{b} = \frac{c}{d}$ que establece que dos razones son iguales.

pirámide Sólido con las siguientes características:

1. Todas las caras, excepto una, se intersecan en un punto llamado *vértice*.
2. La cara sin el vértice se llama *base* y es una región poligonal.
3. Las caras que se encuentran en los vértices se llaman *caras laterales* y son regiones triangulares.



pirámide rectangular

triplete pitágorico Grupo de tres números enteros que satisfacen la ecuación $a^2 + b^2 = c^2$, donde c es el número mayor.

radius **1.** (p. 697) In a circle, any segment with endpoints that are the center of the circle and a point on the circle.
2. (p. 880) In a sphere, any segment with endpoints that are the center and a point on the sphere.

radius of a regular polygon (p. 807) The radius of a circle circumscribed about a polygon.

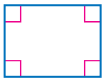
random variable (p. 941) A variable that can assume a set of values, each with fixed probabilities.

rate of change (p. 189) Describes how a quantity is changing over time.

ratio (p. 461) A comparison of two quantities using division.

ray (p. 36) $\overrightarrow{PQ}$ is a ray if it is the set of points consisting of $\overline{PQ}$ and all points S for which Q is between P and S.

rectangle (p. 423) A quadrilateral with four right angles.

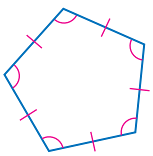


reduction (p. 511) An image that is smaller than the original figure.

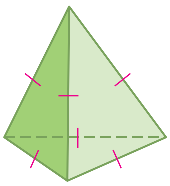
reflection (pp. 296, 623) A transformation representing the flip of a figure over a point, line or plane. A reflection in a line is a function that maps a point to its image such that

- if the point is on the line, then the image and preimage are the same point, or
- if the point is not on the line, the line is the perpendicular bisector of the segment joining the two points.

regular polygon (p. 57) A convex polygon in which all of the sides are congruent and all of the angles are congruent.



regular polyhedron (p. 68) A polyhedron in which all of the faces are regular congruent polygons.



radio **1.** En un círculo, cualquier segmento cuyos extremos son en el centro y un punto del círculo. **2.** En una esfera, cualquier segmento cuyos extremos son el centro y un punto de la esfera.

radio de un polígono regular Radio de un círculo circunscrito alrededor de un polígono.

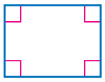
variable aleatoria Variable que puede tomar un conjunto de valores, cada uno con probabilidades fijas.

tasa de cambio Describe cómo cambia una cantidad a través del tiempo.

razón Comparación de dos cantidades mediante división.

rayo $\overrightarrow{PQ}$ es un rayo si se el conjunto de puntos formado por $\overline{PQ}$ y todos los puntos S para los cuales Q se ubica entre P y S.

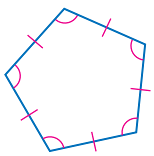
rectángulo Cuadrilátero con cuatro ángulos rectos.



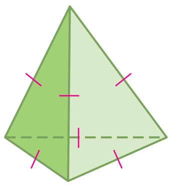
reducción Imagen más pequeña que la figura original.

reflexión Transformación en la cual una figura se "voltea" a través de un punto, una recta o un plano. Una reflexión en una recta es una función que aplica un punto a su imagen, de modo que si el punto yace sobre la recta, entonces la imagen y la preimagen son el mismo punto, o si el punto no yace sobre la recta, la recta es la mediatriz del segmento que une los dos puntos.

polígono regular Polígono convexo cuyos los lados y ángulos son congruentes.



poliedro regular Poliedro cuyas caras son polígonos regulares congruentes.



regular prism (p. 67) A right prism with bases that are regular polygons.

prisma regular Prisma recto cuyas bases son polígonos regulares.

regular pyramid (p. 854) A pyramid with a base that is a regular polygon.

pirámide regular Pirámide cuya base es un polígono regular.

regular tessellation (p. 660) A tessellation formed by only one type of regular polygon.

teselado regular Teselado formado por un solo tipo de polígono regular.

related conditionals (p. 109) Statements that are based on a given conditional statement.

condicionales relacionados Enunciados que se basan en un enunciado condicional dado.

relative frequency (p. 954) In a frequency table, the ratio of the number of observations in a category to the total number of observations.

frecuencia relativa En una tabla de frecuencias, la razón del número de observaciones en una categoría al número total de observaciones.

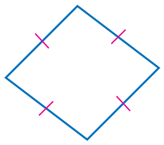
remote interior angles (p. 248) The angles of a triangle that are not adjacent to a given exterior angle.

ángulos internos no adyacentes Ángulos de un triángulo que no son adyacentes a un ángulo exterior dado.

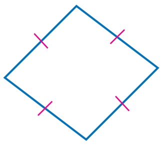
resultant (p. 601) The sum of two vectors.

resultante Suma de dos vectores.

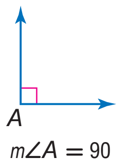
rhombus (p. 430) A quadrilateral with all four sides congruent.



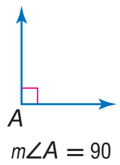
rombo Cuadrilátero con cuatro lados congruentes.



right angle (p. 38) An angle with a degree measure of 90.



ángulo recto Ángulo que mide 90.



right cone (p. 856) A cone with an axis that is also an altitude.

cono recto Cono cuyo eje es también su altura.

right cylinder (p. 848) A cylinder with an axis that is also an altitude.

cilindro recto Cilindro cuyo eje es también su altura.

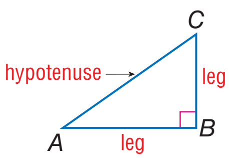
right prism (p. 846) A prism with lateral edges that are also altitudes.

prisma recto Prisma cuyas aristas laterales también son su altura.

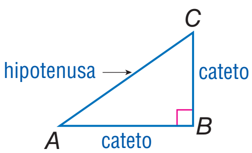
right solid (p. 837) A solid with base(s) that are perpendicular to the edges connecting them or connecting the base and the vertex of the solid.

sólido recto Sólido con base o bases perpendiculares a las aristas, conectándolas entre sí o conectando la base y el vértice del sólido.

right triangle (p. 237) A triangle with a right angle. The side opposite the right angle is called the *hypotenuse*. The other two sides are called *legs*.



triángulo rectángulo Triángulo con un ángulo recto. El lado opuesto al ángulo recto se conoce como *hipotenusa*. Los otros dos lados se llaman *catetos*.



rotation (pp. 296, 640) A transformation that turns every point of a preimage through a specified angle and direction about a fixed point, called the *center of rotation*. A rotation about a fixed point through an angle of $x°$ is a function that maps a point to its image such that

- if the point is the center of rotation, then the image and preimage are the same point, or
- if the point is not the center of rotation, then the image and preimage are the same distance from the center of rotation and the measure of the angle of rotation formed by the preimage, center of rotation, and image points is x.

rotación Transformación en la cual se hace girar cada punto de la preimagen a través de un ángulo y una dirección determinadas alrededor de un punto llamado *centro de rotación*. La rotación de $x°$ es una función que aplica un punto a su imagen, de modo que si el punto es el centro de rotación, entonces la imagen y la preimagen están a la misma distancia del centro de rotación y la medida del ángulo formado por los puntos de la preimagen, centro de rotación e imagen es x.

rotational symmetry (p. 664) If a figure can be rotated less than 360° about a point so that the image and the preimage are indistinguishable, the figure has rotational symmetry.

simetría rotacional Si una imagen se puede girar menos de 360° alrededor de un punto, de modo que la imagen y la preimagen sean idénticas, entonces la figura tiene simetría rotacional.

S

sample space (p. 915) The set of all possible outcomes of an experiment.

espacio muestral El conjunto de todos los resultados posibles de un experimento.

scalar (p. 606) A constant multiplied by a vector.

escalar Constante multiplicada por un vector.

scalar multiplication (p. 606) Multiplication of a vector by a scalar.

multiplicación escalar Multiplicación de un vector por un escalar.

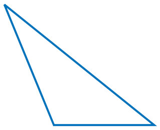
scale factor (p. 470) The ratio of the lengths of two corresponding sides of two similar polygons or two similar solids.

factor de escala Razón entre las longitudes de dos lados correspondientes de dos polígonos o sólidos semejantes.

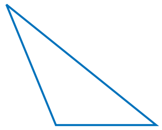
scale factor of dilation (p. 511) The ratio of a length on an image to a corresponding length on the preimage.

factor de escala de homotecia Razon de una longitud en la imagen a una longitud correspondiente en la preimagen.

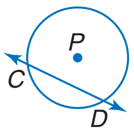
scalene triangle (p. 238) A triangle with no two sides congruent.



triángulo escaleno Triángulo que no tiene dos lados congruentes.

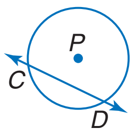


secant (p. 741) Any line that intersects a circle in exactly two points.



$\overleftrightarrow{CD}$ is a secant of $\odot P$.

secante Cualquier recta que interseca un círculo exactamente en dos puntos.

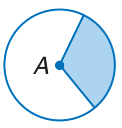


$\overleftrightarrow{CD}$ es una secante de $\odot P$.

secant segment (p. 752) A segment of a secant line that has exactly one endpoint on the circle.

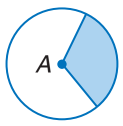
segmento secante Segmento de una recta secante que tiene exactamente un extremo en el círculo.

sector of a circle (p. 799) A region of a circle bounded by a central angle and its intercepted arc.



The shaded region is a sector of $\odot A$.

sector circular Región de un círculo limitada por un ángulo central y su arco de intersección.

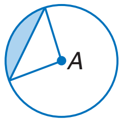


La región sombreada es un sector de $\odot A$.

segment (p. 14) See *line segment*.

segment bisector (p. 29) A segment, line, or plane that intersects a segment at its midpoint.

segment of a circle (p. 803) The region of a circle bounded by an arc and a chord.



The shaded region is a segment of $\odot A$.

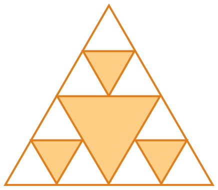
self-similar (p. 509) If any parts of a fractal image are replicas of the entire image, the image is self-similar.

semicircle (p. 707) An arc that measures 180.

semi-regular tessellation (p. 660) A uniform tessellation formed using two or more regular polygons.

sides of an angle (p. 36) The rays of an angle.

Sierpinski Triangle (p. 509) A self-similar fractal described by Waclaw Sierpinski. The figure was named for him.



similar solids (p. 896) Solids that have exactly the same shape, but not necessarily the same size.

similarity ratio (p. 470) The scale factor between two similar polygons

similarity transformation (p. 511) When a figure and its transformation image are similar.

simulation (p. 939) A probability model used to recreate a situation again and again so the likelihood of various outcomes can be estimated.

sine (p. 568) For an acute angle of a right triangle, the ratio of the measure of the leg opposite the acute angle to the measure of the hypotenuse.

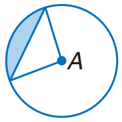
skew lines (p. 173) Lines that do not intersect and are not coplanar.

slant height (p. 854) The height of the lateral side of a pyramid or cone.

segmento Ver *segmento de recta*.

bisector del segmento Segmento, recta o plano que interseca un segmento en su punto medio.

segmento de un círculo Región de un círculo limitada por un arco y una cuerda.



La región sombreada es un segmento de $\odot A$.

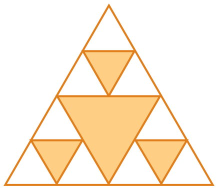
autosemejante Si cualquier parte de una imagen fractal es una réplica de la imagen completa, entonces la imagen es autosemejante.

semicírculo Arco que mide 180.

teselado semiregular Teselado uniforme compuesto por dos o más polígonos regulares.

lados de un ángulo Los rayos de un ángulo.

triángulo de Sierpinski Fractal autosemejante descrito por el matemático Waclaw Sierpinski. La figura se nombró en su honor.



sólidos semejantes Sólidos que tienen exactamente la misma forma, pero no necesariamente el mismo tamaño.

razón de semejanza Factor de escala entre dos polígonos semejantes.

transformación de semejanza cuando una figura y su imagen transformada son semejantes.

simulacro Modelo de la probabilidad que se usa para reconstruir una situación repetidas veces y así poder estimar la probabilidad de varios resultados.

seno Para un ángulo agudo de un triángulo rectángulo, razón entre la medida del cateto opuesto al ángulo agudo a la medida de la hipotenusa.

rectas alabeadas Rectas que no se intersecan y que no son coplanares.

altura oblicua Altura de la cara lateral de una pirámide o un cono.

slope **(p. 188)** For a (nonvertical) line containing two points (x_1, y_1) and (x_2, y_2), the number m given by the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ where $x_2 \neq x_1$.

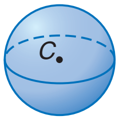
slope-intercept form **(p. 198)** A linear equation of the form $y = mx + b$. The graph of such an equation has slope m and y-intercept b.

solid of revolution **(p. 647)** A three-dimensional figure obtained by rotating a plane figure about a line.

solving a triangle **(p. 591)** Finding the measures of all of the angles and sides of a triangle.

space **(p. 7)** A boundless three-dimensional set of all points.

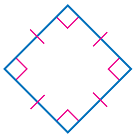
sphere **(p. 67)** In space, the set of all points that are a given distance from a given point, called the *center*.



C is the center of the sphere.

spherical geometry **(p. 889)** The branch of geometry that deals with a system of points, great circles (lines), and spheres (planes).

square **(p. 431)** A quadrilateral with four right angles and four congruent sides.



standard position **(p. 602)** When the initial point of a vector is at the origin.

statement **(p. 99)** Any sentence that is either true or false, but not both.

supplementary angles **(p. 47)** Two angles with measures that have a sum of 180.

surface area **(p. 69)** The sum of the areas of all faces and side surfaces of a three-dimensional figure.

symmetry **(p. 663)** A figure has symmetry if there exists a rigid motion—reflection, translation, rotation, or glide reflection—that maps the figure onto itself.

pendiente Para una recta (no vertical) que contiene dos puntos (x_1, y_1) y (x_2, y_2), tel número m viene dado por la fórmula $m = \frac{y_2 - y_1}{x_2 - x_1}$ donde $x_2 \neq x_1$.

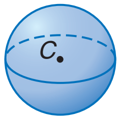
forma pendiente-intersección Ecuación lineal de la forma $y = mx + b$ donde, la pendiente es m y la intersección y es b.

sólido de revolución Figura tridimensional que se obtiene al rotar una figura plana alrededor de una recta.

resolver un triángulo Calcular las medidas de todos los ángulos y todos los lados de un triángulo.

espacio Conjunto tridimensional no acotado de todos los puntos.

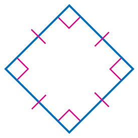
esfera En el espacio, conjunto de todos los puntos a cierta distancia de un punto dado llamado *centro*.



C es el centro de la esfera.

geometría esférica Rama de la geometría que estudia los sistemas de puntos, los círculos mayores (rectas) y las esferas (planos).

cuadrado Cuadrilátero con cuatro ángulos rectos y cuatro lados congruentes.



posición estándar Cuando el punto inicial de un vector es el origen.

enunciado Cualquier suposición que puede ser falsa o verdadera, pero no ambas.

ángulos suplementarios Dos ángulos cuya suma es igual a 180.

área de superficie Suma de las áreas de todas las caras y superficies laterales de una figura tridimensional.

simetría Una figura tiene simetría si existe un movimiento rígido (reflexión, translación, rotación, o reflexión con deslizamiento) que aplica la figura sobre sí misma.

tangent **1.** (p. 568) For an acute angle of a right triangle, the ratio of the measure of the leg opposite the acute angle to the measure of the leg adjacent to the acute angle. **2.** (p. 732) A line in the plane of a circle that intersects the circle in exactly one point. The point of intersection is called the *point of tangency*. **3.** (p. 880) A line that intersects a sphere in exactly one point.

tangente **1.** Para un ángulo agudo de un triángulo rectángulo, razón de la medida del cateto opuesto al ángulo agudo a la medida del cateto adyacente al ángulo agudo. **2.** Recta en el plano de un círculo que interseca el círculo en exactamente un punto. El punto de intersección se conoce como *punto de tangencia*. **3.** Recta que interseca una esfera en exatamente un punto.

tangent segment (p. 752) A segment of a tangent with one endpoint on a circle that is both the exterior and whole segment.

segmento tangente Segmento de la tangente con un extremo en un círculo que es tanto el segmento externo como el segmento completo.

tessellation (p. 660) A pattern that covers a plane by transforming the same figure or set of figures so that there are no overlapping or empty spaces.

teselado Patrón con que se cubre un plano aplicando la misma figura o conjunto de figuras, sin que haya traslapes ni espacios vacíos.

theorem (p. 129) A statement or conjecture that can be proven true by undefined terms, definitions, and postulates.

teorema Enunciado o conjetura que se puede demostrar como verdadera mediante términos geométricos básicos, definiciones y postulados.

topographic map (p. 845) A representation of a three-dimensional surface on a flat piece of paper.

mapa topográfico Representación de una superficie tridimensional sobre una hoja del papel.

traceable network (p. 964) A network in which all of the nodes are connected and each edge is used once when the network is used.

red detectable Red en la cual todos los nodos estén conectados y cada arista se utiliza una vez al usarse la red.

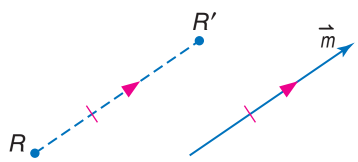
transformation (p. 511) In a plane, a mapping for which each point has exactly one image point and each image point has exactly one preimage point.

transformación En un plano, aplicación para la cual cada punto del plano tiene un único punto de la imagen y cada punto de la imagen tiene un único punto de la preimagen.

translation (pp. 296, 632) A transformation that moves a figure the same distance in the same direction. A translation is a function that maps each point to its image along a vector such that each segment joining a point and its image has the same length as the vector, and this segment is also parallel to the vector.

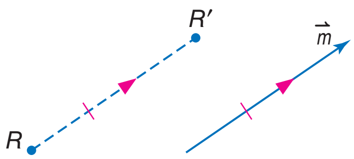
traslación Transformación que mueve una figura la misma distancia en la misma dirección. Una traslación es una función que aplica cada punto a su imagen a lo largo de un vector, de modo que cada segmento que une un punto a su imagen tiene la misma longitud que el vector y este segmento es también paralelo al vector.

translation vector (p. 632) The vector in which a translation maps each point to its image.



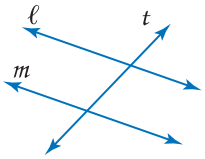
Point *R′*, is a translation of point *R* along translation vector *m*.

vector de traslación Vector en el cual una traslación aplica cada punto a su imagen.



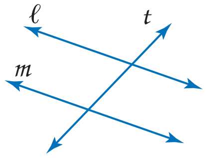
El punta *R′*, es la traslación del punto *R* a lo largo del vector *m* de traslación.

transversal (p. 174) A line that intersects two or more lines in a plane at different points.



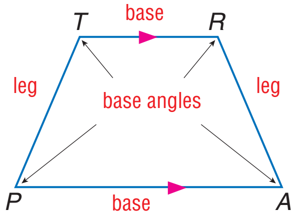
Line *t* is a transversal.

transversal Recta que interseca dos o más rectas en el diferentes puntos del mismo plano.

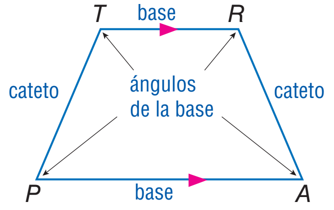


La recta *t* es una transversal.

trapezoid (p. 439) A quadrilateral with exactly one pair of parallel sides. The parallel sides of a trapezoid are called *bases*. The nonparallel sides are called *legs*. The pairs of angles with their vertices at the endpoints of the same base are called *base angles*.



trapecio Cuadrilátero con sólo un par de lados paralelos. Los lados paralelos del trapecio se llaman *bases*. Los lados no paralelos se llaman *catetos*. Los pares de ángulos cuyos vértices coinciden en los extremos de la misma base son los *ángulos de la base*.



tree diagram (p. 915) An organized table of line segments (branches) which shows possible experiment outcomes.

diagrama del árbol Tabla organizada de segmentos de recta (ramas) que muestra los resultados posibles de un experimento.

triangle method (p. 602) A method used to find the resultant of two vectors in which the second vector is connected to the terminal point of the first and the resultant is drawn from the initial point of the first vector to the terminal point of the second vector.

método del triangulo Método para calcular la resultante de dos vectores en cual el segundo vector está conectado al extremo del primer y la resultante se traza desde el punto inicial del primer vector al extremo del segundo vector.

trigonometric ratio (p. 568) A ratio of the lengths of sides of a right triangle.

razón trigonométrica Razón de las longitudes de los lados de un triángulo rectángulo.

trigonometry (p. 568) The study of the properties of triangles and trigonometric functions and their applications.

trigonometría Estudio de las propiedades de los triángulos, de las funciones trigonométricas y sus aplicaciones.

truth table (p. 101) A table used as a convenient method for organizing the truth values of statements.

tabla de validez Tabla que se utiliza para organizar de una manera conveniente los valores de verdaderos de los enunciados.

truth value (p. 99) The truth or falsity of a statement.

valor de verdad Condición de un enunciado de ser verdadero o falso.

two-column proof (p. 137) A formal proof that contains statements and reasons organized in two columns. Each step is called a *statement*, and the properties that justify each step are called *reasons*.

demostración de dos columnas Demostración formal que contiene enunciados y razones organizadas en dos columnas. Cada paso se llama *enunciado* y las propiedades que lo justifican son las *razones*.

two-stage experiment (p. 916) An experiment with two stages or events.

experimento de dos pasos Experimento que consta de dos pasos o eventos.

two-way frequency table (p. 954) A table used to show the frequencies or relative frequencies of data from a survey or experiment classified according to two variables, with the rows indicating one variable and the columns indicating the other.

tabla de doble entrada o de frecuencias Tabla que se usa para mostrar las frecuencias o frecuencias relativas de los datos de una encuesta o experimento clasificado de acuerdo con dos variables, en la cual las hileras indican una variable y las columnas indican la otra variable.

U

undefined term (p. 5) Words, usually readily understood, that are not formally explained by means of more basic words and concepts. The basic undefined terms of geometry are point, line, and plane.

término geométrico básico no definido Palabras que por lo general se entienden fácilmente y que no se explican formalmente mediante palabras o conceptos más básicos. Los términos geométricos básicos no definidos son el punto, la recta y el plano.

uniform tessellations (p. 660) Tessellations containing the same arrangement of shapes and angles at each vertex.

teselado uniforme Teselados con el mismo patrón de formas y ángulos en cada vértice.

V

vector (p. 600) A directed segment representing a quantity that has both magnitude, or length, and direction.

vector Segmento dirigido que representa una cantidad, la cual posee tanto magnitud o longitud como dirección.

vertex angle of an isosceles triangle (p. 285) See *isosceles triangle*.

ángulo del vértice un triángulo isosceles Ver *triángulo isósceles*.

vertex-edge graphs (p. 964) A collection of nodes connected by edges.

gráficas de vértice-arista Colección de nodos conectados por aristas.

vertex of an angle (pp. 36, 67) The common endpoint of an angle.

vértice de un ángulo Extremo común de un ángulo.

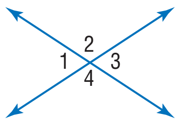
vertex of a polygon (p. 56) The vertex of each angle of a polygon.

vertice de un polígono Vértice de cada ángulo de un polígono.

vertex of a polyhedron (p. 67) The intersection of three edges of a polyhedron.

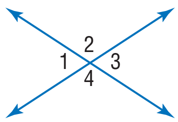
vértice de un poliedro Intersección de las aristas de un poliedro.

vertical angles (p. 46) Two nonadjacent angles formed by two intersecting lines.



∠1 and ∠3 are vertical angles.
∠2 and ∠4 are vertical angles.

ángulos opuestos por el vértice Dos ángulos no adyacentes formados por dos rectas que se intersecan.



∠1 y ∠3 son ángulos opuestos por el vértice.
∠2 y ∠4 son ángulos opuestos por el vértice.

volume (p. 69) A measure of the amount of space enclosed by a three-dimensional figure.

volumen La medida de la cantidad de espacio contiene una figura tridimensional.

W

weight (p. 946) The value assigned to an edge in a vertex-edge graph.

peso Valor asignado a una arista en una gráfica de vértice-arista.

weight of a path (p. 946) The sum of the weights of the edges along a path.

peso de una ruta Suma de los pesos de las aristas a lo largo de una ruta.

weighted vertex-edge graphs (p. 946) A collection of nodes connected by edges in which each edge has an assigned value.

gráficas ponderadas de vértice-arista Colección de nodos conectados por aristas en que cada arista tiene un valor asignado.

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Formulas

Coordinate Geometry

Slope	$m = \dfrac{y_2 - y_1}{x_2 - x_1}$
Distance on a number line:	$d = \lvert a - b \rvert$
Distance on a coordinate plane:	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Distance in space:	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
Distance arc length:	$\ell = \dfrac{x}{360} \cdot 2\pi r$
Midpoint on a number line:	$M = \dfrac{a + b}{2}$
Midpoint on a coordinate plane:	$M = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$
Midpoint in space:	$M = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}, \dfrac{z_1 + z_2}{2}\right)$

Perimeter and Circumference

square	$P = 4s$	**rectangle**	$P = 2\ell + 2w$	**circle**	$C = 2\pi r$ or $C = \pi d$

Area

square	$A = s^2$	**triangle**	$A = \frac{1}{2}bh$
rectangle	$A = \ell w$ or $A = bh$	**regular polygon**	$A = \frac{1}{2}Pa$
parallelogram	$A = bh$	**circle**	$A = \pi r^2$
trapezoid	$A = \frac{1}{2}h(b_1 + b_2)$	**sector of a circle**	$A = \frac{x}{360} \cdot \pi r^2$
rhombus	$A = \frac{1}{2}d_1 d_2$ or $A = bh$		

Lateral Surface Area

prism	$L = Ph$	**pyramid**	$L = \frac{1}{2}P\ell$
cylinder	$L = 2\pi rh$	**cone**	$L = \pi r\ell$

Total Surface Area

prism	$S = Ph + 2B$	**cone**	$S = \pi r\ell + \pi r^2$
cylinder	$S = 2\pi rh + 2\pi r^2$	**sphere**	$S = 4\pi r^2$
pyramid	$S = \frac{1}{2}P\ell + B$		

Volume

cube	$V = s^3$	**pyramid**	$V = \frac{1}{3}Bh$
rectangular prism	$V = \ell wh$	**cone**	$V = \frac{1}{3}\pi r^2 h$
prism	$V = Bh$	**sphere**	$V = \frac{4}{3}\pi r^3$
cylinder	$V = \pi r^2 h$		

Equations for Figures on a Coordinate Plane

slope-intercept form of a line	$y = mx + b$	**circle**	$(x - h)^2 + (y - k)^2 = r^2$
point-slope form of a line	$y - y_1 = m(x - x_1)$		

Trigonometry

Law of Sines	$\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$	**Law of Cosines**	$a^2 = b^2 + c^2 - 2bc \cos A$ $b^2 = a^2 + c^2 - 2ac \cos B$ $c^2 = a^2 + b^2 - 2ab \cos C$
Pythagorean Theorem	$a^2 + b^2 = c^2$		

Symbols

$\neq$	is not equal to
$\approx$	is approximately equal to
$\cong$	is congruent to
$\sim$	is similar to
$\angle$, $\measuredangle$	angle, angles
$m\angle A$	degree measure of $\angle A$
$^\circ$	degree
$\overleftrightarrow{AB}$	line containing points A and B
$\overline{AB}$	segment with endpoints A and B
$\overrightarrow{AB}$	ray with endpoint A containing B
AB	measure of $\overline{AB}$, distance between points A and B
$\sim p$	negation of p, not p
$p \wedge q$	conjunction of p and q
$p \vee q$	disjunction of p and q
$p \longrightarrow q$	conditional statement, if p then q
$p \longleftrightarrow q$	biconditional statement, p if and only if q

$\parallel$	is parallel to
$\nparallel$	is not parallel to
$\perp$	is perpendicular to
$\triangle$	triangle
$>, \geq$	is greater than, is greater than or equal to
$<, \leq$	is less than, is less than or equal to
$\square$	parallelogram
n-gon	polygon with n sides
$a:b$	ratio of a to b
(x, y)	ordered pair
(x, y, z)	ordered triple
$\sin x$	sine of x
$\cos x$	cosine of x
$\tan x$	tangent of x
$\vec{a}$	vector a
$\overrightarrow{AB}$	vector from A to B

$\lvert\overrightarrow{AB}\rvert$	magnitude of the vector from A to B
A'	the image of preimage A
$\rightarrow$	is mapped onto
$\odot A$	circle with center A
π	pi
$\widehat{AB}$	minor arc with endpoints A and B
$\widehat{ABC}$	major arc with endpoints A and C
$m\widehat{AB}$	degree measure of arc AB
$f(x)$	f of x, the value of f at x
$!$	factorial
${}_nP_r$	permutation of n objects taken r at a time
${}_nC_r$	combination of n objects taken r at a time
$P(A)$	probability of A
$P(A\|B)$	the probability of A given that B has already occurred

Measures

Metric	Customary
Length	
1 kilometer (km) = 1000 meters (m) 1 meter = 100 centimeters (cm) 1 centimeter = 10 millimeters (mm)	1 mile (mi) = 1760 yards (yd) 1 mile = 5280 feet (ft) 1 yard = 3 feet 1 yard = 36 inches (in.) 1 foot = 12 inches
Volume and Capacity	
1 liter (L) = 1000 milliliters (mL) 1 kiloliter (kL) = 1000 liters	1 gallon (gal) = 4 quarts (qt) 1 gallon = 128 fluid ounces (fl oz) 1 quart = 2 pints (pt) 1 pint = 2 cups (c) 1 cup = 8 fluid ounces
Weight and Mass	
1 kilogram (kg) = 1000 grams (g) 1 gram = 1000 milligrams (mg) 1 metric ton (t) = 1000 kilograms	1 ton (T) = 2000 pounds (lb) 1 pound = 16 ounces (oz)