Sequences and Series

Find an expression for the nth term of each sequence.

1. $4, 8, 16, 32, 64, \ldots$

2. $1, 3, 7, 15, 31, \ldots$

3. $2, 5, 10, 17, 26, \ldots$

4. $1, -3, 5, -7, 9, \ldots$

5. $\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{5}{11}, \ldots$

6. $\frac{1}{2}, -\frac{1}{4}, \frac{1}{6}, -\frac{1}{8}, \frac{1}{10}, \ldots$

Determine if the **sequence** converges or diverges. If a sequence converges, then find its limit.

7. $(0.2)^n$

8. 2^n

9. $(-0.3)^n$

10. $3+e^{-2n}$

11. $\frac{2^n}{3^n}$

12. $\frac{n}{10}+\frac{10}{n}$

13. $\frac{(-1)^n}{n}$

14. $\frac{2n+1}{n}$

Answers

1) $4 \cdot 2^{n-1}$

2) $2^n - 1$

3) $n^2 + 1$

4) $(-1)^{n+1} \cdot (2n-1)$

5) $\frac{n}{2n+1}$

6) $(-1)^{n+1} \cdot \frac{1}{2n}$

7) 0, converge

8) und, diverge

9) 0, converge

10) 3, converge

11) 0, converge

12) und, diverge

13) 0, converge

14) 2, converge

Find recursive expression for the nth term of each sequence.

15. 1, 3, 5, 7, 9, . . .

16. 3, 5, 9, 17, 33, . . .

The nth Term Divergence Test

If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ is a divergent series.

Note: the test **does not** conclude that a series converges. If the limit is any other number besides 0, then we can only conclude that the series is divergent. If the limit is 0, then there is no conclusion. If the limit is 0, the series is probably convergent, but there are many exceptions such as $1/x$.

Use the nth Term Divergence Test to determine if the **series** converges or diverges.

17. $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n}$

18. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+n}{n^3}$

19. $\displaystyle\sum_{n=1}^{\infty} \frac{1+3n^2+n^3}{4n^3-5n+2}$

20. $\displaystyle\sum_{n=1}^{\infty} \cos(\pi n)$

21. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$

22. $\displaystyle\sum_{n=1}^{\infty} \frac{\sin n}{n}$

23. $\displaystyle\sum_{n=1}^{\infty} \frac{2n+(-1)^n 5}{4n-(-1)^n 3}$

24. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n^3}$

Answers

15) $a_{n-1}+2$, for $n>1$ and $a_1=1$

16) $2a_{n-1}-1$, for $n>1$ and $a_1=3$

17) diverge

18) no conclusion

19) diverge

20) diverge

21) no conclusion

22) no conclusion

23) diverge

24) diverge

Evaluate the sum of each **telescoping series**, and find an expression for the ***n*th Partial Sum** of the series.

25. $\sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+2}$

26. $\sum_{n=1}^{\infty} \frac{2}{n^2 + 4n + 3}$ (Hint: rewrite using partial fractions first)

Answers

25) 3/2, $\dfrac{3n^2+5n}{2n^2+6n+4}$

26) 5/6, $\dfrac{5n^2+13n}{6n^2+30n+36}$

27. The infinite series $\sum_{k=1}^{\infty} a_k$ has nth partial sum $S_n = \frac{n}{3n+1}$ for $n \geq 1$. What is the sum of the series $\sum_{k=1}^{\infty} a_k$?

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) 1 (D) $\frac{3}{2}$ (E) The series diverges.

28. The infinite series $\sum_{k=1}^{\infty} a_k$ has nth partial sum $S_n = (-1)^{n+1}$ for $n \geq 1$. What is the sum of the series $\sum_{k=1}^{\infty} a_k$?

(A) -1 (B) 0 (C) $\frac{1}{2}$ (D) 1 (E) The series diverges.

Answers

27) A

28) E