Warm-up: Derivatives

Name: ______________________________
AP Calculus BC
Warm-up 2

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2. $y = \sin^{-1}\left(5x^2\right)$

3. $y = 6^{\sin x}$

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14. If $x = t^2 - 1$ and $y = 2e^t$, then $\dfrac{dy}{dx} =$

(A) $\dfrac{e^t}{t}$ (B) $\dfrac{2e^t}{t}$ (C) $\dfrac{e^{|t|}}{t^2}$ (D) $\dfrac{4e^t}{2t-1}$ (E) e^t

2. If $x = e^{2t}$ and $y = \sin(2t)$, then $\dfrac{dy}{dx} =$

(A) $4e^{2t}\cos(2t)$ (B) $\dfrac{e^{2t}}{\cos(2t)}$ (C) $\dfrac{\sin(2t)}{2e^{2t}}$ (D) $\dfrac{\cos(2t)}{2e^{2t}}$ (E) $\dfrac{\cos(2t)}{e^{2t}}$

18. For what values of t does the curve given by the parametric equations $x = t^3 - t^2 - 1$ $y = t^4 + 2t^2 - 8t$ have a vertical tangent?

(A) 0 only
(B) 1 only
(C) 0 and $\frac{2}{3}$ only
(D) 0, $\frac{2}{3}$, and 1
(E) No value

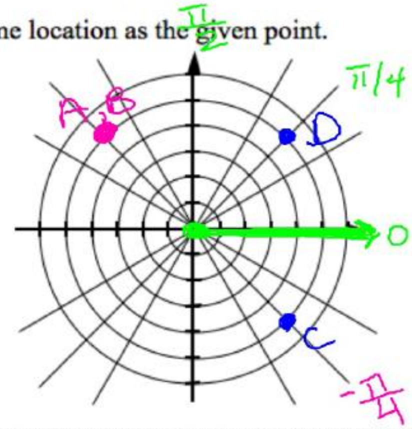
$$\frac{dy}{dt} = 4t^3 + 4t - 8$$

$$\frac{dx}{dt} = 3t^2 - 2t = 0$$
$$t(3t-2) = 0$$
$$t = 0, \frac{2}{3}$$
✓ ✓

2. In the xy-plane, the graph of the parametric equations $x = 5t + 2$ and $y = 3t$, for $-3 \le t \le 3$, is a line segment with slope

(A) $\frac{3}{5}$ (B) $\frac{5}{3}$ (C) 3 (D) 5 (E) 13

Polar Review

Polar coordinates	Plot points using a radius r and an angle θ: ordered pair (r, θ)
Polar axis	A horizontal ray that heads to the right (x-axis)
Pole	Endpoint of the ray that makes up the polar axis (~~y-axis~~)
	Ex8. The ordered pair $\left(-5, -\frac{\pi}{4}\right)$ describes a point on the polar coordinate plane. Determine which representations yield the same location as the given point. (a) $\left(-5, \frac{7\pi}{4}\right)$ (b) $\left(5, -\frac{5\pi}{4}\right)$ (c) $\left(-5, \frac{11\pi}{4}\right)$ (d) $\left(5, \frac{\pi}{4}\right)$ 

Converting from polar to rectangular coordinates	If the point P with polar coordinates $(r,\ \theta)$, the rectangular coordinates $(x,\ y)$ of P are given by: $x = r\cos\theta$ *and* $y = r\sin\theta$
Helpful relationships in converting equations	$r^2 = x^2 + y^2$ $\tan\theta = \dfrac{y}{x}$ *if* $x \neq 0$

Ex9. Convert from rectangular to polar.

(a) $x + 2y = 5$	(b) $y = x^2$	(c) $5xy = 8$
$r\cos\theta + 2r\sin\theta = 5$ $r(\cos\theta + 2\sin\theta) = 5$ $r = \dfrac{5}{\cos\theta + 2\sin\theta}$	$\cancel{r}\sin\theta = r^{\cancel{2}}\cos^2\theta$ $\sin\theta = r\cos^2\theta$ $\dfrac{\sin\theta}{\cos^2\theta} = r$	$5r\cos\theta\, r\sin\theta = 8$ $5r^2\cos\theta\sin\theta = 8$ $r^2 = \dfrac{8}{5\cos\theta\sin\theta}$

Ex10. Convert from polar to rectangular.

(a) $r^2 = 3^2$	(b) $\theta = \dfrac{\pi}{4}$	(c) $r = \csc\theta$
$r^2 = 9$ $x^2 + y^2 = 9$ circle	$\tan\theta = \tan\frac{\pi}{4}$ $\tan\theta = 1$ $\dfrac{y}{x} = 1$ $y = x$ line	$r = \dfrac{1}{\sin\theta}$ $\underbrace{r\sin\theta}_{} = 1$ $y = 1$ horiz. line

Extra Practice: Convert form polar to rectangular and vice versa.

1. $\left(-1, \frac{5\pi}{4}\right)$

6. $r = 3$

2. $(-3, -1.57)$

7. $r = 3\cos\theta$

3. $(0, -5)$

8. $r = 3 - 2\cos\theta,\ \ \theta = 0$

4. $x^2 + y^2 = a^2$

9. $r = 2\csc\theta + 3$

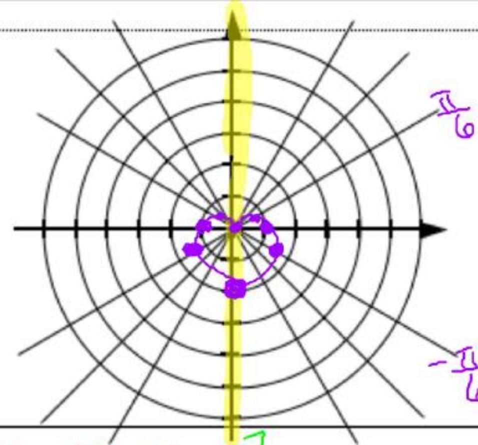
5. $y^2 = 9x$

Graphing Polar Equations	1. Test for symmetry. 2. Create a table of points (only halfway around the circle!); r and θ Plug in θ values to the equation, solve for r. 3. Graph the points. Connect the dots. Consider the *direction* of the curve.
Tests for Symmetry	*Sufficient conditions for symmetry, but not *necessary*. A symmetry test may fail but the graph may still have that specific symmetry. A failure of symmetry simply means you cannot use that specific symmetry for a shortcut in graphing.
Polar axis (x-axis)	Replace θ with $-\theta$. Is the resulting equation equivalent to the original? Recall odd/even functions. cos = even sin = odd
$\theta = \frac{\pi}{2}$ (y-axis)	Replace θ with $\pi - \theta$. Recall trig difference formulas.
Pole (origin)	Replace r with $-r$. Multiply everything by -1.

$\cos(-\theta) = \cos\theta$

$\sin(-\theta) = -\sin\theta$

Ex11. Graph the polar equation: $r = 1 - \sin\theta$



① Polar axis

$r = 1 - \sin(-\theta)$ X

$r = 1 + \sin\theta$

② $\theta = \frac{\pi}{2}$

$r = 1 - \sin(\pi - \theta)$

$r = 1 - [\cancel{\sin\pi\cos\theta} - \underbrace{\cos\pi}_{-1}\sin\theta]$

$r = 1 - \sin\theta$ ✓

③ Pole

X

$r = 1 - \sin\theta$

$-r = 1 - \sin\theta$

$r = -1 + \sin\theta$

Q1 & Q4

θ	$r = 1 - \sin\theta$
$-\pi/2$	$1-(-1) = 2$
$-\pi/6$	$1-(-\frac{1}{2}) = \frac{3}{2}$
0	$1-0 = 1$
$\pi/6$	$1-\frac{1}{2} = \frac{1}{2}$
$\pi/2$	$1-1 = 0$

Polar derivatives	First, determine the x and y equations based on the r equation. Recall: $x = r\cos\theta$ and $y = r\sin\theta$ (Replace the r with the polar equation) $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{f(\theta)\cos\theta + \overset{f'(\theta)}{f'(\theta)}\sin\theta}{-f(\theta)\sin\theta + \underset{f'(\theta)}{f'(\theta)}\cos\theta}$ i.e. Numerator = take derivative of y with respect to θ Denominator = take derivative of x with respect to θ *You will probably need to use product rule!
Polar tangent lines	
Horizontal tangent	$\frac{dy}{d\theta} = 0$ $\quad \frac{dx}{d\theta} \neq 0$
Vertical tangent	$\frac{dx}{d\theta} = 0$ $\quad \frac{dy}{d\theta} \neq 0$
Tangent lines at the pole	If $f(\alpha) = 0$ and $f'(\alpha) \neq 0$, then the line $\theta = \alpha$ is tangent at the pole to the graph of $r = f(\theta)$
FYI	Zeroes of $r = f(\theta)$ can be used to find the tangent line at the pole.
$x = (1+\sin\theta)\cos\theta$ $x = \cos\theta + \sin\theta\cos\theta$	Ex12. Find the points of horizontal and vertical tangency (if any) to the polar curve. $r = 1 + \sin\theta$ $y = (1+\sin\theta)\sin\theta$ $y = \sin\theta + \sin^2\theta$ H.T. $\frac{dy}{d\theta} = \cos\theta + 2\sin\theta\cos\theta = 0$ $\cos\theta(1+2\sin\theta) = 0$ $\cos\theta = 0 \qquad 1 + 2\sin\theta = 0$

$\sin\theta = -\frac{1}{2}$

$\theta = \frac{\pi}{2}, \cancel{\frac{3\pi}{2}}$ $\qquad$ $\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

V.T. $\frac{dx}{d\theta} = -\sin\theta - \sin^2\theta + \cos^2\theta = 0$

$-\sin\theta - \sin^2\theta + (1 - \sin^2\theta) = 0$

$-2\sin^2\theta - \sin\theta + 1 = 0$

$0 = 2\sin^2\theta + \sin\theta - 1$

$2x^2 + x - 1$

$(2x-1)(x+1)$

$2\sin\theta - 1 = 0 \qquad \sin\theta + 1 = 0$

$\sin\theta = \frac{1}{2} \qquad \sin\theta = -1$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$ $\qquad$ $\cancel{\theta = \frac{3\pi}{2}}$

$r = 1 + \sin\theta$

H.T. (r,θ)	V.T. (r,θ)
$\left(2, \frac{\pi}{2}\right)$	$\left(\frac{3}{2}, \frac{\pi}{6}\right)$
$\left(\frac{1}{2}, \frac{7\pi}{6}\right)$	$\left(\frac{3}{2}, \frac{5\pi}{6}\right)$
$\left(\frac{1}{2}, \frac{11\pi}{6}\right)$	

ex. $r = \sin\theta$

$x = r\cos\theta$

$y = r\sin\theta$

$$\frac{dr}{d\theta} = \cos\theta$$

$x = \sin\theta\cos\theta$

$y = \sin^2\theta$

$$\frac{dy}{dx} = ?$$

$$\frac{dy}{dx} = \frac{2\sin\theta\cos\theta}{-\sin^2\theta + \cos^2\theta} = \frac{\sin(2\theta)}{\cos^2\theta - \sin^2\theta}$$

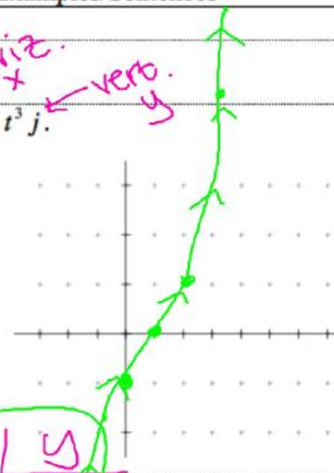
Chapter and Section: Ch. 11 and 12	Name:
	Date:
Lesson Topic: Intro to Vectors and Derivatives	Period:

Questions/Main Ideas/Vocabulary	Notes/Answers/Definitions/Examples/Sentences
Vector valued functions	Plane curves: $x = f(t)$ and $y = g(t)$
	Ex1. Graph the vector curve $r(t) = (t+1)i + t^3 j$.

horiz. x

vert. y

- Express $r(t)$ parametrically.
  $x(t) = t + 1$
  $y(t) = t^3$
- Solve for t.
  $x - 1 = t$
- Substitute t into the y equation.
  $y = (x-1)^3$



t	x	y
-2	-1	-8
-1	0	-1
0	1	0
1	2	1
2	3	8

Extra Practice:

a) Sketch the plane curve represented by r(t) = 2cos*t***i** – 3sin*t***j**, $0 \le t \le 2\pi$

$\frac{x(t)}{2} = \frac{2\cos t}{2}$ $\frac{y(t)}{-3} = \frac{-3\sin t}{-3}$

$\frac{x}{2} = \cos t$ $\frac{y}{-3} = \sin t$

$\cos^2 t + \sin^2 t = 1$ $\left(\frac{x}{2}\right)^2 + \left(\frac{y}{-3}\right)^2 = 1$

b) Sketch the plane curve represented by r(t) = 3*t***i** + *(t – 1)***j**.

$\frac{x^2}{4} + \frac{y^2}{9} = 1$

$x = 3t$ $y = t - 1$

$y + 1 = t$

$x = 3(y+1)$

$x = 3y + 3$

$\frac{x-3}{3} = y$

Summary:

1. How do you convert from parametric to rectangular equations?

2. How do you convert from polar to rectangular and vice versa? (Key formulas)

3. How do you find the derivative and 2nd derivative of a parametric or polar equation?

4. What does the derivative of a parametric or polar equation mean graphically?