APPM 1360 - Calculus 2

Course Objectives:

This course extends the concepts and techniques of single-variable Calculus.

The main objectives are to:

- Improve integration techniques and applications of differential and integral calculus
- Understand sequences and series
- Improve problem solving and critical thinking skills.

This class will form the basis of your set of everyday working skills required for math, engineering and the sciences.

Textbook: *Essential Calculus*, 2^{nd} Edition by James Stewart. We will cover Chapters 6-9. You will also need an access code for WebAssign's online homework system. The access code can also be purchased separately.

Schedule and Topics Covered

Day	Section	Topics
1	Chap 5 Review	Derivatives and Basic Integration
2	6.1	Integration by Parts
3	6.1/6.2	Integration by Parts cont'd. / Trigonometric Integrals
4	6.2	Trigonometric Substitutions
5	6.3	Partial Fractions
6	6.3	Partial Fractions cont'd.
7	6.5	Numerical Integration
8	6.6	Improper Integrals
9	6.6	Improper Integrals cont'd.
10	7.1/7.2	Areas between Curves / Volumes
11	7.2	Volumes
12	7.2/7.3	Volumes by Cylindrical Shells
13	Exam 1 Review	Exam 1 Topics
14	7.3	Volumes by Cylindrical Shells cont'd.
15	7.4	Arc Length
16	7.5	Area of a Surface of Revolution
17	7.6	Applications to Physics/Engineering
18	7.6/7.7	Applications/Differential Equations
19	8.1	Sequences
20	8.1	Sequences cont'd.
21	8.2	Series
22	8.2	Series cont'd.
23	8.3	The Integral and Comparison Tests
24	8.3	The Integral and Comparison Tests cont'd.
25	Exam 2 Review	Exam 2 Topics
26	8.4	Alternating Series, Absolute Convergence, Ratio and Root Tests
27	8.4	Alternating Series, Absolute Convergence, Ratio and Root Tests cont'd.
28	8.5	Power Series
29	8.5	Power Series cont'd.
30	8.6	Representing Functions as Power Series
31	8.6/8.7	Rep. Funcs cont'd./Taylor & Maclaurin Series
32	8.7	Taylor & Maclaurin Series
33	8.7	Taylor & Maclaurin Series cont'd.
34	8.8	Applications.of Taylor Polynomials
35	9.1	Parametric Curves
36	9.2	Calculus of Parametric Curves
37	Exam 3 Review	Exam 3 Topics
38	9.2	Calculus of Parametric Curves cont'd.
39	9.3	Polar Coordinates
40	9.3	Polar Coordinates cont'd.
41	9.4	Area/Length in Polar
42	9.4/Conics	Area/Length in Polar con't.d / Conics
43	Conics	Conics
44	Final Review	Cumulative

6.2	Trigonometric Substitutions	• Review trigonometric identities including 　▫ $\sin^2\theta + \cos^2\theta = 1$ 　▫ $\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$ • Review antiderivatives of trig functions including 　▫ $\int \tan x\,dx = \ln\lvert\sec x\rvert + C$ 　▫ $\int \sec x\,dx = \ln\lvert\sec x + \tan x\rvert + C$ 　▫ $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$ 　▫ $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C$ • Use trigonometric identities to integrate trigonometric functions including 　▫ $\int \sin^m x \cos^n x\,dx$ 　▫ $\int \tan^m x \sec^n x\,dx$ • Perform trigonometric substitutions for $\sqrt{a^2-x^2}, \sqrt{a^2+x^2}, \sqrt{x^2-a^2}$. • Evaluate an integral by completing a square.
6.3	Partial Fractions	• Decompose a rational function into simpler fractions. 　– Denominators will be linear or quadratic factors, possibly repeated. 　– Review factoring of polynomials, including high order polynomials. 　– For improper functions begin by dividing polynomials. • Use the decomposition to integrate the rational function.
6.5	Approximate Integration	• Estimate definite integrals using midpoint rule and trapezoidal rule. • Draw appropriate diagrams to illustrate the approximation. • Determine whether an approximation is an underestimate or overestimate. • Determine error bounds for midpoint and trapezoidal approximations. • *[Skip section on Simpson's Rule.]*

6.6	Improper Integrals	• Evaluate an integral with an infinite interval by rewriting in the form of a limit. • Evaluate an integral with a discontinuous integrand by rewriting in the form of a limit. • Determine the convergence or divergence of $\int_1^\infty \frac{1}{x^p}\,dx$. • Apply the Comparison Test to determine the convergence or divergence of an improper integral.
7.1	Areas between Curves	• Calculate the area of a region bounded by $y = f(x)$ and $y = g(x)$. • Calculate the area of a region bounded by $x = f(y)$ and $x = g(y)$.
7.2	Volumes	• Draw a diagram to illustrate a solid formed by rotating a region about a line $y = k$ or $x = k$. • Find the volume of a solid of revolution using the Disk Method. • Find the volume of a solid of revolution using the Washer Method. • Find the volume of a solid by slicing.
7.3	Volumes by Cylindrical Shells	• Find the volume of a solid of revolution using the Shell Method. • Decide whether Disk/Washer or Shell Method is the best approach for calculating the volume of a given solid.
7.4	Arc Length	• Calculate the length of a curve $y = f(x)$ or $x = g(y)$ on a given interval. • Determine an arc length function.
7.5	Area of a Surface of Revolution	• Find the area of a surface formed by rotating a curve $y = f(x)$ or $x = g(y)$ about a line. – Axis of rotation may be $y = k$ or $x = k$. • Determine whether the differential ds should be expressed in terms of dx or dy.

7.6	Applications to Physics/Engineering	• Calculate the amount of work done when moving an object.
		• Determine moments and centers of mass for
		  – point masses in 1D and 2D systems.
		  – a 2D region bounded by a curve and the x-axis or y-axis.
		  – a 2D region bounded by two curves.
		• Given the centroid of a region, use the Theorem of Pappus to compute the volume of the solid of revolution formed by rotating the region about a line.
		• *[Skip section on Hydrostatic Pressure and Force].*

7.7	Differential Equations	• Recognize and solve separable equations, possibly with initial values. • Solve logistic growth problems. • *[Skip sections on Mixing Problems and Direction Fields.]*
8.1	Sequences	• Calculate the limit of a sequence. • Recognize the geometric $\{r^n\}$ sequence. • Recognize monotonic bounded sequences. • Determine the convergence of an alternating sequence $\{a_n\}$ by examining $\{\lvert a_n \rvert\}$. • Use the Squeeze Theorem to determine the convergence of a sequence. • Determine the convergence of a sequence defined recursively. • Simplify expressions involving factorial.
8.2	Series	• Calculate the partial sums of a series. • Calculate the sum of a series by evaluating the limit of partial sums. • Calculate the sum of a geometric series. • Prove that the harmonic series diverges. • Use the Test for Divergence to demonstrate divergence of a series. • Recognize and calculate the sum of a telescoping series. • Reindex series to begin at any integer value.

8.3	The Integral and Comparison Tests	• Determine the convergence or divergence of a series by – applying the Integral Test. – recognizing a p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$. – using the (Direct) Comparison Test. – using the Limit Comparison Test.
8.4	Alternating Series, Absolute Convergence, Ratio and Root Tests	• Alternating Series 1. Use the Alternating Series Test to determine convergence or divergence. 2. Use the Alternating Series Estimation Theorem to find an upper bound for the error of an approximation. 3. Determine the number of terms needed to ensure a given error bound for an approximation. 4. Prove that the alternating harmonic series converges. • Distinguish between absolute and conditional convergence of a series. • Determine the convergence or divergence of a series by applying the Ratio or Root Test.
8.5	Power Series	• Find the center, radius and interval of convergence of a power series. – Apply the Ratio Test to find an interval of absolute convergence. – Check endpoints separately for possible absolute or conditional convergence. • Radius of convergence may equal 0, ∞ or a constant.
8.6	Representing Functions as Power Series	• Find power series for functions that correspond to geometric series. • Differentiate or integrate a power series to obtain another power series with the same radius of convergence. • Find power series for $\frac{1}{1-x}$ and its derivatives. • Find the power series for $\ln(1+x)$ and $\tan^{-1} x$.

8.7	Taylor & Maclaurin Series	• Find the Taylor or Maclaurin series of a function using its derivatives. • Find the nth order Taylor polynomial of a Taylor series. • Determine the Maclaurin series for e^x, $\sin x$ and $\cos x$,. • Use important Maclaurin series to compute the sums of power series and to find power series for functions. • Evaluate a binomial coefficient $\binom{k}{n}$. • Use the binomial series formula to find Maclaurin series for $(1+x)^k$ for various values of k. • Calculate the product or quotient of power series.
8.8	Applications.of Taylor Polynomials	• Apply Taylor's Formula to find an upper bound for the remainder when a Taylor polynomial $T_n(x)$ is used to approximate a Taylor series. • Use Taylor series to solve physics applications.
9.1	Parametric Curves	• Graph a parametric curve $x = f(t),\ \ y = g(t)$ and indicate direction of movement. • Convert between Cartesian and parametric forms of a curve. • Recognize parametric equations for a circle or ellipse. • Derive the parametric equations for a cycloid.
9.2	Calculus of Parametric Curves	• Find the tangent slope to a parametric curve. • Calculate the area of a region bounded by a parametric curve. • Find the length of a parametric curve.
9.3	Polar Coordinates	• Convert between polar and Cartesian coordinates. • Graph polar curves $r = f(\theta)$. • Recognize common polar curves, including circles, cardioids, lemniscates, n-leaved roses, and spirals. • Calculate the tangent slope to a polar curve.

9.4	Areas and Lengths in Polar Coordinates	• Find the points where two polar curves intersect. • Find the area of a region bounded by a polar curve. • Find the area of a region bounded by two polar curves. • Calculate the length of a polar curve.
Handout	Conic Sections	• Identify the major/minor axes and foci of an ellipse given its equation in Cartesian coordinates. • Identify the major axis, foci, and asymptotes of a hyperbola given its equation in Cartesian coordinates. • Find an equation for a parabola given its focus and directrix. • *[Skip conic sections in polar form.]*