2. Apply the root test to each of the series below. Does it imply that the series converges or diverges, or is the test inconclusive?

A. $\displaystyle\sum_{n=1}^{\infty}\left(\frac{2n^3+34n+5}{n^3+7}\right)^n$

The root test requires us to analyze $\rho = \lim_{n\to\infty}|a_n|^{1/n}$. Then for the above series,

$$\rho = \lim_{n\to\infty}\left|\left(\frac{2n^3+34n+5}{n^3+7}\right)^n\right|^{1/n} = \lim_{n\to\infty}\left|\left(\frac{2n^3+34n+5}{n^3+7}\right)\right| = 2 > 1$$

Since $\rho > 1$, the series **diverges**.

B. $\displaystyle\sum_{n=2}^{\infty}\frac{(\ln(n^2))^n}{n^{2n}}$

$$\rho = \lim_{n\to\infty}\left|\frac{(\ln(n^2))^n}{n^{2n}}\right|^{1/n} = \lim_{n\to\infty}\left|\frac{\ln(n^2)}{n^2}\right| = \lim_{n\to\infty}\frac{\ln(n^2)}{n^2}$$

$$=_{LH} \lim_{n\to\infty}\frac{2n/n^2}{2n} = \lim_{n\to\infty}\frac{1}{n^2} = 0 < 1$$

By the ratio test we conclude that the series **converges**.

C. $\displaystyle\sum_{n=1}^{\infty}\left(\frac{3-2n+10n^2}{10n^2+n+7}\right)^n$

$$\rho = \lim_{n\to\infty}\left|\left(\frac{3-2n+10n^2}{10n^2+n+7}\right)^n\right|^{1/n} = \lim_{n\to\infty}\left|\left(\frac{3-2n+10n^2}{10n^2+n+7}\right)\right| = 1$$

The root test is **inconclusive**.