Series $\sum_{k=0}^{\infty} a_k$ converges / diverges?

Step 1:

$\lim_{k\to\infty} a_k = 0?$

If NO, diverges by Divergence Test.

If Yes, go to

Step 2: Look at a_k

Special Series?

- Telescoping series $a_k = ♥ - ♦$
  $S_n = a_1 + a_2 + \cdots + a_n = \text{formula of n}$, $S_\infty = \lim_{n\to\infty} S_n = ?$
- Geometric series $\sum_{k=0}^{\infty} ar^k = \begin{cases} \text{diverges} & \text{if } |r| \geq 1 \\ \text{converges to } \frac{a}{1-r} & \text{if } |r| < 1 \end{cases}$
- P-series $\sum_{k=0}^{\infty} \frac{1}{k^p} = \begin{cases} \text{diverges} & \text{if } p \leq 1 \\ \text{converges} & \text{if } p > 1 \end{cases}$

Integral Test

a_k looks like an integrable function. e.g. $\frac{(\ln k)^2}{k}$

- $f(x)$ is continuous, positive and decreasing.
- $\int_1^{\infty} f(x)dx$ converges?
- If yes, then series $\sum_{k=0}^{\infty} a_k$ converges. If no, then series $\sum_{k=0}^{\infty} a_k$ diverges.

Ratio Test

a_k has $k!, a^k, k^k$

- $r = \lim_{k\to\infty} \left|\frac{a_{k+1}}{a_k}\right|$
- Series converges if $r < 1$, diverges if $r > 1$, inconclusive if $r = 1$.

Root Test

a_k looks like $(\quad)^k$

- $q = \lim_{k\to\infty} \sqrt[k]{|a_k|}$
- Series converges if $q < 1$, diverges if $q > 1$, inconclusive if $q = 1$.

LCT (Limit Comparison Test)

a_k is comparable to $\frac{1}{k^p}$

- $L = \lim_{k\to\infty} \frac{|a_k|}{\frac{1}{k^p}}$
- If $L = 1$, then $\sum_{k=0}^{\infty} a_k$ and $\sum_{k=0}^{\infty} \frac{1}{k^p}$ both conv. or both div.
- If $L \neq 1$, remember "Big conv $\Rightarrow$ Small conv, Small div $\Rightarrow$ Big div".

Alternating Series $\sum_{k=0}^{\infty}(-1)^k a_k$ diverges/ converges absolutely converges conditionally?

Step 1:

$\lim_{k\to\infty} a_k = 0?$

If NO, diverges by Divergence Test.

If Yes, go to

Step 2: $\sum_{k=0}^{\infty} a_k$ converges?

If YES, $\sum_{k=0}^{\infty}(-1)^k a_k$ **converges absolutely.**

If NO, go to

Step 3: a_k is decreasing, by Alternating Series Test, $\sum_{k=0}^{\infty}(-1)^k a_k$ converges.

Hence, $\sum_{k=0}^{\infty}(-1)^k a_k$ **converges conditionally.**