Limits
The Existence of a Limit The limit of $f(x)$ as x approaches a is L if and only if:

$\lim_{x \to a^-} f(x) = L$ $\lim_{x \to a^+} f(x) = L$

<u>Definition of Limit</u>:

Let f be a function defined on an open interval containing c (except possibly at c) and let L be a real number.

Then $\lim_{x \to a} f(x) = L$

means that for each $\varepsilon > 0$ there exists a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.

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Definition of Limit

Let f be a function defined on an open interval containing c and let L be a real number. The statement:

$$\lim_{x \to a} f(x) = L$$

means that for each $\epsilon > 0$ there exists a $\delta > 0$ such that

$$\text{if } 0 < |x - a| < \delta, \quad \text{then } |f(x) - L| < \epsilon$$

Tip :

Direct substitution: Plug in $f(a)$ and see if it provides a legal answer. If so then L = $f(a)$.

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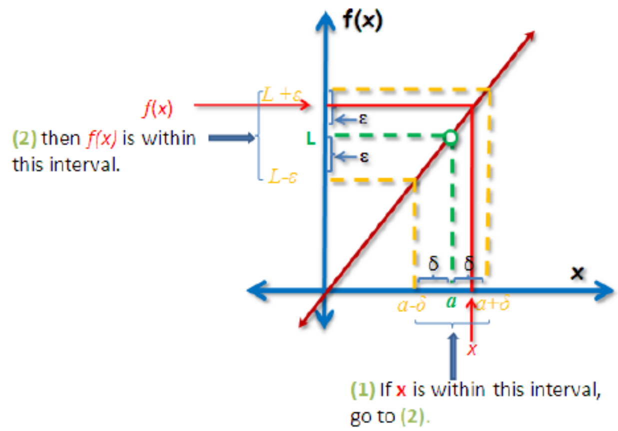

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Two Special Trig Limits

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$$

Limits and Continuity:

A function $y = f(x)$ is continuous at $x = a$ if

i). $f(a)$ exists

ii). $\lim_{x \to a} f(x)$ exists

iii). $\lim_{x \to a} f(x) = f(a)$

Otherwise, f is discontinuous at x = a.

The limit $\lim_{x \to a} f(x)$ exists if and only if both corresponding one-sided limits exist and are equal – that is,

$$\lim_{x \to a} f(x) = L \to \lim_{x \to a^+} f(x) = L = \lim_{x \to a^-} f(x)$$

Limits of Rational Functions as $x \to \pm\infty$

i). $\lim_{x \to \pm\infty} \frac{f(x)}{g(x)} = 0$ if the degree of $f(x) <$ the degree of $g(x)$

Example: $$\lim_{x \to \infty} \frac{x^2 - 2x}{x^3 + 3} = 0$$

ii). $\lim_{x \to \pm\infty} \frac{f(x)}{g(x)}$ is infinite if the degrees of $f(x) >$ the degree of $g(x)$

Example: $$\lim_{x \to \infty} \frac{x^3 + 2x}{x^2 - 8} = \infty$$

iii). $\lim_{x \to \pm\infty} \frac{f(x)}{g(x)}$ is finite if the degree of $f(x) =$ the degree of $g(x)$

Example: $$\lim_{x \to \infty} \frac{2x^2 - 3x + 2}{10x - 5x^2} = -\frac{2}{5}$$