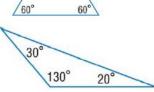
Example 1:

a) Classify the triangle as acute, equiangular, obtuse, or right.

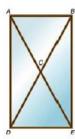


b) Classify the triangle as acute, equiangular, obtuse, or right.



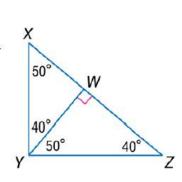
c) The frame of this window design is made up of many triangles.

1) Classify ΔACD.



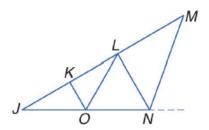
2) Classify ΔADE.

Example 2: Classify \(\Delta XYZ \) as acute, equiangular, obtuse, or right. Explain your reasoning.

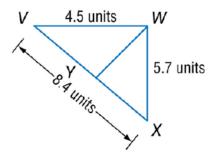


To indicate that sides of a triangle are congruent, an equal number of tick marks are drawn on the corresponding sides.

Example 3: The triangle truss shown is modeled for steel construction. Classify ΔJMN , ΔJKO , and ΔOLN as equilateral, isosceles, or scalene.

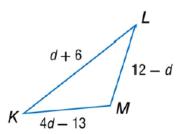


Example 4: If point *Y* is the midpoint of \overline{VX} , and WY = 3.0 units, classify ΔVWY as equilateral, isosceles, or scalene. Explain your reasoning.



Example 5:

a) Find the measures of the sides of isosceles triangle *KLM* with base \overline{KL} .



b) Find the value of x and the measures of each side of an equilateral triangle ABC if AB = 6x - 8, BC = 7 + x, and AC = 13 - x.

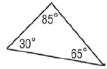
SECTION 4.1 PRACTICE

For numbers 1-3, classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*.

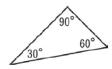
1.



2.



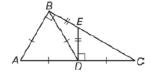
3.



For numbers 4-7, classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*.

 $4.\Delta ABD$

5. Δ*ABC*



6. Δ*EDC*

7. Δ*BDC*

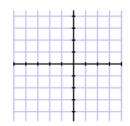
For numbers 8 and 9, for each triangle, find the value of x and the measure of each side.

8. $\triangle FGH$ is an equilateral triangle with FG = x + 5, GH = 3x - 9, and FH = 2x - 2.

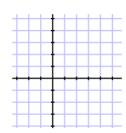
9. $\triangle LMN$ is an isosceles triangle, with LM = LN, LM = 3x - 2, LN = 2x + 1, and MN = 5x - 2.

For numbers 10 - 12, find the measures of the sides of ΔKPL and classify each triangle by its sides.

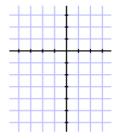
10. K(-3, 2), P(2, 1), L(-2, -3)



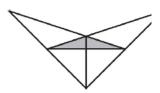
11. K(5, -3), P(3, 4), L(-1, 1)



12. K(-2, -6), P(-4, 0), L(3, -1)



13. Diana entered the design at the right in a logo contest sponsored by a wildlife environmental group. Use a protractor. How many right angles are there?

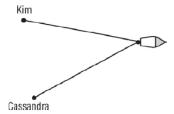


14. Paul is standing in front of a museum exhibition. When he turns his head 60° to the left, he can see a statue by Donatello. When he turns his head 60° to the right, he can see a statue by Della Robbia. The two statues and Paul form the vertices of a triangle. Classify this triangle as acute, right, or obtuse.

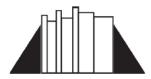
15. Marsha cuts a rectangular piece of paper in half along a diagonal. The result is two triangles. Classify these triangles as acute, right, or obtuse.



16. Kim and Cassandra are waterskiing. They are holding on to ropes that are the same length and tied to the same point on the back of a speed boat. The boat is going full speed ahead and the ropes are fully taut. Kim, Cassandra, and the point where the ropes are tied on the boat form the vertices of a triangle. The distance between Kim and Cassandra is never equal to the length of the ropes. Classify the triangle as equilateral, isosceles, or scalene.

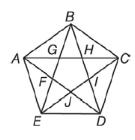


17. Two bookends are shaped like right triangles. The bottom side of each triangle is exactly half as long as the slanted side of the triangle. If all the books between the bookends are removed and they are pushed together, they will form a single triangle. Classify the triangle that can be formed as equilateral, isosceles, or scalene.



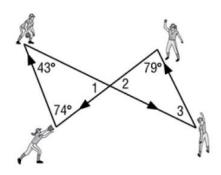
18. Suzanne saw this pattern on a pentagonal floor tile. She noticed many different kinds of triangles were created by the lines on the tile.

a) Identify five triangles that appear to be acute isosceles triangles.

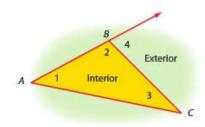


b) Identify five triangles that appear to be obtuse isosceles triangles.

Example 1: SOFTBALL The diagram shows the path of the softball in a drill developed by four players. Find the measure of each numbered angle.



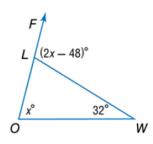
Exterior Angle: an angle formed by one side of the triangle and the extension of an adjacent side. Each exterior angle of a triangle has two remote interior angles that are not adjacent to the exterior angle.



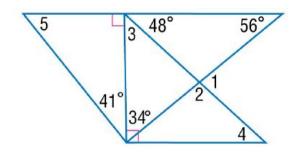
 $\angle 4$ is an exterior angle of $\triangle ABC$.

Its two remote interior angles are $\angle 1$ and $\angle 3$.

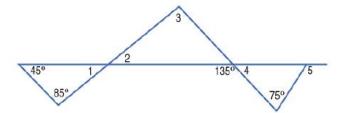
Example 2: Find the measure of $\angle FLW$ in the fenced flower garden shown.



Example 3: a) Find the measure of each numbered angle.



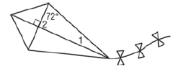
b) Find $m \angle 3$.



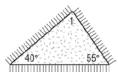
SECTION 4.2 PRACTICE

For numbers 1 and 2, find the measure of each numbered angle.

1.



2.

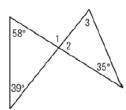


For numbers 3 - 5, find each measure.

 $3. m \angle 1$

 $4. m\angle 2$

 $5. m \angle 3$



For numbers 6 - 9, find each measure.

6. $m \angle 1$

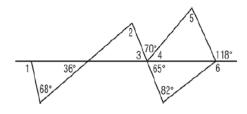
7. $m \angle 4$

 $8. m \angle 3$

9. *m*∠2

10. *m*∠5

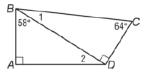
11. *m*∠6



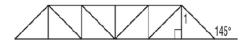
For numbers 12 and 13, find each measure.

12. *m*∠1

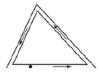
13. *m*∠2



14. The diagram shows an example of the Pratt Truss used in bridge construction. Use the diagram to find $m \angle 1$.

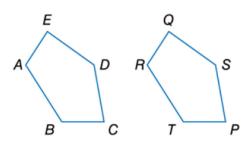


15. Eric walks around a triangular path. At each corner, he records the measure of the angle he creates. He makes one complete circuit around the path. What is the sum of the three angle measures that he wrote down?



Example 1:

a) Show that the polygons are congruent by identifying all of the congruent corresponding parts. Then write a congruence statement.



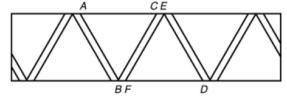
b) The support beams on the fence form congruent triangles. In the figure $\triangle ABC \cong \triangle DEF$, which of the following congruence statements correctly identifies corresponding angles or sides?

a)
$$\angle ABC \cong \angle EFD$$

b)
$$\angle BAC \cong \angle DFE$$

c)
$$\overline{BC} \cong \overline{DE}$$

d)
$$\overline{AC} \cong \overline{DF}$$

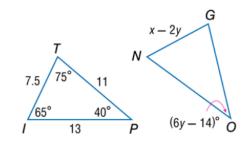


The phrase "if and only if" in the congruent polygon definition means that both the conditional and converse are true. So, if two polygons are congruent, then their corresponding parts are congruent.

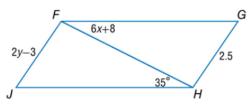
For triangles we say Corresponding Parts of Congruent Triangles are Congruent or CPCTC.

Example 2:

a) In the diagram, $\triangle ITP \cong \triangle NGO$. Find the values of x and y.



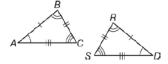
b) In the diagram, $\Delta FHJ \cong \Delta HFG$. Find the values of x and y.



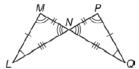
SECTION 4.3 PRACTICE

For numbers 1 and 2, show that the polygons are congruent by identifying all congruent corresponding parts. Then write a congruence statement.

1.

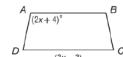


2.



For numbers 3 and 4, polygon $ABCD \cong \text{polygon } PQRS$.

3. Find the value of x.



 $R = \frac{12}{80^{\circ}}$ S

4. Find the value of y.

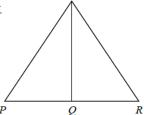
5. Write a two-column proof.

Given:
$$\angle P \cong \angle R$$

$$\frac{\angle PSQ}{PQ} \cong \angle RSQ$$

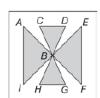
$$\frac{PQ}{PS} \cong \frac{RQ}{RS}$$

Prove: $\triangle PQS \cong \triangle RQS$



Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5. ∠ <i>PQS</i> ≅ ∠ <i>RQS</i>	5.
6.	6. Reflexive Property
7.	7.

- 6. Use the quilting diagram to the right.
- a) Indicate the triangles that appear to be congruent.



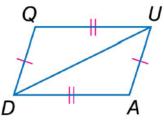
b) Name the congruent angles and congruent sides of a pair of congruent triangles.

Example 1: Write a flow proof.

Given: $\overline{QU} \cong \overline{AD}$

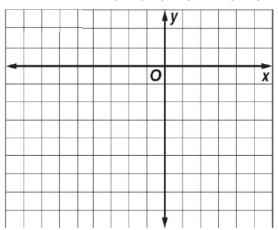
 $\overline{QD}\cong \overline{AU}$

Prove: $\Delta QUD \cong \Delta ADU$



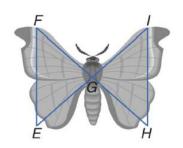
Example 2: $\triangle DVW$ has vertices D(-5,-1), V(-1,-2), and W(-7,-4). $\triangle LPM$ has vertices L(1,-5), P(2,-1), and M(4,-7).

a) Graph both triangles on the same coordinate plane.



- b) Use your graph to make a conjecture as to whether the triangles are congruent. Explain your reasoning.
- c) Write a logical argument that uses coordinate geometry to support the conjecture you made in part b.

Example 3: The wings of one type of moth form two triangles. Write a two-column proof to prove that $\Delta FEG \cong \Delta HIG$ if $\overline{EI} \cong \overline{FH}$, and G is the midpoint of both \overline{EI} and \overline{FH} .



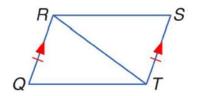
Statements	Reasons
1.	1.
2.	2.
3. $\overline{EG} \cong \overline{IG}$	3.
4.	4. Given
5.	5. Midpoint Theorem
6. ∠ <i>FGE</i> ≅ ∠ <i>HGI</i>	6.
7.	7.

Example 4: Write a paragraph proof.

Given: $\overline{RQ} \parallel \overline{TS}$

 $\overline{RQ} \cong \overline{TS}$

Prove: $\angle Q \cong \angle S$

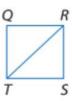


Section 4.4a Homework

Complete the following proofs.

1. Given: $\overline{QR} \cong \overline{SR}$, $\overline{ST} \cong \overline{QT}$ Prove: $\Delta QRT \cong \Delta SRT$

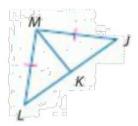
Statements	Reasons
1.	1.
2.	2.
3.	3. Reflexive
4.	4.



2. Given: $\overline{MJ} \cong \overline{ML}$, K is the midpoint of \overline{JL} Prove: $\Delta MJK \cong \Delta MLK$

Statements	Reasons
1.	1. Given
2.	2. Given
3.	3. Midpoint Theorem
4.	4.

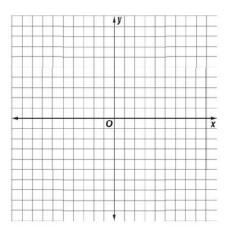
5.



3. Determine whether $\triangle MNO \cong \triangle QRS$. Explain.

5.

$$M(0,-3), N(1,4), O(3,1)$$
 $Q(4,-1), R(6,1), S(9,-1)$



4. Is it possible to prove that the triangles are congruent? If so, state the postulate/theorem. If not, write "not possible".

a)



b



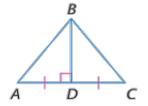
Section 4.4b Homework

Complete the following proof:

1. Given: $\overline{BD} \perp \overline{AC}$, \overline{BD} bisects \overline{AC} .

Prove: $\triangle ABD \cong \triangle CBD$

Statements	Reasons
1.	1. Given
2. $\angle ADB$ and $\angle CDB$ are right angles	2.
3.	3. All right angles are ≅
4. \overline{BD} bisects \overline{AC}	4.
5.	5. Definition of bisect
6. $\overline{BD} \cong \overline{BD}$	6.
7.	7.



2. Given: R is the midpoint of \overline{QS} and \overline{PT}

Prove: $\Delta PRQ \cong \Delta TRS$

Statements	Reasons
1.	1. Given
2.	2. Midpoint Theorem
3. R is the midpoint of \overline{PT}	3.
4. $\overline{PR} \cong \overline{TR}$	4.
5. ∠ <i>PRQ</i> ≅ ∠ <i>TRS</i>	5.
6.	6.



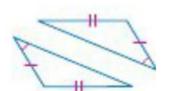
3. Given: ΔXYZ is equilateral, \overline{WY} bisects $\angle XYZ$

Prove: $\overline{XW} \cong \overline{ZW}$

Statements	Reasons
1.	1. Given
2.	2. Definition of equilateral
3. WY bisects ∠XYZ	3.
4. ∠ <i>XYW</i> ≅ ∠ <i>ZYW</i>	4.
5.	5. Reflexive Property
6. $\Delta XYW \cong \Delta ZYW$	6.
7	7.



4. Is it possible to prove that the triangles are congruent? If so, state the postulate/theorem. If not, write "not possible".



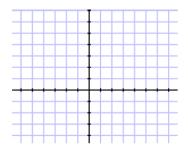




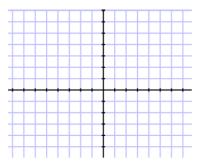
SECTION 4.4 PRACTICE

For numbers 1 and 2, determine whether $\Delta DEF \cong \Delta PQR$ given the coordinates of the vertices. Explain.

1. D(-6, 1), E(1, 2), F(-1, -4), P(0, 5), Q(7, 6), R(5, 0)



 $2.\ D(-7,-3),\ E(-4,-1),\ F(-2,-5),\ P(2,-2),\ Q(5,-4),\ R(0,-5)$



3. Write a flow proof.

Given: $\overline{RS} \cong \overline{TS}$

V is the midpoint of \overline{RT}

Prove: $\Delta RSV \cong \Delta TSV$



For numbers 4-6, determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove congruence, write *not possible*.

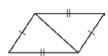
4.



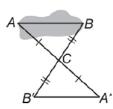
5



6.



7. To measure the width of a sinkhole on his property, Harmon marked off congruent triangles as shown in the diagram. How does he know that the lengths A'B' and AB are equal?



8. Tyson had three sticks of lengths 24 inches, 28 inches, and 30 inches. Is it possible to make two noncongruent triangles using the same three sticks? Explain.

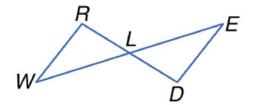
9. Sonia made a sheet of baklava. She has markings on her pan so that she can cut them into large squares. After she cuts the pastry in squares, she cuts them diagonally to form two congruent triangles. Which postulate could you use to prove the two triangles congruent?
10. Carl had a piece of cake in the shape of an isosceles triangle with angles 26°, 77°, and 77°. He wanted to divide it into two equal parts, so he cut it through the middle of the 26° angle to the midpoint of the opposite side. He says that because he is dividing it at the midpoint of a side, the two pieces are congruent. Is this enough information? Explain.
11. Tammy installs bathroom tiles. Her current job requires tiles that are equilateral triangles and all the tiles have to be congruent to each other. She has a big sack of tiles all in the shape of equilateral triangles. Although she knows that all the tiles are equilateral, she is not sure they are all the same size. What must she measure on each tile to be sure they are congruent? Explain.
12. An investigator at a crime scene found a triangular piece of torn fabric. The investigator remembered that one of the suspects had a triangular hole in their coat. Perhaps it was a match. Unfortunately, to avoid tampering with evidence, the investigator did not want to touch the fabric and could not fit it to the coat directly. a) If the investigator measures all three side lengths of the fabric and the hole, can the investigator make a conclusion about whether or not the hole could have been filled by the fabric?

b) If the investigator measures two sides of the fabric and the included angle and then measures two sides of the hole and the included angle can be determine if it is a match? Explain.

Example 1: Write a two-column proof.

Given: L is the midpoint of \overline{WE}

 $\overline{WR} \parallel \overline{ED}$ Proof: $\Delta WRL \cong \Delta EDL$

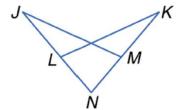


Statements	Reasons
1.	1. Given
2. $\overline{WL} \cong \overline{EL}$	2.
3.	3.
$4. \angle W \cong \angle E$	4.
5. ∠ <i>WLR</i> ≅ ∠ <i>ELD</i>	5.
6.	6.

Example 2: Write a paragraph proof.

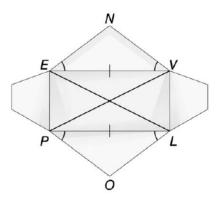
Given: $\angle NKL \cong \angle NJM$

 $\frac{\overline{KL} \cong \overline{JM}}{LN \cong \overline{MN}}$ Proof: $\overline{LN} \cong \overline{MN}$



You can use congruent triangles to measure distances that are difficult to measure directly.

Example 3: Barbara designs a paper template for a certain envelope. She designs the top and bottom flaps to be isosceles triangles that have congruent bases and base angles. If EV = 8 cm and the height of the isosceles triangle is 3 cm, find PO.



Example 4: Determine whether each pair of triangles is congruent. If yes, state the postulate/theorem that apples.

a)



b)



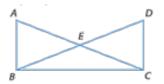
c)



Example 5: Complete the proof.

Given: $\overline{AB} \perp \overline{BC}$, $\overline{DC} \perp \overline{BC}$, $\overline{AC} \cong \overline{BD}$

Prove: $\triangle ABC \cong \triangle DCB$



1.
2.
3. Given
4. Definition of ⊥
5.
6. Reflexive Property
7.

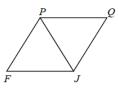
Section 4.5a Homework

Complete the following proofs.

1. Given: $\angle QPJ \cong \angle FJP$, $\angle QJP \cong \angle FPJ$

Prove: $\triangle QPJ \cong \triangle FJP$

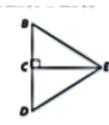
Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.



2. Given: $\overline{\textit{CE}}$ bisects $\angle \textit{BED}$, $\angle \textit{BCE}$ and $\angle \textit{ECD}$ are right angles.

Prove: $\Delta ECB \cong \Delta ECD$

Statements	Reasons
1.	1. Given
2.	2. Definition of bisect
3.	3. Given
4.	4. All right angles are ≅
5.	5. Reflexive Property
6.	6.



3. Given: $\overline{MS} \cong \overline{RQ}$, $\overline{MS} \parallel \overline{RQ}$

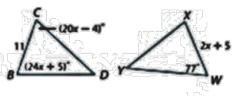
Prove: $\triangle MSP \cong \triangle RQP$

Statements	Reasons
1.	1. Given
2.	2. Given
3. ∠ <i>MSP</i> ≅ ∠ <i>RQP</i>	3.
4.	4. Alternate Interior ∠s Theorem
5.	5.



4. Find the value of the variable that yields the given congruent triangles.

 $\Delta BCD \cong \Delta WXY$



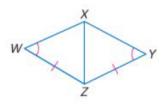
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Complete the following proofs.

1. Given: $\angle W \cong \angle Y$, $\overline{WZ} \cong \overline{YZ}$, \overline{XZ} bisects $\angle WXY$

Prove: $\Delta XWZ \cong \Delta XYZ$

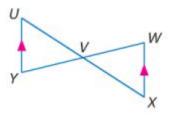
Statements	Reasons
1.	1. Given
2.	2. Given
3.	3. Given
4. ∠ <i>WX</i> Z ≅ ∠ <i>YX</i> Z	4.
5.	5.



2. Given: V is the midpoint of \overline{YW} , $\overline{UY} \parallel \overline{XW}$

Prove: $\Delta UVY \cong \Delta XVW$

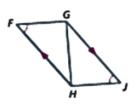
Statements	Reasons
1.	1. Given
2. $\overline{YV} \cong \overline{WV}$	2.
3.	3. Given
4. ∠ <i>YUV</i> ≅ ∠ <i>VXW</i>	4.
5. ∠ <i>UVY</i> ≅ ∠ <i>XVW</i>	5.
6.	6.



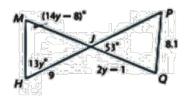
3. Given: $\angle F \cong \angle J$, $\overline{FH} \parallel \overline{GJ}$

Prove: $\overline{FH} \cong \overline{JG}$

Statements	Reasons
1.	1. Given
2.	2. Given
3. ∠ <i>FHG</i> ≅ ∠ <i>JGH</i>	3.
4.	4. Reflexive
5. $\triangle FHG \cong \triangle JGH$	5.
6.	6.



4. Find the value of the variable that yields the given congruent triangles; $\Delta MHJ \cong \Delta PQJ$



SECTION 4.5 PRACTICE

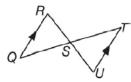
For numbers 1 and 2, write the specified type of proof.

1. Write a flow proof.

Given: S is the midpoint of \overline{QT}

 $\overline{QR} \parallel \overline{TU}$

Prove: $\triangle QSR \cong \Delta TSU$

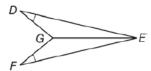


2. Write a paragraph proof.

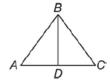
Given: $\angle D \cong \angle F$

 \overline{GE} bisects $\angle DEF$.

Prove: $\overline{DG} \cong \overline{FG}$



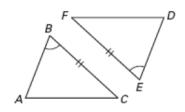
For Numbers 3 and 4, use the following information. An architect used the window design in the diagram when remodeling an art studio. \overline{AB} and \overline{CB} each measure 3 feet.



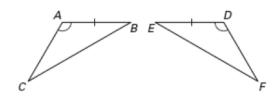
- 3. Suppose *D* is the midpoint of \overline{AC} . Determine whether $\triangle ABD \cong \triangle CBD$. Justify your answer.
- 4. Suppose $\angle A \cong \angle C$. Determine whether $\triangle ABD \cong \triangle CBD$. Justify your answer.
- 5. Two door stops have cross-sections that are right triangles. They both have a 20° angle and the length of the side betwe and 20° angles are equal. Are the cross sections congruent? Explain.

I. State the third congruence that must be given to prove that $\triangle ABC \cong \triangle DEF$, using the given shortcut. (Determine the third piece of information that would have to be marked in order to use the given shortcut)

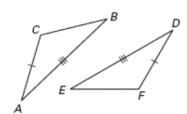
1. *ASA* ≅



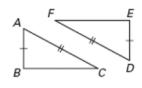
2. *AAS* ≅



3. *SSS* ≅

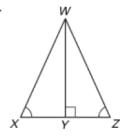


4. *SAS* ≅ _____

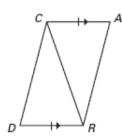


II. Is it possible to prove that the triangles are congruent? If so, which shortcut can you use?

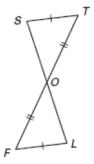
5.



6.



7.

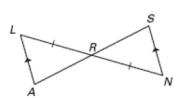


III. Complete the proof by supplying the reasons.

8. Given: $\overline{LA} \mid \mid \overline{SN} , \overline{LR} \cong \overline{NR}$

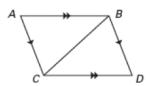
Prove: $\Delta LAR \cong \Delta NSR$

Statements	Reasons
1. <i>LA</i> <i>SN</i>	1.
2. $\angle ALR \cong \angle SNR$	2.
3. $\overline{LR} \cong \overline{NR}$	3.
4. $\angle LRA \cong \angle NRS$	4.
5. $\Delta LAR \cong \Delta NSR$	3.



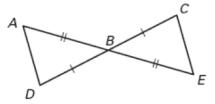
9. Given: $\overline{AB} \mid \mid \overline{CD}, \overline{AC} \mid \mid \overline{BD}$ Prove: $\triangle ABC \cong \triangle DCB$

Statements	Reasons
1. $\overline{AB} \mid \mid \overline{CD}$	1.
2. ∠ <i>ABC</i> ≅ ∠ <i>BCD</i>	2. Alt. Int. ∠s
3. $\overline{AC} \mid \mid \overline{BD}$	3.
4. ∠ <i>ACB</i> ≅ ∠ <i>DBC</i>	4.
5. $\overline{BC} \cong \overline{BC}$	5.
6. $\triangle ABC \cong \triangle DCB$	6.



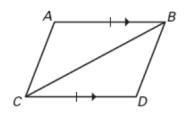
10. Given: *B* is the midpoint of \overline{AE} , *B* is the midpoint of \overline{CD} Prove: $\angle D \cong \angle C$

Statements	Reasons
1. B is the midpoint of \overline{AE} 2. $\overline{AB} \cong \overline{BE}$ 3. B is the midpoint of \overline{CD} 4. $\overline{DB} \cong \overline{BC}$ 5. $\angle ABD \cong \angle CBE$ 6. $\triangle ABD \cong \triangle EBC$	1. 2. 3. 4. 5. 6. 7
7.	1



11. Given: $\overline{AB} \mid \mid \overline{CD}$, $\overline{AB} \cong \overline{CD}$ Prove: $\triangle ABC \cong \triangle DCB$

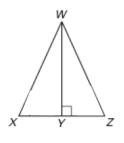
Statements	Reasons
1. $\overline{AB} \mid \mid \overline{CD}$	1.
2. $\angle ABC \cong \angle BCD$	2.
$3. \overline{AB} \cong \overline{CD}$	3.
4.	5.
$5. \ \Delta ABC \cong \Delta DCB$	



12. Given: $\overline{WY} \perp \overline{XZ}$, $\overline{XW} \cong \overline{ZW}$

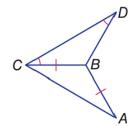
Prove: $\angle X \cong \angle Z$

VC. ∠1 ≡ ∠2	
Statements	Reasons
$1. \overline{WY} \perp \overline{XZ}$	1.
2. $\angle WYZ$ and $\angle WYX$ are right angles	2.
3.	3. Given
4.	 Reflexive Property
5. $\Delta WXY \cong \Delta WZY$	5.
6.	6.

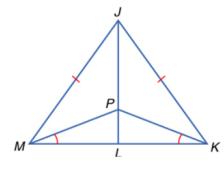


Example 1:

a) Name two unmarked congruent angles.



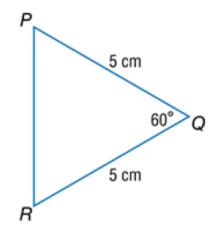
- b) Name two unmarked congruent segments.
- c) Which statement correctly names two congruent angles?
- a) $\angle PJM \cong \angle PMJ$
- b) $\angle JMK \cong \angle JKM$
- c) $\angle KJP \cong \angle JKP$
- d) $\angle PML \cong \angle PLK$



Example 2:

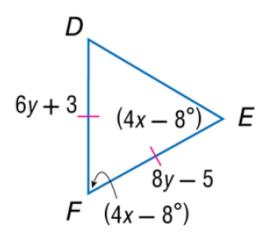
a) Find $m \angle R$

b) Find PR.



You can use the properties of equilateral triangles and algebra to find missing values.

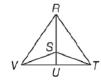
Example 3: Find the value of each variable.



SECTION 4.6 PRACTICE

For numbers 1 - 4, refer to the figure at the right.

1. If $\overline{RV} \cong \overline{RT}$, name two congruent angles.



- 2. If $\overline{RS} \cong \overline{SV}$, name two congruent angles.
- 3. If $\angle SRT \cong \angle STR$, name two congruent segments.
- 4. If $\angle STV \cong \angle SVT$, name two congruent segments.

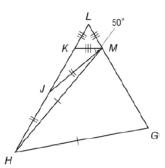
For numbers 5-9, find each measure.

5. *m∠KML*

6. *m∠HMG*

7. *m∠GHM*

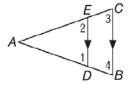
8. If $m \angle HJM = 145^{\circ}$, find $m \angle MHJ$



- 9. If $m \angle G = 67^{\circ}$, find $m \angle GHM$
- 10. Write a two-column proof.

Given: $\overline{DE} \parallel \overline{BC}$

$$\frac{\angle 1 \cong \angle 2}{AB \cong AC}$$
Prove: $AB \cong AC$



11. A pennant for the sports teams at Lincoln High School is in the shape of an isosceles triangle. If the measure of the vertex angle is 18° , find the measure of each base angle.

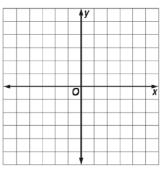


Section 4.8 Notes: Triangles and Coordinate Proof

Coordinate proofs use figures in the coordinate plane and algebra to prove geometric concepts. The first step in a coordinate proof is placing the figure on the coordinate plane.

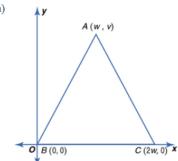
Example 1:

a) Position and label right triangle XYZ with leg $\overline{XZ}\,\,d$ units long on the coordinate plane.

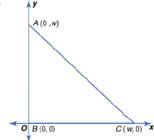


b) Which would be the best way to position and label equilateral triangle ABC with side \overline{BC} w units long on the coordinate plane?

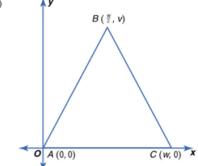
a)



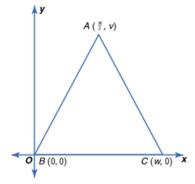
b)



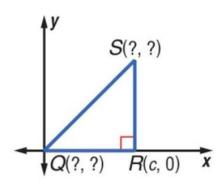
c)



d)



Example 2: Name the missing coordinates of isosceles right triangle QRS.

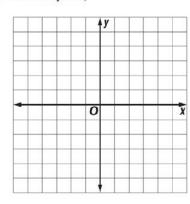


Example 3: Write a coordinate proof to prove that the segment that joins the vertex angle of an isosceles triangle to the midpoint of its base is perpendicular to the base. (Hint: First draw and label an isosceles triangle on the coordinate plane.)

Given: \(\Delta XYZ \) is isosceles.

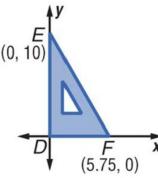
$$\overline{XW} \cong \overline{WZ}$$

Prove: $\overline{YW} \perp \overline{XZ}$



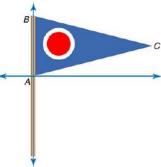
Example 4:

a) Write a coordinate proof to prove that the outside of this drafter's tool is shaped like a right triangle. The length of one side is 10 inches and the length of another side is 5.75 inches.



b) Tracy wants to write a coordinate proof to prove this flag is shaped like an isosceles triangle. The altitude is 16 inches and the base is 10 inches. What ordered pair should she use for point C?

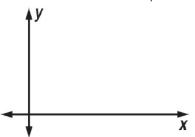
- a) (10, 10)
- b) (10, 5)
- c) (16, 10)
- d) (16, 5)



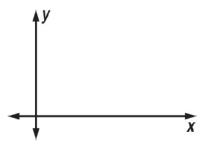
SECTION 4.8 PRACTICE

For numbers 1-3, position and label each triangle on the coordinate plane.

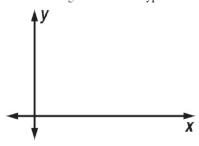
1. equilateral ΔSWY with sides $\frac{1}{4}a$ units long.



2. isosceles $\triangle BLP$ with base \overline{BL} 3b units long.

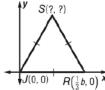


3. isosceles right ΔDGJ with hypotenuse \overline{DJ} and legs 2a units long.

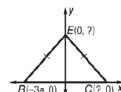


For numbers 4 - 6, name the missing coordinates of each triangle.

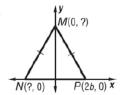
4.



5.



6.



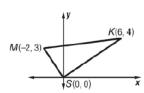
For numbers 7 and 8, use the following information.

Karina lives 6 miles east and 4 miles north of her high school. After school she works part time at the mall in a music store. The mall is 2 miles west and 3 miles north of the school.

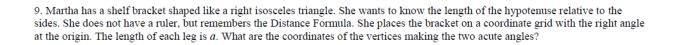
7. Write a coordinate proof to prove that Karina's high school, her home, and the mall are at the vertices of a right triangle.

Given: ΔSKM

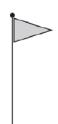
Prove: $\triangle SKM$ is a right triangle.



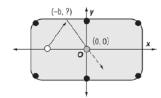
8. Find the distance between the mall and Karina's home.



10. A flag is shaped like an isosceles triangle. A designer would like to make a drawing of the flag on a coordinate plane. She positions it so that the base of the triangle is on the *y*-axis with one endpoint located at (0, 0). She locates the tip of the flag at $\left(a, \frac{b}{2}\right)$ What are the coordinates of the third vertex?



11. The figure shows a situation on a billiard table. What are the coordinates of the cue ball before it is hit and the point where the cue ball hits the edge of the table?



12. The entrance to Matt's tent makes an isosceles triangle. If placed on a coordinate grid with the base on the x-axis and the left corner at the origin, the right corner would be at (6, 0) and the vertex angle would be at (3, 4). Prove that it is an isosceles triangle.

13. An engineer is designing a roadway. Three roads intersect to form a triangle. The engineer marks two points of the triangle at (-5, 0) and (5, 0) on a coordinate plane.

a) Describe the set of points in the coordinate plane that could not be used as the third vertex of the triangle.

b) Describe the set of points in the coordinate plane that would make the vertex of an isosceles triangle together with the two congruent sides.

c) Describe the set of points in the coordinate plane that would make a right triangle with the other two points if the right angle is located at (-5, 0).