CHAPTER 2 REASONING AND PROOF

Postulates 2.1 – 2.7

2.1 Through any two points, there is exactly one line.

2.2 Through any three points not on the same line, there is exactly one plane.

2.3 A line contains at least two points.

2.4 A plane contains at least three points not on the same line.

2.5 If two points lie in a plane, then the entire line containing those points lies in that plane.

2.6 If two lines intersect, then their intersection is exactly one point.

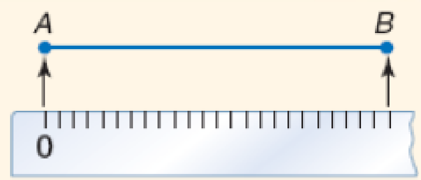
2.7 If two planes intersect, then their intersection is a line.

Theorem 2.1 Midpoint Theorem

If M is the midpoint of $\overline{AB}$, then $\overline{AM} \cong \overline{MB}$.

Postulate 2.8 Ruler Postulate

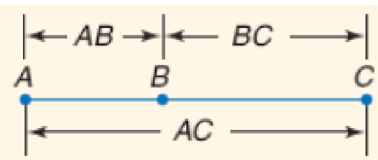
The points on any line or line segment can be paired with real numbers so that, given any two points A and B on a line, A corresponds to zero, and B corresponds to a positive real number.



Postulate 2.9 Segment Addition Postulate

If A, B, and C are collinear and B is between A and C, then $AB + BC = AC$.

If $AB + BC = AC$, then B is between A and C.



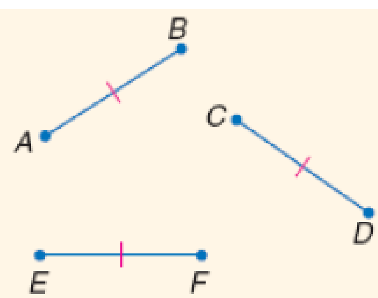
Theorem 2.2 Segment Congruence

Congruence of segments is reflexive, symmetric, and transitive.

Reflexive Property $\overline{AB} \cong \overline{AB}$

Symmetric Property If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.

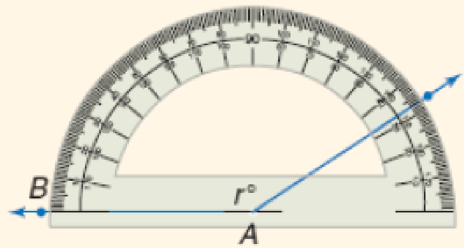
Transitive Property If $\overline{AB} \cong \overline{CD}$, and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.



CHAPTER 2 REASONING AND PROOF

Postulate 2.10 Protractor Postulate

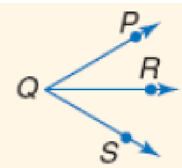
Given $\overrightarrow{AB}$ and a number r between 0 and 180, there is exactly one ray with endpoint A, extending on either side of $\overrightarrow{AB}$, such that the measure of the angle formed is r.



Postulate 2.11 Angle Addition Postulate

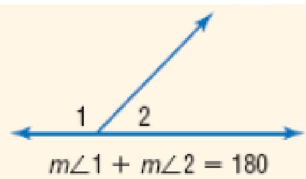
If R is in the interior of $\angle PQS$, then $m\angle PQR + m\angle RQS = m\angle PQS$.

If $m\angle PQR + m\angle RQS = m\angle PQS$, then R is in the interior of $\angle PQS$.

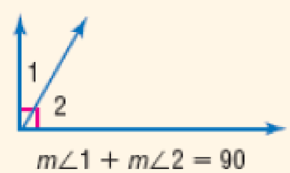


Theorems 2.3 and 2.4 Supplement and Complement Theorems

2.3 Supplement Theorem If two angles form a linear pair, then they are supplementary angles.



2.4 Complement Theorem If the noncommon sides of two adjacent angles form a right angle, then the angles are complementary angles.



Theorem 2.5

Congruence of angles is reflexive, symmetric, and transitive.

Reflexive Property $\angle 1 \cong \angle 1$

Symmetric Property If $\angle 1 \cong \angle 2$, then $\angle 2 \cong \angle 1$.

Transitive Property If $\angle 1 \cong \angle 2$, and $\angle 2 \cong \angle 3$, then $\angle 1 \cong \angle 3$.

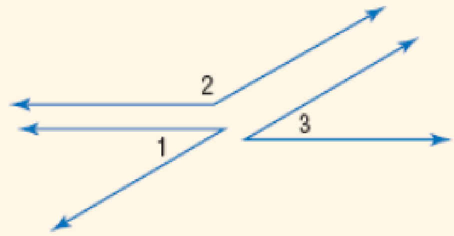
CHAPTER 2 REASONING AND PROOF

Theorems 2.6 and 2.7

2.6 Angles supplementary to the same angle or to congruent angles are congruent.

Abbreviation: $\angle$s suppl. to same $\angle$ or $\cong$ $\angle$s are $\cong$.

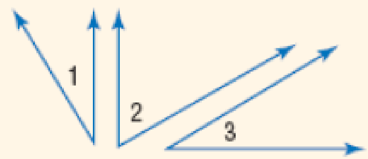
Example: If $m\angle 1 + m\angle 2 = 180$ and $m\angle 2 + m\angle 3 = 180$, then $\angle 1 \cong \angle 3$.



2.7 Angles complementary to the same angle or to congruent angles are congruent.

Abbreviation: $\angle$s compl. to same $\angle$ or $\cong$ $\angle$s are $\cong$.

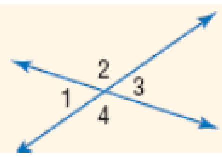
Example: If $m\angle 1 + m\angle 2 = 90$ and $m\angle 2 + m\angle 3 = 90$, then $\angle 1 \cong \angle 3$.



Theorem 2.8 Vertical Angle Theorem

If two angles are vertical angles, then they are congruent.

Abbreviation: Vert. $\angle$s are $\cong$.



$\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$

Theorems 2.9 – 2.13 Right Angle Theorems

2.9 Perpendicular lines intersect to form four right angles.

2.10 All right angles are congruent.

2.11 Perpendicular lines form congruent adjacent angles.

2.12 If two angles are congruent and supplementary, then each angle is a right angle.

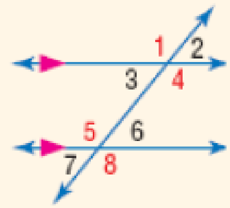
2.13 If two congruent angles form a linear pair, then they are right angles.

CHAPTER 3 PARALLEL AND PERPENDICULAR LINES

Postulate 3.1 Corresponding Angles Postulate

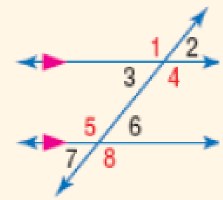
If two parallel lines are cut by a transversal, then each pair of corresponding angles is congruent.

Examples: $\angle 1 \cong \angle 5$, $\angle 2 \cong \angle 6$, $\angle 3 \cong \angle 7$, $\angle 4 \cong \angle 8$



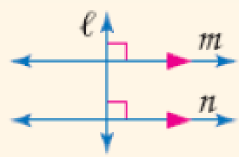
CHAPTER 3 PARALLEL AND PERPENDICULAR LINES

Theorem 3.1 – 3.3 Parallel Lines and Angle Pairs

Theorems	Examples	Model
3.1 Alternate Interior Angles If two parallel lines are cut by a transversal, then each pair of alternate interior angles is congruent.	$\angle 4 \cong \angle 5$ $\angle 3 \cong \angle 6$	
3.2 Consecutive Interior Angles If two parallel lines are cut by a transversal, then each pair of consecutive interior angles is supplementary.	$\angle 4$ and $\angle 6$ are supplementary. $\angle 3$ and $\angle 5$ are supplementary.	
3.3 Alternate Exterior Angles If two parallel lines are cut by a transversal, then each pair of alternate exterior angles is congruent.	$\angle 1 \cong \angle 8$ $\angle 2 \cong \angle 7$	

Theorem 3.4 Perpendicular Transversal Theorem

In a plane, if a line is perpendicular to one of two parallel lines, then it is perpendicular to the other.



Postulate 3.2 and 3.3 Parallel and Perpendicular Lines

3.2 Two nonvertical lines have the same slope if and only if they are parallel.

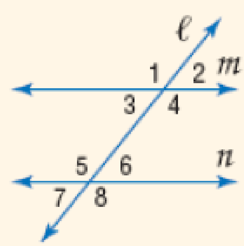
3.3 Two nonvertical lines are perpendicular if and only if the product of their slopes is -1.

Postulate 3.4

If two lines in a plane are cut by a transversal so that corresponding angles are congruent, then the lines are parallel.

Abbreviation: *If corr. ∠s are ≅, then lines are ||.*

Examples: If $\angle 1 \cong \angle 5$, $\angle 2 \cong \angle 6$, $\angle 3 \cong \angle 7$, or $\angle 4 \cong \angle 8$, then $m \parallel n$.

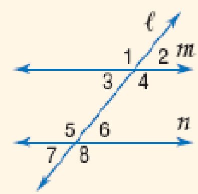
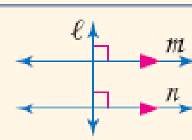


CHAPTER 3 PARALLEL AND PERPENDICULAR LINES

Postulate 3.5 Parallel Postulate

If given a line and a point not on the line, then there exists exactly one line through the point that is parallel to the given line.

Theorems 3.5 – 3.8 Proving Lines Parallel

	Theorems	Examples	Models
3.5	If two lines in a plane are cut by a transversal so that a pair of alternate exterior angles is congruent, then the two lines are parallel. **Abbreviation:** *If alt. ext.* $\measuredangle$ *are* $\cong$, *then lines are* $\parallel$.	If $\angle 1 \cong \angle 8$ or if $\angle 2 \cong \angle 7$, then $m \parallel n$.	
3.6	If two lines in a plane are cut by a transversal so that a pair of consecutive interior angles is supplementary, then the lines are parallel. **Abbreviation:** *If cons. int.* $\measuredangle$ *are suppl., then lines are* $\parallel$.	If $m\angle 3 + m\angle 5$ are supplementary or if $m\angle 4$ and $m\angle 6$ are supplementary then $m \parallel n$.	
3.7	If two lines in a plane are cut by a transversal so that a pair of alternate interior angles is congruent, then the lines are parallel. **Abbreviation:** *If alt. int.* $\measuredangle$ *are* $\cong$, *then lines are* $\parallel$.	If $\angle 3 \cong \angle 6$ or if $\angle 4 \cong \angle 5$, then $m \parallel n$.	
3.8	In a plane, if two lines are perpendicular to the same line, then they are parallel. **Abbreviation:** *If 2 lines are* $\perp$ *to the same line, then lines are* $\parallel$.	If $\ell \perp m$ and $\ell \perp n$, then $m \parallel n$.	

Theorem 3.9

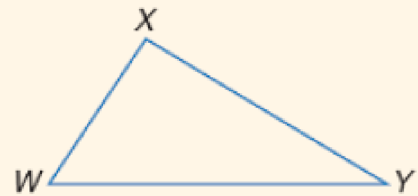
In a plane, if two lines are equidistant from a third line, then the two lines are parallel to each other.

CHAPTER 4 CONGRUENT TRIANGLES

Theorem 4.1 Angle Sum Theorem

The sum of the measures of the angles of a triangle is 180.

Example: $m\angle W + m\angle X + m\angle Y = 180$



CHAPTER 4 CONGRUENT TRIANGLES

Theorem 4.2 Third Angle Theorem

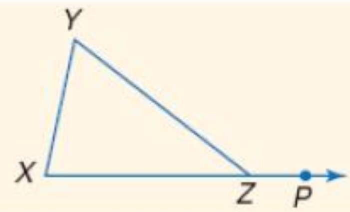
If two angles of one triangle are congruent to two angles of a second triangle, then the third angles of the triangles are congruent.

Example: If $\angle A \cong \angle F$ and $\angle C \cong \angle D$, then $\angle B \cong \angle E$.

Theorem 4.3 Exterior Angle Theorem

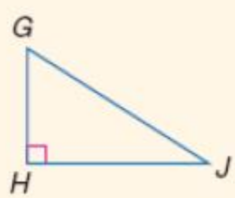
The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.

Example: $m\angle X + m\angle Y = m\angle YZP$



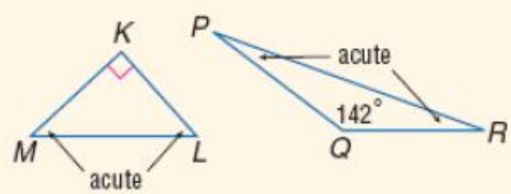
Corollaries 4.1 and 4.2

4.1 The acute angles of a right triangle are complementary.



Example: $m\angle G + m\angle J = 90$

4.2 There can be at most one right or obtuse angle in a triangle.



Theorem 4.4 Properties of Triangle Congruence

Congruence of triangles is reflexive, symmetric, and transitive.

Reflexive

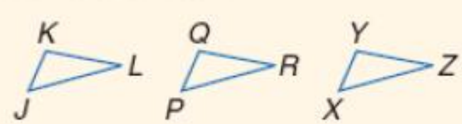
$\triangle JKL \cong \triangle JKL$

Symmetric

If $\triangle JKL \cong \triangle PQR$, then $\triangle PQR \cong \triangle JKL$.

Transitive

If $\triangle JKL \cong \triangle PQR$, and $\triangle PQR \cong \triangle XYZ$, then $\triangle JKL \cong \triangle XYZ$.

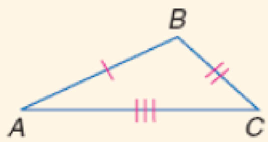


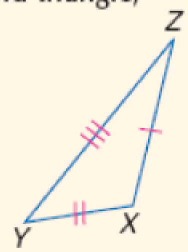
CHAPTER 4 CONGRUENT TRIANGLES

Postulate 4.1 Side-Side-Side Congruence (SSS)

If the sides of one triangle are congruent to the sides of a second triangle, then the triangles are congruent.

Abbreviation: *SSS*



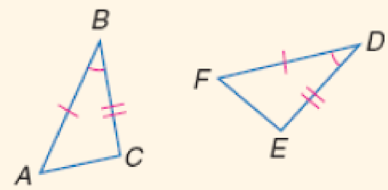


$\triangle ABC \cong \triangle ZXY$

Postulate 4.2 Side-Angle-Side Congruence (SAS)

If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.

Abbreviation: *SAS*



$\triangle ABC \cong \triangle FDE$

Postulate 4.3 Angle-Side-Angle Congruence (ASA)

Congruence of triangles is reflexive, symmetric, and transitive.

Reflexive

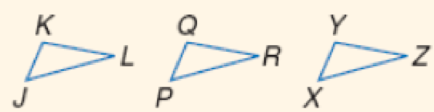
$\triangle JKL \cong \triangle JKL$

Symmetric

If $\triangle JKL \cong \triangle PQR$, then $\triangle PQR \cong \triangle JKL$.

Transitive

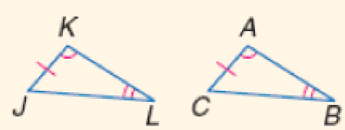
If $\triangle JKL \cong \triangle PQR$, and $\triangle PQR \cong \triangle XYZ$, then $\triangle JKL \cong \triangle XYZ$.



Theorem 4.5 Angle-Angle-Side Congruence (AAS)

If two angles and a nonincluded side of one triangle are congruent to the corresponding two angles and side of a second triangle, then the two triangles are congruent.

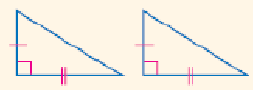
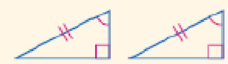
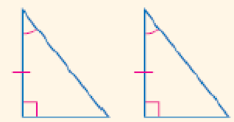
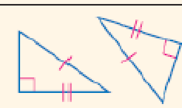
Abbreviation: *AAS*



Example: $\triangle JKL \cong \triangle CAB$

CHAPTER 4 CONGRUENT TRIANGLES

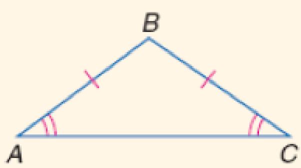
Theorem 4.6-4.8 and Postulate 4.4 Right Triangle Congruence

Theorems	Abbreviation	Example
4.6 Leg-Leg Congruence If the legs of one right triangle are congruent to the corresponding legs of another right triangle, then the triangles are congruent.	**LL**	
4.7 Hypotenuse-Angle Congruence If the hypotenuse and acute angle of one right triangle are congruent to the hypotenuse and corresponding acute angle of another right triangle, then the two triangles are congruent.	**HA**	
4.8 Leg-Angle Congruence If one leg and an acute angle of one right triangle are congruent to the corresponding leg and acute angle of another right triangle, then the triangles are congruent.	**LA**	
Postulate		
4.4 Hypotenuse-Leg Congruence If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and corresponding leg of another right triangle, then the triangles are congruent.	**HL**	

Theorem 4.9 Isosceles Triangle Theorem

If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

Example: If $\overline{AB} \cong \overline{CB}$, then $\angle A \cong \angle C$.

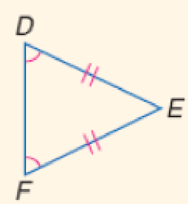


Theorem 4.10

If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

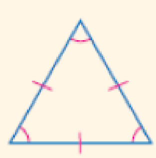
Abbreviation: *Conv. of Isos.* △ *Th.*

Example: If $\angle D \cong \angle F$, then $\overline{DE} \cong \overline{FE}$.

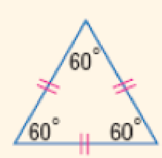


Corollaries 4.3 and 4.4

4.3 A triangle is equilateral if and only if it is equiangular.



4.4 Each angle of an equilateral triangle measures 60°.

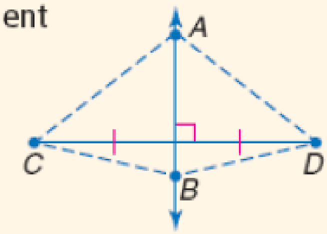


CHAPTER 5 RELATIONSHIPS IN TRIANGLES

Theorem 5.1 and 5.2 Points on Perpendicular Bisectors

5.1 Any point on the perpendicular bisector of a segment is equidistant from the endpoints of the segment.

Example: If $\overline{AB} \perp \overline{CD}$ and $\overline{AB}$ bisects $\overline{CD}$, then $AC = AD$ and $BC = BD$.



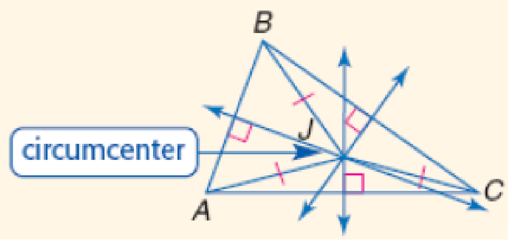
5.2 Any point equidistant from the endpoints of a segment lies on the perpendicular bisector of the segment.

Example: If $AC = AD$, then A lies on the perpendicular bisector of $\overline{CD}$. If $BC = BD$, then B lies on the perpendicular bisector of $\overline{CD}$.

Theorem 5.3

The circumcenter of a triangle is equidistant from the vertices of the triangle.

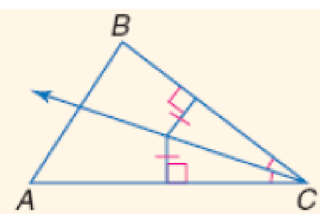
Example: If J is the circumcenter of $\triangle ABC$, then $AJ = BJ = CJ$.



Theorems 5.4 and 5.5 Points on Angle Bisectors

5.4 Any point on the angle bisector is equidistant from the sides of the angle.

5.5 Any point equidistant from the sides of an angle lies on the angle bisector.

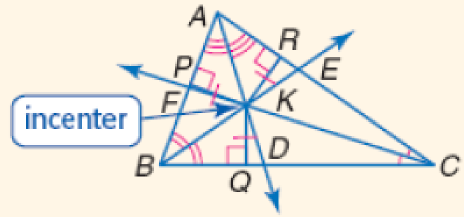


CHAPTER 5 RELATIONSHIPS IN TRIANGLES

Theorem 5.6 Incenter Theorem

The incenter of a triangle is equidistant from each side of the triangle.

Example: If K is the incenter of $\triangle ABC$, then $KP = KQ = KR$.

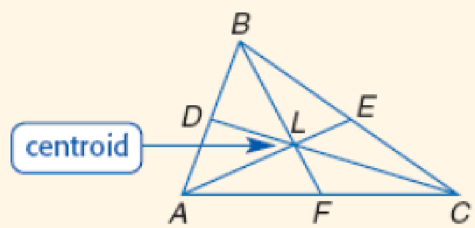


Theorem 5.7 Centroid Theorem

The centroid of a triangle is located two thirds of the distance from a vertex to the midpoint of the side opposite the vertex on a median.

Example: If L is the centroid of $\triangle ABC$,

$AL = \frac{2}{3}AE$, $BL = \frac{2}{3}BF$, and $CL = \frac{2}{3}CD$.

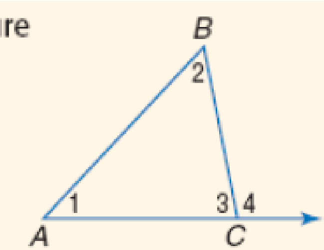


Theorem 5.8 Exterior Angle Inequality Theorem

If an angle is an exterior angle of a triangle, then its measure is greater than the measure of either of its corresponding remote interior angles.

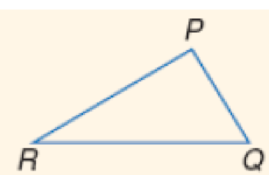
Example: $m\angle 4 > m\angle 1$

$m\angle 4 > m\angle 2$



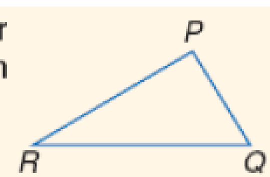
Theorem 5.9

If one side of a triangle is longer than another side, then the angle opposite the longer side has a greater measure than the angle opposite the shorter side.



Theorem 5.10

If one angle of a triangle has a greater measure than another angle, then the side opposite the greater angle is longer than the side opposite the lesser angle.

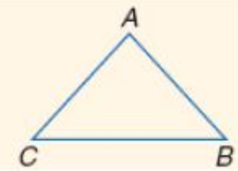


CHAPTER 5 RELATIONSHIPS IN TRIANGLES

Theorem 5.11 Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

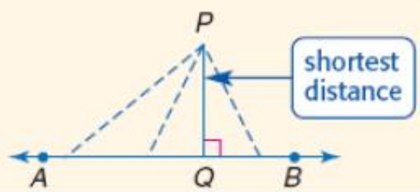
Examples:
$AB + BC > AC$
$BC + AC > AB$
$AC + AB > BC$



Theorem 5.12

The perpendicular segment from a point to a line is the shortest segment from the point to the line.

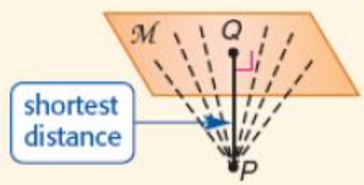
Example: $\overline{PQ}$ is the shortest segment from P to $\overleftrightarrow{AB}$.



Corollary 5.1

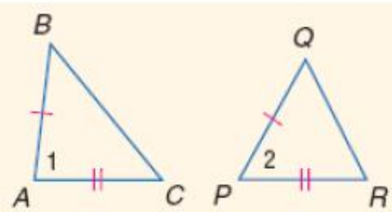
The perpendicular segment from a point to a plane is the shortest segment from the point to the plane.

Example: $\overline{QP}$ is the shortest segment from P to Plane $\mathcal{M}$.



Theorem 5.13 SAS Inequality/Hinge Theorem

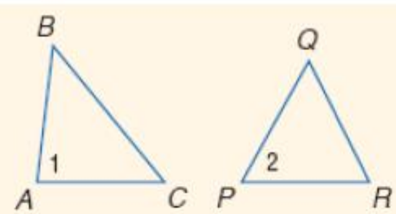
Two sides of a triangle are congruent to two sides of another triangle. If the included angle in the first triangle has a greater measure than the included angle in the second triangle, then the third side of the first triangle is longer than the third side of the second triangle.



Example: Given $\overline{AB} \cong \overline{PQ}$, $\overline{AC} \cong \overline{PR}$, if $m\angle 1 > m\angle 2$, then $BC > QR$.

Theorem 5.14 SSS Inequality

If two sides of a triangle are congruent to two sides of another triangle and the third side in one triangle is longer than the third side in the other, then the angle between the pair of congruent sides in the first triangle is greater than the corresponding angle in the second triangle.



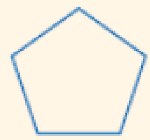
Example: Given $\overline{AB} \cong \overline{PQ}$, $\overline{AC} \cong \overline{PR}$, if $BC > QR$, then $m\angle 1 > m\angle 2$.

CHAPTER 6 QUADRILATERALS

Theorem 6.1 Interior Angle Sum Theorem

If a convex polygon has n sides and S is the sum of the measures of its interior angles, then $S = 180(n - 2)$.

Example:

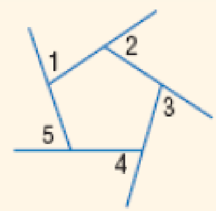


$n = 5$
$S = 180(n - 2)$
$= 180(5 - 2)$ or 540

Theorem 6.2 Exterior Angle Sum Theorem

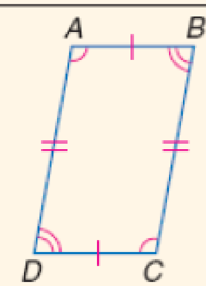
If a polygon is convex, then the sum of the measures of the exterior angles, one at each vertex, is 360.

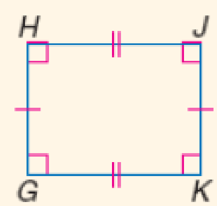
Example: $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 = 360$



Theorems 6.3 - 6.6

		Examples
6.3	Opposite sides of a parallelogram are congruent. **Abbreviation:** *Opp. sides of ▱ are ≅.*	$\overline{AB} \cong \overline{DC}$ $\overline{AD} \cong \overline{BC}$
6.4	Opposite angles in a parallelogram are congruent. **Abbreviation:** *Opp ∠s of ▱ are ≅.*	$\angle A \cong \angle C$ $\angle B \cong \angle D$
6.5	Consecutive angles in a parallelogram are supplementary. **Abbreviation:** *Cons. ∠s in ▱ are suppl.*	$m\angle A + m\angle B = 180$ $m\angle B + m\angle C = 180$ $m\angle C + m\angle D = 180$ $m\angle D + m\angle A = 180$
6.6	If a parallelogram has one right angle, it has four right angles. **Abbreviation:** *If ▱ has 1 rt. ∠, it has 4 rt. ∠s.*	$m\angle G = 90$ $m\angle H = 90$ $m\angle J = 90$ $m\angle K = 90$





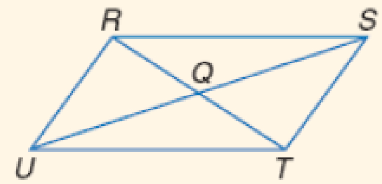
CHAPTER 6 QUADRILATERALS

Theorem 6.7

The diagonals of a parallelogram bisect each other.

Abbreviation: *Diag. of ▱ bisect each other.*

Example: $\overline{RQ} \cong \overline{QT}$ and $\overline{SQ} \cong \overline{QU}$

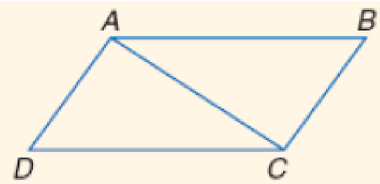


Theorem 6.8

Each diagonal of a parallelogram separates the parallelogram into two congruent triangles.

Abbreviation: *Diag. separates ▱ into 2 ≅ △s.*

Example: $\triangle ACD \cong \triangle CAB$

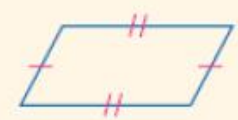


CHAPTER 6 QUADRILATERALS

Theorems 6.9 – 6.12 Proving Parallelograms

6.9 If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

Abbreviation: *If both pairs of opp. sides are ≅, then quad. is ▱.*

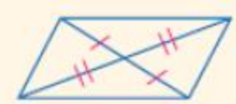


6.10 If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

Abbreviation: *If both pairs of opp. ∠s are ≅, then quad. is ▱.*

6.11 If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

Abbreviation: *If diag. bisect each other, then quad. is ▱.*



6.12 If one pair of opposite sides of a quadrilateral is both parallel and congruent, then the quadrilateral is a parallelogram.

Abbreviation: *If one pair of opp. sides is ∥ and ≅, then the quad. is a ▱.*

Theorem 6.13

If a parallelogram is a rectangle, then the diagonals are congruent.

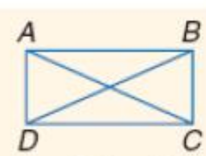
Abbreviation: *If ▱ is rectangle, diag. are ≅.*

$\overline{AC} \cong \overline{BD}$

Theorem 6.14

If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

Abbreviation: *If diagonals of ▱ are ≅, ▱* is a rectangle.

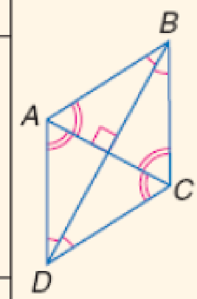


$\overline{AC} \cong \overline{BD}$

CHAPTER 6 QUADRILATERALS

Theorems 6.15 – 6.17 Rhombus

		Examples
6.15	The diagonals of a rhombus are perpendicular.	$\overline{AC} \perp \overline{BD}$
6.16	If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus. (Converse of Theorem 6.15)	If $\overline{BD} \perp \overline{AC}$ then $\square ABCD$ is a rhombus.
6.17	Each diagonal of a rhombus bisects a pair of opposite angles.	$\angle DAC \cong \angle BAC \cong \angle DCA \cong \angle BCA$ $\angle ABD \cong \angle CBD \cong \angle ADB \cong \angle CDB$



Theorem 6.18 and 6.19 Isosceles Trapezoid

6.18 Each pair of base angles of an isosceles trapezoid are congruent.

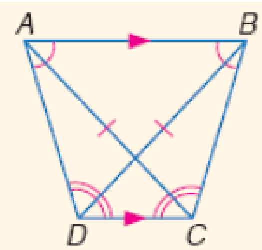
6.19 The diagonals of an isosceles trapezoid are congruent.

Example:

$\angle DAB \cong \angle CBA$

$\angle ADC \cong \angle BCD$

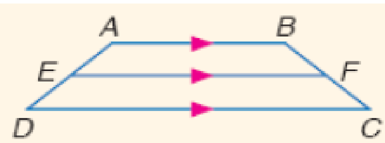
$\overline{AC} \cong \overline{BD}$



Theorem 6.20

The median of a trapezoid is parallel to the bases, and its measure is one-half the sum of the measures of the bases.

Example: $EF = \frac{1}{2}(AB + DC)$

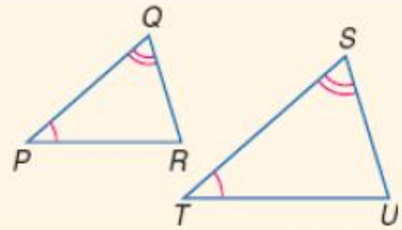


CHAPTER 7 Proportions and similarity

Postulate 7.1 Angle-Angle (AA) Similarity

If the two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

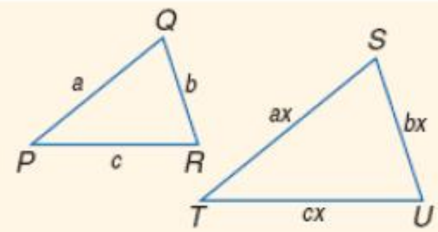
Example: $\angle P \cong \angle T$ and $\angle Q \cong \angle S$, so $\triangle PQR \sim \triangle TSU$.



Theorem 7.1 and 7.2

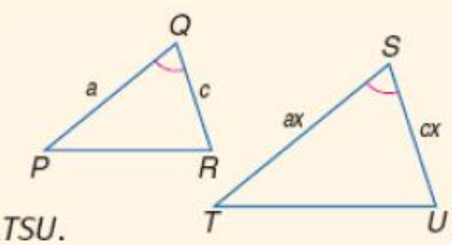
7.1 Side-Side-Side (SSS) Similarity If the measures of the corresponding sides of two triangles are proportional, then the triangles are similar.

Example: $\frac{PQ}{ST} = \frac{QR}{SU} = \frac{RP}{UT}$, so $\triangle PQR \sim \triangle TSU$.



7.2 Side-Angle-Side (SAS) Similarity If the measures of two sides of a triangle are proportional to the measures of two corresponding sides of another triangle and the included angles are congruent, then the triangles are similar.

Example: $\frac{PQ}{ST} = \frac{QR}{SU}$ and $\angle Q \cong \angle S$, so $\triangle PQR \sim \triangle TSU$.



Theorem 7.3

Similarity of triangles is reflexive, symmetric, and transitive.

Examples:

Reflexive: $\triangle ABC \sim \triangle ABC$

Symmetric: If $\triangle ABC \sim \triangle DEF$, then $\triangle DEF \sim \triangle ABC$.

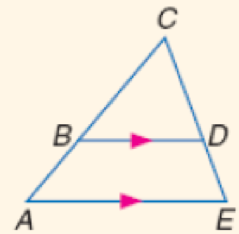
Transitive: If $\triangle ABC \sim \triangle DEF$ and $\triangle DEF \sim \triangle GHI$, then $\triangle ABC \sim \triangle GHI$.

CHAPTER 7 Proportions and similarity

Theorem 7.4 Triangle Proportionality Theorem

If a line is parallel to one side of a triangle and intersects the other two sides in two distinct points, then it separates these sides into segments of proportional lengths.

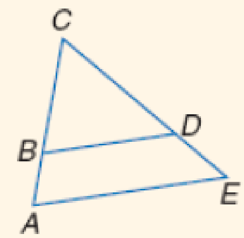
Example: If $\overline{BD} \parallel \overline{AE}$, $\frac{BA}{CB} = \frac{DE}{CD}$.



Theorem 7.5 Converse of the Triangle Proportionality Theorem

If a line intersects two sides of a triangle and separates the sides into corresponding segments of proportional lengths, then the line is parallel to the third side.

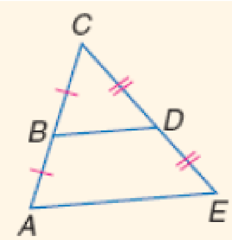
Example: If $\frac{BA}{CB} = \frac{DE}{CD}$, then $\overline{BD} \parallel \overline{AE}$.



Theorem 7.6 Triangle Midsegment Theorem

A midsegment of a triangle is parallel to one side of the triangle, and its length is one-half the length of that side.

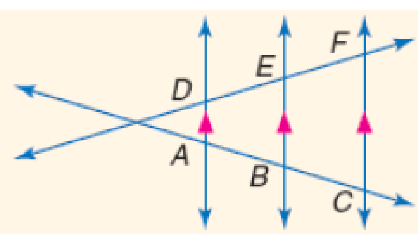
Example: If B and D are midpoints of $\overline{AC}$ and $\overline{EC}$, respectively, $\overline{BD} \parallel \overline{AE}$ and $BD = \frac{1}{2}AE$.



Corollaries 7.1 and 7.2

7.1 If three or more parallel lines intersect two transversals, then they cut off the transversals proportionally.

Example: If $\overleftrightarrow{DA} \parallel \overleftrightarrow{EB} \parallel \overleftrightarrow{FC}$, then $\frac{AB}{BC} = \frac{DE}{EF}$, $\frac{AC}{DF} = \frac{BC}{EF}$, and $\frac{AC}{BC} = \frac{DF}{EF}$.



7.2 If three or more parallel lines cut off congruent segments on one transversal then they cut off congruent segments on every transversal.

Example: If $\overline{AB} \cong \overline{BC}$, then $\overline{DE} \cong \overline{EF}$.

CHAPTER 7 Proportions and similarity

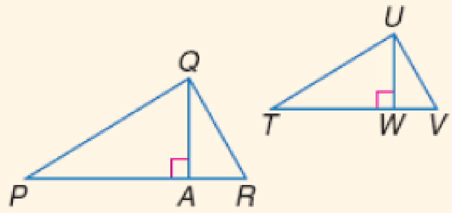
Theorem 7.7 Proportional Perimeters Theorem

If two triangles are similar, then the perimeters are proportional to the measures of corresponding sides.

Theorem 7.8 – 7.10 Special Segments of Similar Triangles

7.8 If two triangles are similar, then the measures of the corresponding altitudes are proportional to the measures of the corresponding sides.

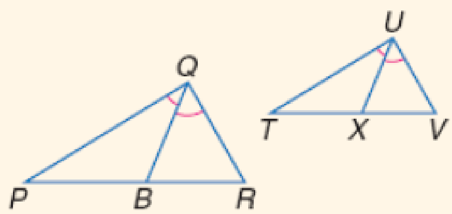
Abbreviation: *~ △s have corr. altitudes proportional to the corr. sides.*



$$\frac{QA}{UW} = \frac{PR}{TV} = \frac{QR}{UV} = \frac{PQ}{TU}$$

7.9 If two triangles are similar, then the measures of the corresponding angle bisectors of the triangles are proportional to the measures of the corresponding sides.

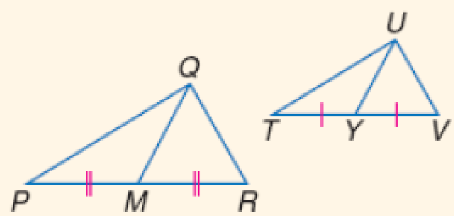
Abbreviation: *~ △s have corr. ∠ bisectors proportional to the corr. sides.*



$$\frac{QB}{UX} = \frac{PR}{TV} = \frac{QR}{UV} = \frac{PQ}{TU}$$

7.10 If two triangles are similar, then the measures of the corresponding medians are proportional to the measures of the corresponding sides.

Abbreviation: *~ △s have corr. medians proportional to the corr. sides.*

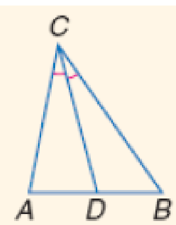


$$\frac{QM}{UY} = \frac{PR}{TV} = \frac{QR}{UV} = \frac{PQ}{TU}$$

Theorem 7.11 Angle Bisector Theorem

An angle bisector in a triangle separates the opposite side into segments that have the same ratio as the other two sides.

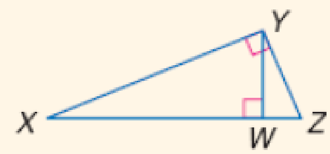
Example: $\frac{AD}{DB} = \frac{AC}{BC}$ ← segments with vertex *A* ← segments with vertex *B*



CHAPTER 8 RIGHT TRIANGLES AND TRIGONOMETRY

Theorem 8.1

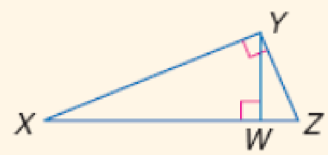
If the altitude is drawn from the vertex of the right angle of a right triangle to its hypotenuse, then the two triangles formed are similar to the given triangle and to each other.



Example: $\triangle XYZ \sim \triangle XWY \sim \triangle YWZ$

Theorem 8.2

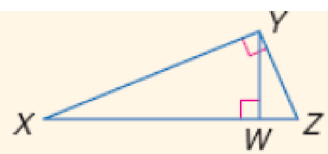
The measure of an altitude drawn from the vertex of the right angle of a right triangle to its hypotenuse is the geometric mean between the measures of the two segments of the hypotenuse.



Example: *YW* is the geometric mean of *XW* and *ZW*.

Theorem 8.3

If the altitude is drawn from the vertex of the right angle of a right triangle to its hypotenuse, then the measure of a leg of the triangle is the geometric mean between the measures of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

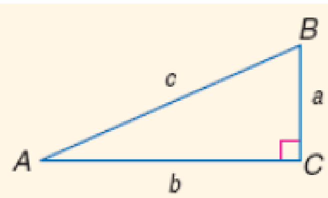


Example: $\frac{XZ}{XY} = \frac{XY}{XW}$ and $\frac{XZ}{YZ} = \frac{YZ}{WZ}$

Theorem 8.4 Pythagorean Theorem

In a right triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse.

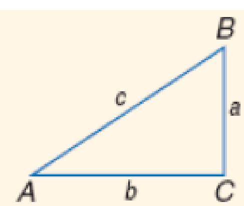
Symbols: $a^2 + b^2 = c^2$



Theorem 8.5 Converse of the Pythagorean Theorem

If the sum of the squares of the measures of two sides of a triangle equals the square of the measure of the longest side, then the triangle is a right triangle.

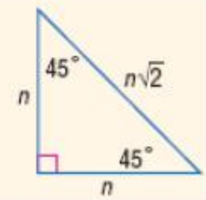
Symbols: If $a^2 + b^2 = c^2$, then $\triangle ABC$ is a right triangle.



CHAPTER 8 RIGHT TRIANGLES AND TRIGONOMETRY

Theorem 8.6

In a 45°-45°-90° triangle, the length of the hypotenuse is $\sqrt{2}$ times the length of a leg.

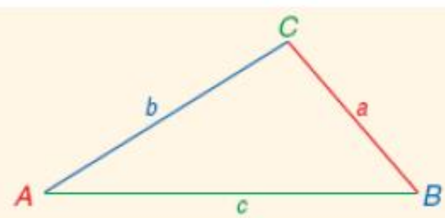


Theorem 8.7

In a 30°-60°-90° triangle, the length of the hypotenuse is twice the length of the shorter leg, and the length of the longer leg is $\sqrt{3}$ times the length of the shorter leg.

Theorem 8.8 Law of Sines

Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of the sides opposite the angles with measures A, B, and C, respectively. Then $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.



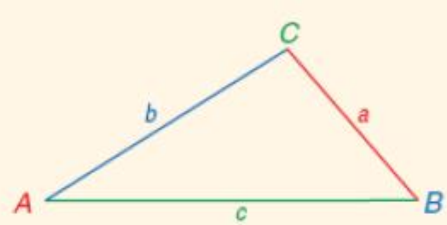
Theorem 8.9 Law of Cosines

Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of sides opposite angles A, B, and C, respectively. Then the following equations are true.

$a^2 = b^2 + c^2 - 2bc \cos A$

$b^2 = a^2 + c^2 - 2ac \cos B$

$c^2 = a^2 + b^2 - 2ab \cos C$



CHAPTER 9 TRANSFORMATIONS

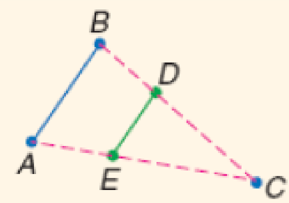
Theorem 9.1 and Corollary 9.1

Theorem 9.1 In a given rotation, if A is the preimage, A'' is the image, and P is the center of rotation, then the measure of the angle of rotation $\angle APA''$ is twice the measure of the acute or right angle formed by the intersecting lines of reflection.

Corollary 9.1 Reflecting an image successively in two perpendicular lines results in a 180° rotation.

Theorem 9.2

If a dilation with center C and a scale factor of r transforms A to E and B to D, then $ED = |r|(AB)$.



Theorem 9.3

If $P(x, y)$ is the preimage of a dilation centered at the origin with a scale factor r, then the image is $P'(rx, ry)$.

CHAPTER 10 CIRCLES

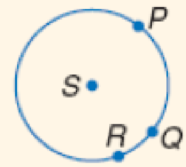
Theorem 10.1

In the same or in congruent circles, two arcs are congruent if and only if their corresponding central angles are congruent.

Postulate 10.1 Arc Addition Postulate

The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

Example: In $\odot S$, $m\widehat{PQ} + m\widehat{QR} = m\widehat{PQR}$.



CHAPTER 10 CIRCLES

Theorem 10.2

In a circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

Abbreviations:

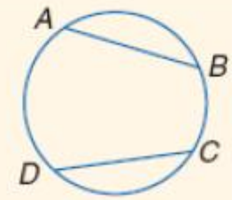
In ⊙, 2 minor arcs are ≅, corr. chords are ≅.

In ⊙, 2 chords are ≅, corr. minor arcs are ≅.

Examples:

If $\overline{AB} \cong \overline{CD}$, $\widehat{AB} \cong \widehat{CD}$.

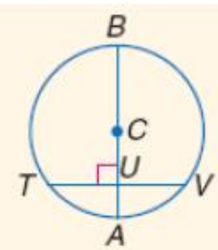
If $\widehat{AB} \cong \widehat{CD}$, $\overline{AB} \cong \overline{CD}$.



Theorem 10.3

In a circle, if a diameter (or radius) is perpendicular to a chord, then it bisects the chord and its arc.

Example: If $\overline{BA} \perp \overline{TV}$, then $\overline{UT} \cong \overline{UV}$ and $\widehat{AT} \cong \widehat{AV}$.



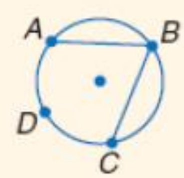
Theorem 10.4

In a circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

Theorem 10.5 Inscribed Angle Theorem

If an angle is inscribed in a circle, then the measure of the angle equals one-half the measure of its intercepted arc (or the measure of the intercepted arc is twice the measure of the inscribed angle).

Example: $m\angle ABC = \frac{1}{2}(m\widehat{ADC})$ or $2(m\angle ABC) = m\widehat{ADC}$



CHAPTER 10 CIRCLES

Theorem 10.6

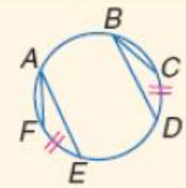
If two inscribed angles of a circle (or congruent circles) intercept congruent arcs or the same arc, then the angles are congruent.

Abbreviations:

Inscribed ∡ of ≅ arcs are ≅.

Inscribed ∡ of same arc are ≅.

$\angle DAC \cong \angle DBC$

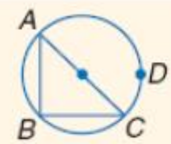


$\angle FAE \cong \angle CBD$

Theorem 10.7

If the inscribed angle of a triangle intercepts a semicircle, the angle is a right angle.

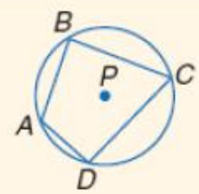
Example: $\widehat{ADC}$ is a semicircle, so $m\angle ABC = 90$.



Theorem 10.8

If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.

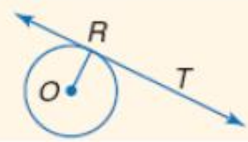
Example: Quadrilateral *ABCD* is inscribed in ⊙*P*.
∠*A* and ∠*C* are supplementary.
∠*B* and ∠*D* are supplementary.



Theorem 10.9

If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.

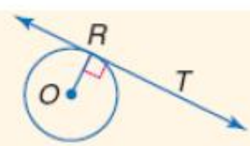
Example: If $\overleftrightarrow{RT}$ is a tangent, $\overline{OR} \perp \overleftrightarrow{RT}$.



Theorem 10.10

If a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is tangent to the circle.

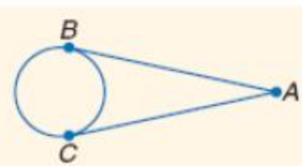
Example: If $\overline{OR} \perp \overleftrightarrow{RT}$, $\overleftrightarrow{RT}$ is a tangent.



Theorem 10.11

If two segments from the same exterior point are tangent to a circle, then they are congruent.

Example: $\overline{AB} \cong \overline{AC}$



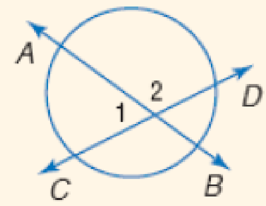
CHAPTER 10 CIRCLES

Theorem 10.12

If two secants intersect in the interior of a circle, then the measure of an angle formed is one-half the sum of the measure of the arcs intercepted by the angle and its vertical angle.

Examples: $m\angle 1 = \frac{1}{2}(m\widehat{AC} + m\widehat{BD})$

$m\angle 2 = \frac{1}{2}(m\widehat{AD} + m\widehat{BC})$

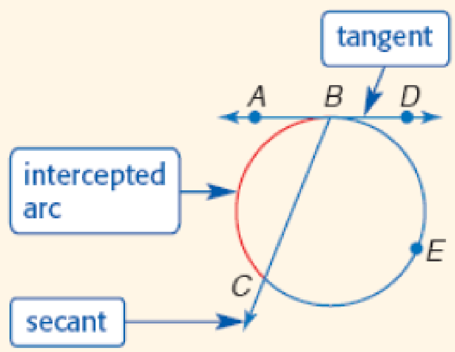


Theorem 10.13

If a secant and a tangent intersect at the point of tangency, then the measure of each angle formed is one-half the measure of its intercepted arc.

Examples: $m\angle ABC = \frac{1}{2}m\widehat{BC}$

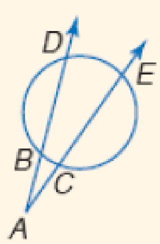
$m\angle DBC = \frac{1}{2}m\widehat{BEC}$



Theorem 10.14

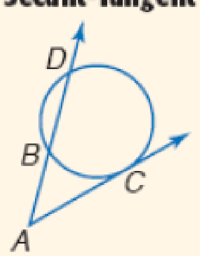
If two secants, a secant and a tangent, or two tangents intersect in the exterior of a circle, then the measure of the angle formed is one-half the positive difference of the measures of the intercepted arcs.

Two Secants



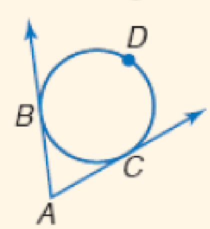
$m\angle A = \frac{1}{2}(m\widehat{DE} - m\widehat{BC})$

Secant-Tangent



$m\angle A = \frac{1}{2}(m\widehat{DC} - m\widehat{BC})$

Two Tangents



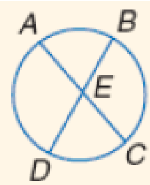
$m\angle A = \frac{1}{2}(m\widehat{BDC} - m\widehat{BC})$

CHAPTER 10 CIRCLES

Theorem 10.15

If two chords intersect in a circle, then the products of the measures of the segments of the chords are equal.

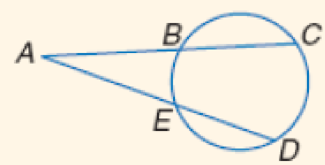
Example: $AE \cdot EC = BE \cdot ED$



Theorem 10.16

If two secant segments are drawn to a circle from an exterior point, then the product of the measures of one secant segment and its external secant segment is equal to the product of the measures of the other secant segment and its external secant segment.

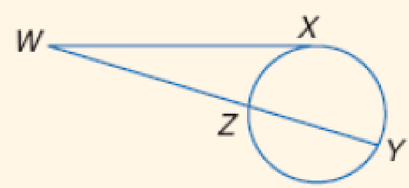
Example: $AB \cdot AC = AE \cdot AD$



Theorem 10.17

If a tangent segment and a secant segment a are drawn to circle from an exterior point, then the square of the measure of the tangent segment is equal to the product of the measures of the secant segment and its external secant segment.

Example: $WX \cdot WX = WZ \cdot WY$



CHAPTER 11 AREA OF POLYGONS AND CIRCLES

Postulate 11.1

Congruent figures have equal areas.

Postulate 11.2

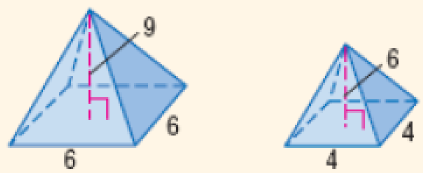
The area of a region is the sum of the areas of all of its nonoverlapping parts.

CHAPTER 13 VOLUME

Theorem 13.1

If two solids are similar with a scale factor of $a:b$, then the surface areas have a ratio of $a^2:b^2$, and the volumes have a ratio of $a^3:b^3$.

Example:



Scale factor 3:2

Ratio of surface areas $3^2:2^2$ or 9:4

Ratio of volumes $3^3:2^3$ or 27:8