

Trigonometric Identities

Trig Identities

Prove each identity:

1.
$$\sec x - \tan x \sin x = \frac{1}{\sec x}$$

2.
$$\frac{1 + \cos x}{\sin x} = \csc x + \cot x$$

3.
$$\frac{\sec \theta \sin \theta}{\tan \theta + \cot \theta} = \sin^2 \theta$$

4.
$$\frac{\sec \theta}{\cos \theta} - \frac{\tan \theta}{\cot \theta} = 1$$

5.
$$\cos^2 y - \sin^2 y = 1 - 2\sin^2 y$$

6.
$$\csc^2 \theta \tan^2 \theta - 1 = \tan^2 \theta$$

7.
$$\frac{\sec^2 \theta}{\sec^2 \theta - 1} = \csc^2 \theta$$

8.
$$\tan^2 x \sin^2 x = \tan^2 x - \sin^2 x$$

Trigonometric Identities

Answers

Trig Identities worksheet 3.4 name:

Prove each identity:

1. $\sec x - \tan x \sin x = \frac{1}{\sec x}$

$$\begin{aligned} \frac{1}{\cos x} - \frac{\sin x \cdot \sin x}{\cos x} &= \\ \frac{1 - \sin^2 x}{\cos x} &= \\ \frac{\cos^2 x}{\cos x} &= \\ \cos x &= \frac{1}{\sec x} = \checkmark \end{aligned}$$

2. $\frac{1 + \cos x}{\sin x} = \csc x + \cot x$

$$\begin{aligned} \frac{1}{\sin x} + \frac{\cos x}{\sin x} &= \\ \csc x + \cot x &= \checkmark \end{aligned}$$

3. $\frac{\sec \theta \sin \theta}{\tan \theta + \cot \theta} = \sin^2 \theta$

$$\begin{aligned} \frac{\frac{1}{\cos \theta} \cdot \frac{\sin \theta}{1}}{\tan \theta + \cot \theta} &= \frac{\tan \theta}{\tan \theta + \cot \theta} \\ \frac{\tan \theta}{\frac{1}{\sin \theta} + \frac{1}{\cos \theta}} &= \frac{\tan \theta}{\frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta}} = \frac{\tan \theta}{\frac{1}{\sin \theta \cos \theta}} = \end{aligned}$$

4. $\left(\frac{\sec \theta}{\cos \theta}\right) \frac{\tan \theta}{\cot \theta} = 1$

$$\begin{aligned} \frac{1}{\cos^2 \theta} - \frac{\sin^2 \theta}{\cos^2 \theta} &= \frac{1 - \sin^2 \theta}{\cos^2 \theta} = \\ \frac{\sin^2 \theta}{\cos^2 \theta} \cdot \frac{\sin \theta \cos \theta}{\cos^2 \theta} &= \frac{\sin^2 \theta}{\cos^2 \theta} \\ \frac{\cos^2 \theta}{\cos^2 \theta} &= 1 = 1 \end{aligned}$$

5. $\cos^2 y - \sin^2 y = 1 - 2\sin^2 y$

$$\begin{aligned} 1 - \sin^2 y + \sin^2 y &= \\ 1 - 2\sin^2 y &= \checkmark \end{aligned}$$

6. $\csc^2 \theta \tan^2 \theta - 1 = \tan^2 \theta$

$$\begin{aligned} (1 + \cot^2 \theta)(\tan^2 \theta) - 1 &= \\ \tan^2 \theta + \cancel{\cot^2 \theta \cdot \tan^2 \theta} - 1 &= \\ \tan^2 \theta + 1 - 1 &= \\ \tan^2 \theta &= \checkmark \end{aligned}$$

7. $\frac{\sec^2 \theta}{\sec^2 \theta - 1} = \csc^2 \theta$

$$\begin{aligned} \frac{\sec^2 \theta}{\tan^2 \theta} &= \\ \frac{1}{\cos^2 \theta} \div \frac{\cos^2 \theta}{\sin^2 \theta} &= \\ \csc^2 \theta &= \checkmark \end{aligned}$$

8. $\frac{\tan^2 x \sin^2 x}{(\sec^2 x - 1)(\sin^2 x)} = \tan^2 x - \sin^2 x$

$$\begin{aligned} \sec^2 x \cdot \sin^2 x - \sin^2 x &= \\ \frac{1}{\cos^2 x} \cdot \sin^2 x - \sin^2 x &= \\ \tan^2 x - \sin^2 x &= \checkmark \end{aligned}$$

Trigonometric Identities

Answers

| | |
|--|---|
| <p>9. $(\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2 = 2$</p> $\begin{aligned} & \cancel{\sin^2 \theta + 2\sin \theta \cos \theta + \cos^2 \theta} \\ & + \cancel{\sin^2 \theta - 2\sin \theta \cos \theta + \cos^2 \theta} \\ & \quad 1 + 1 = \\ & \quad 2 = \checkmark \end{aligned}$ | <p>10. $(\sin \theta + \cos \theta)(\tan \theta + \cot \theta) = \sec \theta + \csc \theta$</p> $\begin{aligned} & \sin \theta \tan \theta + \sin \theta \cot \theta + \cos \theta \tan \theta \\ & + \cos \theta \cot \theta = \\ & \left(\frac{\sin^2 \theta}{\cos \theta} + \cos \theta \right) + \left(\sin \theta + \frac{\cos^2 \theta}{\sin \theta} \right) \\ & \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta} + \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta} \end{aligned}$ |
| <p>11. $\frac{\tan \theta - 1}{\tan \theta + 1} = \frac{1 - \cot \theta}{1 + \cot \theta}$</p> $\begin{aligned} & \frac{\sin \theta}{\cos \theta} - \frac{\cos \theta}{\cos \theta} = \\ & \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\cos \theta} \end{aligned}$ | <p>12. $\frac{1 - \tan^2 x}{1 + \tan^2 x} = 1 - 2 \sin^2 x$</p> $\begin{aligned} & \frac{1 - \tan^2 x}{\sec^2 x} \\ & \frac{1}{\sec^2 x} - \frac{\tan^2 x}{\sec^2 x} \stackrel{?}{=} \frac{1}{\sec^2 x} \\ & \cos^2 x - \sin^2 x = \\ & 1 - \sin^2 x - \sin^2 x \end{aligned}$ |
| <p>13. $\frac{\cos x + 1}{\sin^3 x} = \frac{\csc x}{1 - \cos x}$</p> $\begin{aligned} & \frac{\cos x + 1}{\sin x (1 - \cos^2 x)} = \\ & \sin x (1 - \cos x) (1 + \cos x) \end{aligned}$ | <p>14. $\csc^4 x - \cot^4 x = \csc^2 x + \cot^2 x$</p> $\begin{aligned} & (\csc^2 x - \cot^2 x)(\csc^2 x + \cot^2 x) = \\ & (1 + \cot^2 x)(1 + \csc^2 x) \\ & 1 (\csc^2 x + \cot^2 x) = \checkmark \end{aligned}$ |
| <p>15. $\frac{\tan \theta}{\sec \theta} + \frac{\cot \theta}{\csc \theta} = \sin \theta + \cos \theta$</p> $\begin{aligned} & \frac{s}{c} \div \frac{a}{b} = \frac{b}{a} * \frac{c}{b} \\ & \sin \theta + \cos \theta \end{aligned}$ | <p>16. $\frac{\sin y + \tan y}{1 + \sec y} = \sin y$</p> $\begin{aligned} & \frac{\sin y + \frac{\sin y}{\cos y}}{1 + \frac{1}{\cos y}} = \\ & \frac{\sin y + \frac{1}{\cos y}}{1 + \frac{1}{\cos y}} = \\ & \frac{\sin y \cos y + 1}{\cos y + 1} = \frac{\sin(\cos y + 1)}{\sin y} = \checkmark \end{aligned}$ |

Trigonometric Identities

Prove the following identity

$$1) 2 \cos \theta \tan \theta \csc \theta = 2$$

$$2) 6 \cos \theta \left(\frac{1}{\cos \theta} - \frac{\cot \theta}{\csc \theta} \right) = 6 \sin^2 \theta$$

$$3) 7 \frac{\cot^2 \theta}{\csc \theta} \sec^2 \theta = 7 \tan \theta \cos \theta \csc^2 \theta$$