Parametric Coordinates and Equations
…. Set 7

Parametric Equations

Up until this point we have been representing a graph by a single equation involving two variables x and y. Today we will introduce the use of a third variable, t, to represent a curve in the plane.

Why??

- Rectangular equations can identify where an object has been but do not tell you When the object was at a given point (x,y) on the path. To determine this time, we will introduce a third variable t, called a parameter. We will write both x and y as functions of t to obtain parametric equations.
- Because of this, parametric equations are useful for modeling situations involving position, velocity, and acceleration.

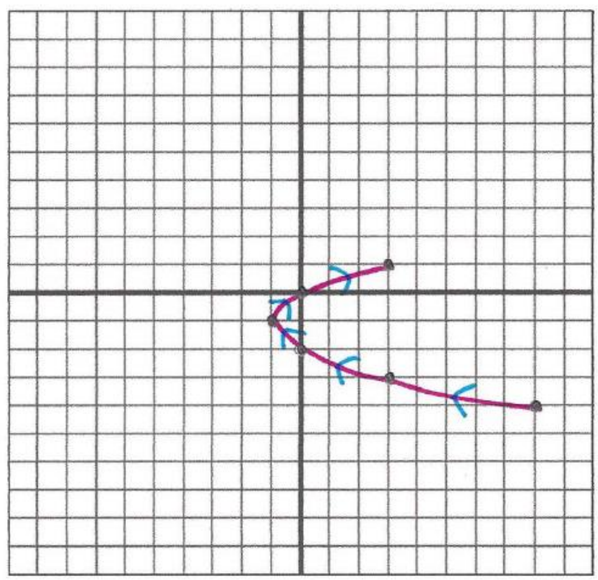
Plane Curve: If f and g are continuous functions of t on an interval I, the set of ordered pairs $(f(t), g(t))$ is a plane curve C. The equations $x = f(t)$ and $y = g(t)$ are parametric equations for C and t is the parameter.

Part One: Sketching a Plane Curve

Example 1: Sketch the curve represented by the parametric equations

$x = t^2 - 2t$ and $y = t - 2$ $\quad -2 \le t \le 3$

t	x	y	(x,y)
-2	$(-2)^2-2(-2)$	$-2-2$	$(8,-4)$
-1	$(-1)^2-2(-1)$	$-1-2$	$(3,-3)$
0	$(0)^2-2(0)$	$0-2$	$(0,-2)$
1	$(1)^2-2(1)$	$1-2$	$(-1,-1)$
2	$(2)^2-2(2)$	$2-2$	$(0,0)$
3	$(3)^2-2(3)$	$3-2$	$(3,1)$



★ place arrows to indicate the direction of the increasing values of t.

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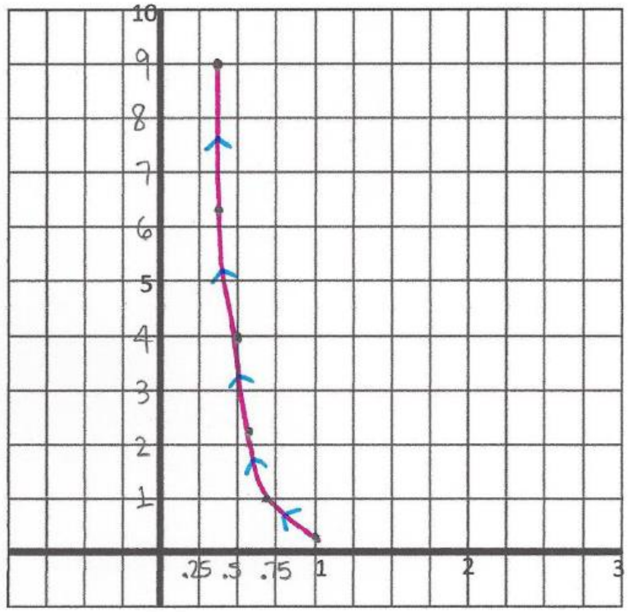
Example 2: Sketch the curve represented by the parametric equations

$x = \frac{1}{\sqrt{t}}$ and $y = \frac{t^2}{4}$ $1 \le t \le 6$

t	x	y	(x,y)
1	$\frac{1}{1} = 1$	$\frac{1^2}{4} = \frac{1}{4}$	(1, .25)
2	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{2^2}{4} = \frac{4}{4} = 1$	(.71, 1)
3	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	$\frac{3^2}{4} = \frac{9}{4}$	(.58, 2.25)
4	$\frac{1}{\sqrt{4}} = \frac{1}{2}$	$\frac{4^2}{4} = \frac{16}{4} = 4$	(.5, 4)
5	$\frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$	$\frac{5^2}{4} = \frac{25}{4}$	(.45, 6.25)
6	$\frac{1}{\sqrt{6}} = \frac{\sqrt{6}}{6}$	$\frac{6^2}{4} = \frac{36}{4} = 9$	(.41, 9)



Part Two: Eliminating the Parameter (Find a rectangular equation)

To make graphing easier, sometimes it is helpful to eliminate the parameter to write a rectangular equation (in x and y) that has the same graph.

Steps:

1) Solve for t in one of the parametric equations
2) Substitute into the other parametric equation
3) Simplify

Example 3:

a) Create a table of x- and y- values by adjusting the domain of the given functions. Then sketch the curve represented by the equations: $x = \sqrt{t-2}$ and $y = 1 + t$ *Parametric Equations*

b) Find the rectangular equation by eliminating the parameter. Sketch its graph.

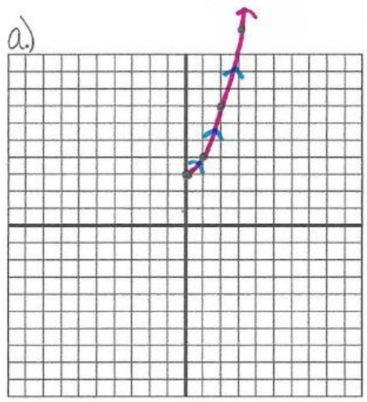
c) How do the graphs differ?

c.) When you graph a rectangular equation, you lose the domain restrictions and you lose the direction in which the graph is travelling as time increases.

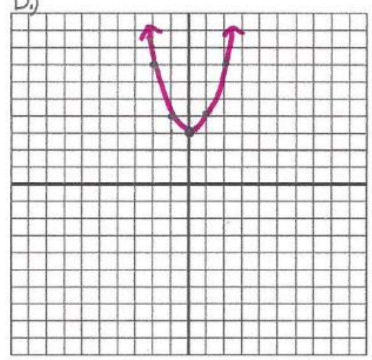
a.) Domain: $t \geq 2$

t	x	y	(x,y)
2	0	3	(0,3)
3	1	4	(1,4)
6	2	7	(2,7)
11	3	12	(3,12)

a.)



b.)



b.) $x = \sqrt{t-2}$
$x^2 = t-2$
$x^2 + 2 = t$
$t = x^2 + 2$

$y = 1 + t$
$y = 1 + x^2 + 2$
$\boxed{y = x^2 + 3}$

Rectangular Equation

x	y
-2	4
-1	1
0	0
1	1
2	4

Vertex: (0,3)

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You Try: Eliminate the Parameter and graph.

$x = 3 + 2t$ and $y = -1 + 5t$

$\frac{x-3}{2} = \frac{2t}{2}$

$t = \frac{x-3}{2}$

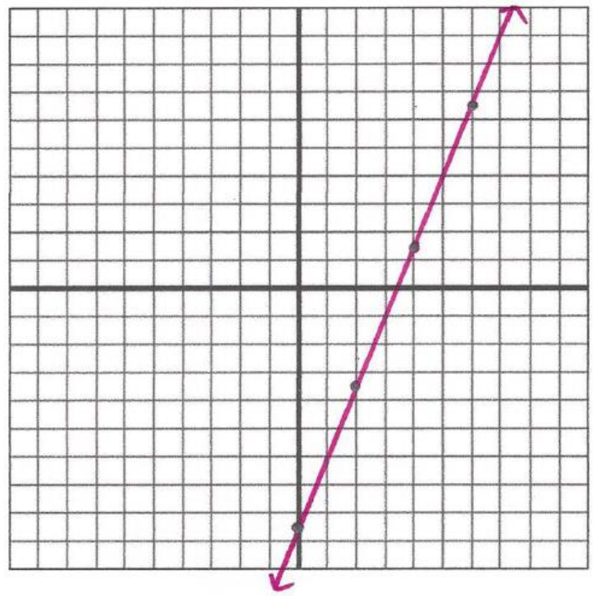
$y = -1 + 5\left(\frac{x-3}{2}\right)$

$y = -1 + \frac{5x}{2} - \frac{15}{2}$

$y = \frac{-2}{2} + \frac{5x}{2} - \frac{15}{2}$

$\boxed{y = \frac{5}{2}x - \frac{17}{2}}$

$y = \frac{5}{2}x - 8\frac{1}{2}$



Example 4: **Eliminating an Angle Parameter**

Sketch the curve represented by

$\frac{x}{2} = \frac{2\cos\theta}{2}$ and $\frac{y}{3} = \frac{3\sin\theta}{3}$ for $0 \le \theta \le 2\pi$.

$\frac{x}{2} = \cos\theta$ $\qquad \frac{y}{3} = \sin\theta$

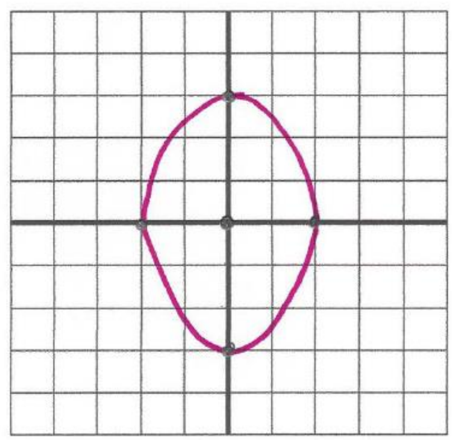
$\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{y}{3}\right)^2 + \left(\frac{x}{2}\right)^2 = 1$

* vertical ellipse

$\frac{y^2}{9} + \frac{x^2}{4} = 1 \longrightarrow \boxed{\frac{x^2}{\underset{b^2}{4}} + \frac{y^2}{\underset{a^2}{9}} = 1}$ $\quad a=3$, $b=2$

center: (0,0)



You Try: Eliminate the Parameter and graph.

$\frac{x}{4} = \frac{4\cos\theta}{4}$ and $\frac{y}{-4} = \frac{-4\sin\theta}{-4}$ for $0 \le \theta \le 2\pi$.

$\frac{x}{4} = \cos\theta$ $\qquad \frac{y}{-4} = \sin\theta$

$\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{y}{-4}\right)^2 + \left(\frac{x}{4}\right)^2 = 1$

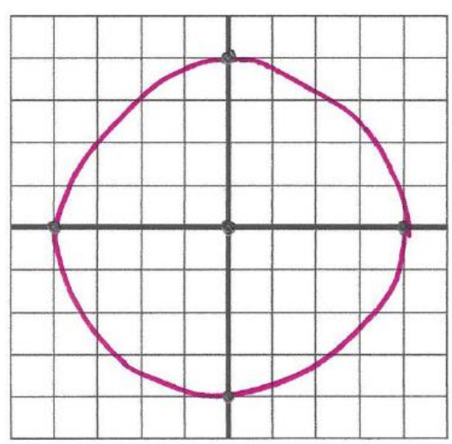
$\frac{y^2}{16} + \frac{x^2}{16} = 1$

$\boxed{\frac{x^2}{16} + \frac{y^2}{16} = 1}$

center: (0,0)
$r = 4$

or $\boxed{x^2 + y^2 = 16}$

*circle



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Part Three: Finding Parametric Equations for a Graph **** Parametric Equations come in PAIRS!****

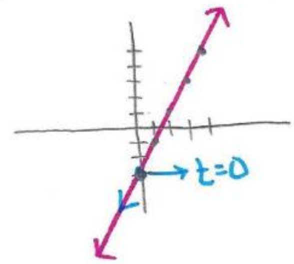
Example 5: Write parametric equations for the line $y = 2x - 3$ with $\underbrace{x = -4t}_{\text{Substitute in for } x.}$.

$y = 2(-4t) - 3$

$y = -8t - 3$

$$\boxed{\begin{aligned} x &= -4t \\ y &= -8t-3 \end{aligned}}$$

t	x	y	(x,y)
-2	8	13	(8,13)
-1	4	5	(4,5)
0	0	-3	(0,-3)
1	-4	-11	(-4,-11)
2	-8	-19	(-8,-19)



You Try: Write parametric equations for the line $y = -4x + 1$ with $x = \frac{t}{2}$.

$y = -4\left(\frac{t}{2}\right) + 1$

$y = -2t + 1$

$$\boxed{\begin{aligned} x &= \frac{t}{2} \\ y &= -2t+1 \end{aligned}}$$

★Note

In the linear equation $y = 2x - 3$

__x__ is the INDEPENDENT variable

__y__ is the DEPENDENT variable

In parametric equations

__t__ is the INDEPENDENT variable

__x__ & __y__ are the DEPENDENT variables
