Exploring Limits … Set 1

1. Evaluate this limit using the Limit Laws. Show each step.

$$\lim_{x \to 5}(2x^2 - 3x + 4)$$

Exploring Limits … Set 1

Answers

Notice that the limits on this worksheet can be evaluated using direct substitution, but the purpose of the problems here is to give you practice at using the Limit Laws.

1. Evaluate this limit using the Limit Laws. Show each step.

$$\lim_{x\to 5}(2x^2 - 3x + 4)$$

Solution:
Using the Limit Laws, rewrite the limit.

$$\lim_{x\to 5}(2x^2 - 3x + 4) = \lim_{x\to 5}2x^2 - \lim_{x\to 5}3x + \lim_{x\to 5}4$$

$$= 2\lim_{x\to 5}x^2 - 3\lim_{x\to 5}x + \lim_{x\to 5}4$$

$$= 2\left(\lim_{x\to 5}x\right)^2 - 3\lim_{x\to 5}x + \lim_{x\to 5}4$$

$$= 2(5)^2 - 3(5) + 4 = 39$$

Answer: 39

Exploring Limits … Set 1

2. Evaluate this limit using the Limit Laws. Show each step.

$$\lim_{x \to -2} \left(\frac{x^3 + 2x^2 - 1}{5 - 3x} \right)$$

Exploring Limits … Set 1

Answers

Notice that the limits on this worksheet can be evaluated using direct substitution, but the purpose of the problems here is to give you practice at using the Limit Laws.

2. Evaluate this limit using the Limit Laws. Show each step.

$$\lim_{x\to -2}\left(\frac{x^3+2x^2-1}{5-3x}\right)$$

Solution:

Using the Limit Laws, rewrite the limit.

$$\lim_{x\to -2}\left(\frac{x^3+2x^2-1}{5-3x}\right)=\frac{\lim_{x\to -2} x^3+\lim_{x\to -2} 2x^2-\lim_{x\to -2} 1}{\lim_{x\to -2} 5-\lim_{x\to -2} 3x}$$

$$=\frac{\left(\lim_{x\to -2} x\right)^3+2\left(\lim_{x\to -2} x\right)^2-\lim_{x\to -2} 1}{\lim_{x\to -2} 5-3\lim_{x\to -2} x}$$

$$=\frac{(-2)^3+2(-2)^2-1}{5-3(-2)}=\frac{-8+8-1}{5+6}=-\frac{1}{11}$$

Answer: $-\frac{1}{11}$

Exploring Limits … Set 1

3. Evaluate this limit using the Limit Laws. Show each step.

$$\lim_{h \to 6} \left(\frac{\sqrt{3+h} - 2}{h} \right)$$

Exploring Limits … Set 1

Answers

Notice that the limits on this worksheet can be evaluated using direct substitution, but the purpose of the problems here is to give you practice at using the Limit Laws.

3. Evaluate this limit using the Limit Laws. Show each step.

$$\lim_{h\to 6}\left(\frac{\sqrt{3+h}-2}{h}\right)$$

Solution:

Using the Limit Laws, rewrite the limit.

$$\lim_{h\to 6}\left(\frac{\sqrt{3+h}-2}{h}\right)=\frac{\lim\limits_{h\to 6}\sqrt{3+h}-\lim\limits_{h\to 6}2}{\lim\limits_{h\to 6}h}$$

$$\frac{\sqrt{\lim\limits_{h\to 6}(3+h)}-\lim\limits_{h\to 6}2}{\lim\limits_{h\to 6}h}=\frac{\sqrt{\lim\limits_{h\to 6}3+\lim\limits_{h\to 6}h}-\lim\limits_{h\to 6}2}{\lim\limits_{h\to 6}h}=\frac{\sqrt{3+6}-2}{6}=\frac{3-2}{6}=\frac{1}{6}$$

Answer: $\frac{1}{6}$

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4. Evaluate this limit using the Limit Laws. Show each step.

$$\lim_{x \to 1} \left(\frac{x^2 - 1}{x + 1} \right)$$

Exploring Limits … Set 1

Answers

Notice that the limits on this worksheet can be evaluated using direct substitution, but the purpose of the problems here is to give you practice at using the Limit Laws.

4. Evaluate this limit using the Limit Laws. Show each step.

$$\lim_{x\to 1}\left(\frac{x^2-1}{x+1}\right)$$

Solution:

The given limit can be factored and simplified.

$$\lim_{x\to 1}\left(\frac{x^2-1}{x+1}\right)=\lim_{x\to 1}\frac{(x-1)(x+1)}{x+1}=\lim_{x\to 1}(x-1)$$

Using the Limit Laws, rewrite the limit.

$$\lim_{x\to 1}(x-1)=\lim_{x\to 1}x-\lim_{x\to 1}1=1-1=0$$

Without factoring and simplifying, using the Limit Laws, rewrite the limit.

$$\lim_{x\to 1}\left(\frac{x^2-1}{x+1}\right)=\frac{\lim\limits_{x\to 1}x^2-\lim\limits_{x\to 1}1}{\lim\limits_{x\to 1}x+\lim\limits_{x\to 1}1}=\frac{\left(\lim\limits_{x\to 1}x\right)^2-\lim\limits_{x\to 1}1}{\lim\limits_{x\to 1}x+\lim\limits_{x\to 1}1}=\frac{1^2-1}{1+1}=\frac{0}{2}=0$$

Answer: 0

5. Evaluate this limit using the Limit Laws. Show each step.

$$\lim_{x \to 6} \left(\frac{\sqrt{x^2 + 4x + 4} - 3}{x} \right)$$

Exploring Limits … Set 1

Answers

5. Evaluate this limit using the Limit Laws. Show each step.

$$\lim_{x\to 6}\left(\frac{\sqrt{x^2+4x+4}-3}{x}\right)$$

Solution:

The given limit can be factored and simplified.

$$\lim_{x\to 6}\left(\frac{\sqrt{x^2+4x+4}-3}{x}\right)=\lim_{x\to 6}\left(\frac{\sqrt{(x+2)^2}-3}{x}\right)$$

$$=\lim_{x\to 6}\left(\frac{x+2-3}{x}\right)=\lim_{x\to 6}\left(\frac{x-1}{x}\right)$$

Using the Limit Laws, rewrite the limit.

$$\lim_{x\to 6}\left(\frac{x-1}{x}\right)=\frac{\lim\limits_{x\to 6}x-\lim\limits_{x\to 6}1}{\lim\limits_{x\to 6}x}=\frac{6-1}{6}=\frac{5}{6}$$

Without factoring and simplifying, using the Limit Laws, rewrite the limit.

$$\frac{\lim\limits_{x\to 6}\sqrt{x^2+4x+4}-\lim\limits_{x\to 6}3}{\lim\limits_{x\to 6}x}=\frac{\sqrt{\lim\limits_{x\to 6}x^2+\lim\limits_{x\to 6}4x+\lim\limits_{x\to 6}4}-\lim\limits_{x\to 6}3}{\lim\limits_{x\to 6}x}$$

$$=\frac{\sqrt{\left(\lim\limits_{x\to 6}x\right)^2+4\lim\limits_{x\to 6}x+\lim\limits_{x\to 6}4}-\lim\limits_{x\to 6}3}{\lim\limits_{x\to 6}x}=\frac{\sqrt{6^2+4(6)+4}-3}{6}$$

$$=\frac{\sqrt{64}-3}{6}=\frac{8-3}{6}=\frac{5}{6}$$

Answer: $\frac{5}{6}$

Exploring Limits … Set 1

6. Evaluate this limit using the Limit Laws. Show each step.

$$\lim_{x \to 4}(5x^3 - 3x^2 + x - 6)$$

Exploring Limits … Set 1

Answers

6. Evaluate this limit using the Limit Laws. Show each step.

$$\lim_{x\to 4}(5x^3 - 3x^2 + x - 6)$$

Solution:

Using the Limit Laws, rewrite the limit.

$$\lim_{x\to 4}(5x^3 - 3x^2 + x - 6) = \lim_{x\to 4} 5x^3 - \lim_{x\to 4} 3x^2 + \lim_{x\to 4} x - \lim_{x\to 4} 6$$

$$= 5\left(\lim_{x\to 4} x\right)^3 - 3\left(\lim_{x\to 4} x\right)^2 + \lim_{x\to 4} x - \lim_{x\to 4} 6$$

$$= 5(4)^3 - 3(4)^2 + 4 - 6 = 270$$

Answer: 270

Exploring Limits … Set 1

7. Evaluate this limit using the Limit Laws. Show each step.

$$\lim_{x \to -1} (x^4 - 3x)(x^2 + 5x + 3)$$

Exploring Limits … Set 1

Answers

7. Evaluate this limit using the Limit Laws. Show each step.

$$\lim_{x \to -1} (x^4 - 3x)(x^2 + 5x + 3)$$

Solution:

Using the Limit Laws, rewrite the limit.

$$\lim_{x \to -1} (x^4 - 3x)(x^2 + 5x + 3)$$

$$= \left(\lim_{x \to -1} x^4 - \lim_{x \to -1} 3x\right) \cdot \left(\lim_{x \to -1} x^2 + \lim_{x \to -1} 5x + \lim_{x \to -1} 3\right)$$

$$= \left[\left(\lim_{x \to -1} x\right)^4 - 3 \lim_{x \to -1} x\right] \cdot \left[\left(\lim_{x \to -1} x\right)^2 + 5 \lim_{x \to -1} x + \lim_{x \to -1} 3\right]$$

$$= [(-1)^4 - 3(-1)] \cdot [(-1)^2 + 5(-1) + 3]$$

$$= [1 + 3] \cdot [1 - 5 + 3] = 4 \cdot (-1) = -4$$

Answer: -4

Exploring Limits … Set 1

8. Evaluate this limit using the Limit Laws. Show each step.

$$\lim_{x \to -2} \left(\frac{x^4 - 4}{2x^2 - 3x + 2} \right)$$

Answers

8. Evaluate this limit using the Limit Laws. Show each step.

$$\lim_{x\to -2}\left(\frac{x^4-4}{2x^2-3x+2}\right)$$

Solution:

Using the Limit Laws, rewrite the limit.

$$\lim_{x\to -2}\left(\frac{x^4-4}{2x^2-3x+2}\right)=\frac{\lim\limits_{x\to -2}x^4-\lim\limits_{x\to -2}4}{\lim\limits_{x\to -2}2x^2-\lim\limits_{x\to -2}3x+\lim\limits_{x\to -2}2}$$

$$=\frac{\left(\lim\limits_{x\to -2}x\right)^4-\lim\limits_{x\to -2}4}{2\left(\lim\limits_{x\to -2}x\right)^2-3\lim\limits_{x\to -2}x+\lim\limits_{x\to -2}2}$$

$$=\frac{(-2)^4-4}{2(-2)^2-3(-2)+2}=\frac{16-4}{8+6+2}=\frac{12}{16}=\frac{3}{4}$$

Answer: $\frac{3}{4}$

Exploring Limits … Set 1

9. Evaluate this limit using the Limit Laws. Show each step.

$$\lim_{x \to 2} \sqrt{x^4 - 3x - 6}$$

Exploring Limits … Set 1

Answers

9. Evaluate this limit using the Limit Laws. Show each step.

$$\lim_{x\to 2}\sqrt{x^4 - 3x - 6}$$

Solution:

Using the Limit Laws, rewrite the limit.

$$\lim_{x\to 2}\sqrt{x^4 - 3x - 6} = \sqrt{\lim_{x\to 2}x^4 - \lim_{x\to 2}3x - \lim_{x\to 2}6}$$

$$= \sqrt{\left(\lim_{x\to 2}x\right)^4 - 3\lim_{x\to 2}x - \lim_{x\to 2}6}$$

$$= \sqrt{2^4 - 3(2) - 6} = \sqrt{4} = 2$$

Answer: 2

Exploring Limits … Set 1

10. Evaluate this limit using the Limit Laws. Show each step.

$$\lim_{x \to 2} \left(\frac{x^4 - 2}{x^3 - 3x + 5} \right)^2$$

Answers

10. Evaluate this limit using the Limit Laws. Show each step.

$$\lim_{x\to 2}\left(\frac{x^4-2}{x^3-3x+5}\right)^2$$

Solution:

Using the Limit Laws, rewrite the limit.

$$\lim_{x\to 2}\left(\frac{x^4-2}{x^3-3x+5}\right)^2=\left(\frac{\lim\limits_{x\to 2}x^4-\lim\limits_{x\to 2}2}{\lim\limits_{x\to 2}x^3-\lim\limits_{x\to 2}3x+\lim\limits_{x\to 2}5}\right)^2$$

$$=\left[\frac{\left(\lim\limits_{x\to 2}x\right)^4-\lim\limits_{x\to 2}2}{\left(\lim\limits_{x\to 2}x\right)^3-3\lim\limits_{x\to 2}x+\lim\limits_{x\to 2}5}\right]^2=\left[\frac{2^4-2}{2^3-3(2)+5}\right]^2=\left[\frac{14}{7}\right]^2=2^2=4$$

Answer: 4

Exploring Limits … Set 1

11. Evaluate this limit using the Limit Laws. Show each step.

$$\lim_{x \to 8}\left(1 + \sqrt[3]{x}\right)(2 - 9x^2 + x^3)$$

Answers

11. Evaluate this limit using the Limit Laws. Show each step.

$$\lim_{x\to 8}\left(1+\sqrt[3]{x}\right)\left(2-9x^2+x^3\right)$$

Solution:

Using the Limit Laws, rewrite the limit.

$$\lim_{x\to 8}\left(1+\sqrt[3]{x}\right)\left(2-9x^2+x^3\right)=\left[\lim_{x\to 8}1+\lim_{x\to 8}\sqrt[3]{x}\right]\cdot\left[\lim_{x\to 8}2-\lim_{x\to 8}9x^2+\lim_{x\to 8}x^3\right]$$

$$=\left[\lim_{x\to 8}1+\sqrt[3]{\lim_{x\to 8}x}\right]\cdot\left[\lim_{x\to 8}2-9\left(\lim_{x\to 8}x\right)^2+\left(\lim_{x\to 8}x\right)^3\right]$$

$$=\left[1+\sqrt[3]{8}\right]\cdot\left[2-9(8)^2+8^3\right]$$

$$=(1+2)(2-576+512)=-186$$

Answer: -186

12. Evaluate this limit using the Limit Laws. Show each step.

$$\lim_{x \to 2} \sqrt{\frac{2x^2 + 1}{3x - 2}}$$

Answers

12. Evaluate this limit using the Limit Laws. Show each step.

$$\lim_{x \to 2} \sqrt{\frac{2x^2+1}{3x-2}}$$

Solution:

Using the Limit Laws, rewrite the limit.

$$\lim_{x \to 2} \sqrt{\frac{2x^2+1}{3x-2}} = \sqrt{\lim_{x \to 2} \frac{2x^2+1}{3x-2}}$$

$$= \sqrt{\frac{\lim_{x \to 2} 2x^2 + \lim_{x \to 2} 1}{\lim_{x \to 2} 3x - \lim_{x \to 2} 2}} = \sqrt{\frac{2\left(\lim_{x \to 2} x\right)^2 + \lim_{x \to 2} 1}{3\lim_{x \to 2} x - \lim_{x \to 2} 2}} = \sqrt{\frac{2(2)^2+1}{3(2)-2}} = \sqrt{\frac{9}{4}} = \frac{3}{2}$$

Answer: $\frac{3}{2}$

Exploring Limits … Set 1

13. Evaluate this limit using the Limit Laws. Show each step. Give an exact answer.

$$\lim_{x \to \frac{\pi}{3}} (2\sin x + 3\cos x + 4\tan x)$$

Exploring Limits … Set 1

Answers

13. Evaluate this limit using the Limit Laws. Show each step. Give an exact answer.

$$\lim_{x \to \frac{\pi}{3}} (2\sin x + 3\cos x + 4\tan x)$$

Solution:

Using the Limit Laws, rewrite the limit.

$$\lim_{x \to \frac{\pi}{3}} (2\sin x + 3\cos x + 4\tan x) = 2\lim_{x \to \frac{\pi}{3}} \sin x + 3\lim_{x \to \frac{\pi}{3}} \cos x + 4\lim_{x \to \frac{\pi}{3}} \tan x$$

$$= 2\sin\left(\frac{\pi}{3}\right) + 3\cos\left(\frac{\pi}{3}\right) + 4\tan\left(\frac{\pi}{3}\right)$$

$$= 2\left(\frac{\sqrt{3}}{2}\right) + 3\left(\frac{1}{2}\right) + 4\left(\sqrt{3}\right) = 5\sqrt{3} + \frac{3}{2}$$

Answer: $5\sqrt{3} + \frac{3}{2}$