Chain Rule … Set 2

Chain Rule Practice Problems WS #1

Finding a Derivative **In Exercises 7–34, find the derivative of the function.**

Chain Rule: $\frac{d}{dx}[f(g(x))] = f'[g(x)] * g'(x)$

1. $y = (5x - 8)^4$

2) $y = (4x - 1)^3$

3) $y = 5(2 - x^3)^4$

4) $g(x) = 3(4 - 9x)^4$

5) $f(t) = \sqrt{5 - t}$

6) $y = \sqrt[3]{6x^2 + 1}$

7) $f(x) = \sqrt{x^2 - 4x + 2}$

8) $y = 2\sqrt[4]{9 - x^2}$

Chain Rule … Set 2

Answers

Finding a Derivative In Exercises 7–34, find the derivative of the function.

Chain Rule: $\frac{d}{dx}[f(g(x))] = f'[g(x)] * g'(x)$

1. $y = (5x - 8)^4$

outside: $(\ )^4$
inside: $5x-8$

$y' = 4(\ \ )^3 \cdot (5)$

$y' = 4(5x-8)^3 \cdot 5$

$\boxed{y' = 20(5x-8)^3}$

2) $y = (4x - 1)^3$

outside: $(\ )^3$
inside: $4x-1$

$y' = 3(4x-1)^2 \cdot (4)$

$\boxed{y' = 12(4x-1)^2}$

3) $y = 5(2 - x^3)^4$

outside: $5(\ )^4$
inside: $2-x^3$

$y' = 5 \cdot 4(\ \ )^3 \cdot (-3x^2)$

$y' = 20(2-x^3)^3 \cdot -3x^2$

$\boxed{y' = -60x^2(2-x^3)^3}$

4) $g(x) = 3(4 - 9x)^4$

outside: $3(\ )^4$
inside: $4-9x$

$g'(x) = 3 \cdot 4(\ \ )^3 \cdot (-9)$

$g'(x) = 12(4-9x)^3(-9)$

$\boxed{g'(x) = -108(4-9x)^3}$

5) $f(t) = \sqrt{5 - t}$

outside: $(\ )^{1/2}$
inside: $5-t$

$f(t) = (5-t)^{1/2}$

$f'(t) = \frac{1}{2}(\ \ )^{-1/2}(-1)$

$f'(t) = \frac{1}{2}(5-t)^{-1/2}(-1)$

$\boxed{f'(t) = \frac{-1}{2(5-t)^{1/2}}}$

6) $y = \sqrt[3]{6x^2 + 1}$

outside: $(\ \ )^{1/3}$
inside: $6x^2+1$

$y = (6x^2+1)^{1/3}$

$y' = \frac{1}{3}(\ \ )^{-2/3}(12x)$

$y' = \frac{1}{3}(6x^2+1)^{-2/3} \cdot 12x$

$\boxed{y' = \frac{4x}{(6x^2+1)^{2/3}}}$

7) $f(x) = \sqrt{x^2 - 4x + 2}$

outside: $(\ )^{1/2}$
inside: x^2-4x+2

$f(x) = (x^2-4x+2)^{1/2}$

$f'(x) = \frac{1}{2}(\ \ )^{-1/2} \cdot (2x-4)$

$f'(x) = \frac{1}{\cancel{2}}(x^2-4x+2)^{-1/2} \cdot \cancel{2}(x-2)$

$\boxed{f'(x) = \frac{x-2}{(x^2-4x+2)^{1/2}}}$

8) $y = 2\sqrt[4]{9 - x^2}$

outside: $2(\ )^{1/4}$
inside: $9-x^2$

$y = 2(9-x^2)^{1/4}$

$y' = 2 \cdot \frac{1}{4}(\ \ )^{-3/4} \cdot (-2x)$

$y' = \frac{2}{4}(9-x^2)^{-3/4}(-2x)$

$\boxed{y' = \frac{-x}{(9-x^2)^{3/4}}}$

Chain Rule … Set 2

Find the derivative of the function below:

Chain Rule: $\frac{d}{dx}[f(g(x))] = f'[g(x)] * g'(x)$

9) $y = \dfrac{1}{x-2}$

10) $y = \dfrac{1}{\sqrt{3x+5}}$

11) $y = \dfrac{x}{\sqrt{x^2+1}}$

12) $y = \dfrac{x}{\sqrt{x^4+4}}$

13) $g(x) = \left(\dfrac{x+5}{x^2+2}\right)^2$

14) $g(x) = \left(\dfrac{3x^2-2}{2x+3}\right)^3$

Chain Rule … Set 2

Answers

Find the derivative of the function below:

Chain Rule: $\frac{d}{dx}[f(g(x))] = f'[g(x)] * g'(x)$

9) $y = \dfrac{1}{x-2}$

outside: $(\ )^{-1}$
inside: $x-2$

$y = (x-2)^{-1}$

$y' = -1(x-2)^{-2}(1)$

$\boxed{y' = \dfrac{-1}{(x-2)^2}}$

10) $y = \dfrac{1}{\sqrt{3x+5}}$

outside: $(\ )^{-1/2}$
inside: $3x+5$

$y = \dfrac{1}{(3x+5)^{1/2}}$

$y = (3x+5)^{-1/2}$

$y' = \frac{-1}{2}(\quad)^{-3/2}(3)$

$y' = \frac{-1}{2}(3x+5)^{-3/2}(3)$

$\boxed{y' = \dfrac{-3}{2(3x+5)^{3/2}}}$

11) $y = \dfrac{x}{\sqrt{x^2+1}}$

1) quotient
2) chain
outside: $(\ )^{1/2}$
inside: x^2+1

$y = \dfrac{x}{(x^2+1)^{1/2}}$

$y' = \dfrac{(1)(x^2+1)^{1/2} - x \cdot \frac{1}{2}(x^2+1)^{-1/2}(2x)}{[(x^2+1)^{1/2}]^2}$

$y' = \dfrac{(x^2+1)^{1/2} - \dfrac{x^2}{(x^2+1)^{1/2}}}{x^2+1} \cdot \dfrac{(x^2+1)^{1/2}}{}$

$y' = \dfrac{x^2+1-x^2}{(x^2+1)(x^2+1)^{1/2}}$

$\boxed{y' = \dfrac{1}{(x^2+1)^{3/2}}}$

12) $y = \dfrac{x}{\sqrt{x^4+4}}$

1) quotient
2) chain:
outside: $(\ )^{1/2}$
inside: x^4+4

$y = \dfrac{x}{(x^4+4)^{1/2}}$

$y' = \dfrac{(1)(x^4+4)^{1/2} - (x)\cdot\frac{1}{2}(x^4+4)^{-1/2}(4x^3)}{[(x^4+4)^{1/2}]^2}$

$y' = \dfrac{(x^4+4)^{1/2} - \dfrac{2x^4}{(x^4+4)^{1/2}}}{x^4+4} \cdot (x^4+4)^{1/2}$

$y' = \dfrac{x^4+4-2x^4}{(x^4+4)^{3/2}}$

$\boxed{y' = \dfrac{4-x^4}{(x^4+4)^{3/2}}}$

13) $g(x) = \left(\dfrac{x+5}{x^2+2}\right)^2$

1) chain
outside: $(\ )^2$
inside: $\dfrac{x+5}{x^2+2}$
2) quotient

$g'(x) = 2\left[\dfrac{x+5}{x^2+2}\right]^1 \left[\dfrac{(1)(x^2+2)-(x+5)(2x)}{(x^2+2)^2}\right]$

$g'(x) = \dfrac{2(x+5)(x^2+2-2x^2-10x)}{(x^2+2)^3}$

$\boxed{g'(x) = \dfrac{2(x+5)(-x^2-10x+2)}{(x^2+2)^3}}$

14) $g(x) = \left(\dfrac{3x^2-2}{2x+3}\right)^3$

1) chain
outside: $(\ )^3$
inside: $\dfrac{3x^2-2}{2x+3}$
2) quotient

$g'(x) = 3\left[\dfrac{3x^2-2}{2x+3}\right]^2 \left[\dfrac{(6x)(2x+3)-(3x^2-2)(2)}{(2x+3)^2}\right]$

$g'(x) = \dfrac{3(3x^2-2)^2(12x^2+18x-6x^2+4)}{(2x+3)^2(2x+3)^2}$

$\boxed{g'(x) = \dfrac{3(3x^2-2)^2(6x^2+18x+4)}{(2x+3)^4}}$