Logarithms Rule … Set 2

Worksheet: Derivatives of exponential and logarithmic functions

Find the derivatives of the following functions.

1. $f(x) = 2^x$ Hint: Write 2^x as $e^{\ln(2^x)}$, which is the same as $e^{(\ln 2)x}$.

2. $f(x) = a^x$, where a is any positive number.

3. $f(x) = \log_2(x)$ Hint: use the change of base formula to write in terms of natural log.

4. $f(x) = \log_a(x)$, where a is any positive number.

5. $f(x) = e^{2x}\cos(x)$

6. $f(x) = \dfrac{\ln x}{x}$

7. $f(x) = \ln(2x)$

8. $f(x) = \ln(1/x)$

9. $f(x) = x\ln(x) - x$

10. $f(x) = x^x$ Hint: see hint for #1.

For the following functions, find all critical points and classify each critical point as either a local maximum, a local minimum, or neither.

11. $f(x) = xe^{-x}$

12. $f(x) = e^x + e^{-2x}$

13. $f(x) = x\ln(x)$

Logarithms Rule … Set 2

Answers

Find the derivatives of the following functions.

1. $f(x) = 2^x$ Hint: Write 2^x as $e^{\ln(2^x)}$, which is the same as $e^{(\ln 2)x}$.

   Answer: $f'(x) = \ln(2) \cdot 2^x$

2. $f(x) = a^x$, where a is any positive number.

   Answer: $f'(x) = \ln(2) \cdot 2^x$

3. $f(x) = \log_2(x)$ Hint: use the change of base formula to write in terms of natural log.

   Answer: $f'(x) = \frac{1}{(\ln 2)x}$

4. $f(x) = \log_a(x)$, where a is any positive number.

   Answer: $f'(x) = \frac{1}{(\ln a)x}$

5. $f(x) = e^{2x}\cos(x)$

   Answer: $f'(x) = -e^{2x}\sin(x) + 2e^{2x}\cos(x)$

6. $f(x) = \dfrac{\ln x}{x}$

   Answer: $f'(x) = \frac{1-\ln x}{x^2}$

7. $f(x) = \ln(2x)$

   Answer: $f'(x) = 1/x$. Can you explain why $\ln(2x)$ and $\ln(x)$ have the same derivative?

8. $f(x) = \ln(1/x)$

   Answer: $f'(x) = -1/x$

9. $f(x) = x\ln(x) - x$

   Answer: $f'(x) = \ln(x)$

10. $f(x) = x^x$ Hint: see hint for #1.

    Answer: $f'(x) = x^x(1 + \ln x)$

For the following functions, find all critical points and classify each critical point as either a local maximum, a local minimum, or neither.

11. $f(x) = xe^{-x}$

    Answer: There is one critical point, $(1, \frac{1}{e})$. It is a local maximum (it's actually the global maximum).

12. $f(x) = e^x + e^{-2x}$

    Answer: There is one critical point, $\left(\frac{\ln 2}{3}, \sqrt[3]{2} + \sqrt[3]{\frac{1}{4}}\right)$. It is a local minimum (it's actually the global minimum).

13. $f(x) = x\ln(x)$

    Answer: There is one critical point, $(\frac{1}{e}, -\frac{1}{e})$. It is a local minimum (it's actually the global minimum). (By the way, the domain of the function is $(0, \infty)$.)