Differentiation Summary … Set 1

1. Show that $\frac{d}{dx}\left[\cos^{-1}(x)\right] = \frac{-1}{\sqrt{1-x^2}}$.

2. Show that $\frac{d}{dx}\left[\csc^{-1}(x)\right] = \frac{-1}{|x|\sqrt{x^2-1}}$.

3. Show that $\frac{d}{dx}\left[\cot^{-1}(x)\right] = \frac{-1}{1+x^2}$.

Find the derivatives of the given functions.

4. $\sin^{-1}\left(\sqrt{2x}\right)$

5. $\ln\left(\tan^{-1}(x)\right)$

6. $e^x \tan^{-1}(x)$

7. $\tan^{-1}(\pi x)$

8. $\sec^{-1}(\pi x)$

9. $\ln\left(\sin^{-1}(x)\right)$

10. $\cos^{-1}(\pi x)$

11. $\sec^{-1}\left(x^5\right)$

12. $e^{\tan^{-1}(\pi x)}$

13. $\tan^{-1}\left(\ln(x)\right) + \pi$

14. $\tan^{-1}\left(x\sin(x)\right)$

15. $x\sin^{-1}\left(\ln(x)\right)$

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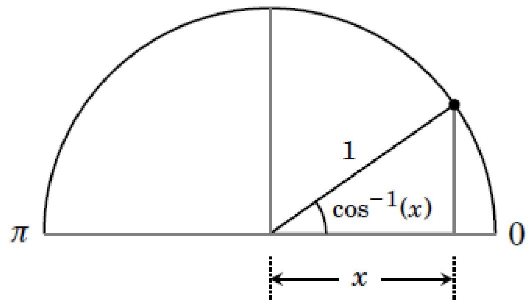
Answers

1. Show that $\frac{d}{dx}\left[\cos^{-1}(x)\right] = \frac{-1}{\sqrt{1-x^2}}$.

By the inverse rule, $\frac{d}{dx}\left[\cos^{-1}(x)\right] = \frac{1}{-\sin\left(\cos^{-1}(x)\right)}$.

Now we simplify the denominator. From the standard diagram for $\cos^{-1}(x)$ we get $\sin\left(\cos^{-1}(x)\right) = \frac{\text{OPP}}{\text{HYP}} = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$. With this, the above becomes $\frac{d}{dx}\left[\cos^{-1}(x)\right] = \frac{-1}{\sqrt{1-x^2}}$.



3. Show that $\frac{d}{dx}\left[\cot^{-1}(x)\right] = \frac{-1}{1+x^2}$.

Suggestion: Verify the identity $\cot^{-1}(x) = \frac{\pi}{2} - \tan^{-1}(x)$. Then differentiate both sides of this.

5. $\frac{d}{dx}\left[\ln\left(\tan^{-1}(x)\right)\right] = \frac{1}{\tan^{-1}(x)}\frac{d}{dx}\left[\tan^{-1}(x)\right] = \frac{1}{\tan^{-1}(x)}\frac{1}{1+x^2} = \frac{1}{\tan^{-1}(x)\left(1+x^2\right)}$

7. $\frac{d}{dx}\left[\tan^{-1}(\pi x)\right] = \frac{\pi}{1+(\pi x)^2} = \frac{\pi}{1+\pi^2 x^2}$

9. $\frac{d}{dx}\left[\ln\left(\sin^{-1}(x)\right)\right] = \frac{1}{\sin^{-1}(x)}\frac{d}{dx}\left[\sin^{-1}(x)\right] = \frac{1}{\sin^{-1}(x)}\frac{1}{\sqrt{1-x^2}} = \frac{1}{\sin^{-1}(x)\sqrt{1-x^2}}$

11. $\frac{d}{dx}\left[\sec^{-1}\left(x^5\right)\right] = \frac{1}{|x^5|\sqrt{\left(x^5\right)^2-1}}5x^4 = \frac{5x^4}{|x^5|\sqrt{x^{10}-1}} = \frac{5}{|x|\sqrt{x^{10}-1}}$

13. $\frac{d}{dx}\left[\tan^{-1}(\ln(x))+\pi\right] = \frac{1}{1+\left(\ln(x)\right)^2}\frac{1}{x} = \frac{1}{x+x\left(\ln(x)\right)^2}$

15. $\frac{d}{dx}\left[x\sin^{-1}(\ln(x))\right] = 1\cdot\sin^{-1}(\ln(x)) + x\frac{d}{dx}\left[\sin^{-1}(\ln(x))\right]$

$$= \sin^{-1}(\ln(x)) + x\frac{1}{\sqrt{1-(\ln(x))^2}}\frac{1}{x} = \sin^{-1}(\ln(x)) + \frac{1}{\sqrt{1-(\ln(x))^2}}$$