90 Minutes—No Calculator

Note: In this examination, ln x denotes the natural logarithm of x (that is, logarithm to the base e).

1. Which of the following defines a function f for which $f(-x) = -f(x)$?

(A) $f(x) = x^2$ (B) $f(x) = \sin x$ (C) $f(x) = \cos x$

(D) $f(x) = \log x$ (E) $f(x) = e^x$

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2. $\ln(x-2) < 0$ if and only if

(A) $x < 3$ (B) $0 < x < 3$ (C) $2 < x < 3$

(D) $x > 2$ (E) $x > 3$

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3. If $\begin{cases} f(x) = \dfrac{\sqrt{2x+5} - \sqrt{x+7}}{x-2}, & \text{for } x \neq 2, \\ f(2) = k \end{cases}$ and if f is continuous at $x = 2$, then $k =$

(A) 0 (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) 1 (E) $\frac{7}{5}$

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4. $\displaystyle\int_0^8 \frac{dx}{\sqrt{1+x}} =$

(A) 1 (B) $\frac{3}{2}$ (C) 2 (D) 4 (E) 6

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5. If $3x^2 + 2xy + y^2 = 2$, then the value of $\frac{dy}{dx}$ at $x = 1$ is

(A) -2 (B) 0 (C) 2 (D) 4 (E) not defined

Answers

1. B Sine is the only odd function listed. $\sin(-x) = -\sin(x)$.

2. C $\ln t < 0$ for $0 < t < 1 \Rightarrow \ln(x-2) < 0$ for $2 < x < 3$.

3. B Need to have $\lim_{x\to 2} f(x) = f(2) = k$.

$$k = \lim_{x\to 2}\frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2} = \lim_{x\to 2}\frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}\cdot\frac{\sqrt{2x+5}+\sqrt{x+7}}{\sqrt{2x+5}+\sqrt{x+7}}$$

$$= \lim_{x\to 2}\frac{2x+5-(x+7)}{x-2}\cdot\frac{1}{\sqrt{2x+5}+\sqrt{x+7}} = \lim_{x\to 2}\frac{1}{\sqrt{2x+5}+\sqrt{x+7}} = \frac{1}{6}$$

4. D $\int_0^8 \frac{dx}{\sqrt{1+x}} = 2\sqrt{1+x}\Big|_0^8 = 2(3-1) = 4$

5. E Using implicit differentiation, $6x + 2xy' + 2y + 2y\cdot y' = 0$. Therefore $y' = \frac{-2y-6x}{2x+2y}$.

When $x = 1$, $3 + 2y + y^2 = 2 \Rightarrow 0 = y^2 + 2y + 1 = (y+1)^2 \Rightarrow y = -1$

Therefore $2x + 2y = 0$ and so $\frac{dy}{dx}$ is not defined at $x = 1$.

6. What is $\lim_{h\to 0} \dfrac{8\left(\frac{1}{2}+h\right)^8 - 8\left(\frac{1}{2}\right)^8}{h}$?

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) The limit does not exist.

(E) It cannot be determined from the information given.

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7. For what value of k will $x+\frac{k}{x}$ have a relative maximum at $x=-2$?

(A) –4 (B) –2 (C) 2 (D) 4 (E) None of these

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8. If $p(x)=(x+2)(x+k)$ and if the remainder is 12 when $p(x)$ is divided by $x-1$, then $k=$

(A) 2 (B) 3 (C) 6 (D) 11 (E) 13

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9. When the area in square units of an expanding circle is increasing twice as fast as its radius in linear units, the radius is

(A) $\frac{1}{4\pi}$ (B) $\frac{1}{4}$ (C) $\frac{1}{\pi}$ (D) 1 (E) π

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10. The set of all points (e^t, t), where t is a real number, is the graph of $y=$

(A) $\frac{1}{e^x}$ (B) $e^{\frac{1}{x}}$ (C) $xe^{\frac{1}{x}}$ (D) $\frac{1}{\ln x}$ (E) $\ln x$

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11. The point on the curve $x^2+2y=0$ that is nearest the point $\left(0,-\frac{1}{2}\right)$ occurs where y is

(A) $\frac{1}{2}$ (B) 0 (C) $-\frac{1}{2}$ (D) -1 (E) none of the above

Answers

6. B This is the derivative of $f(x)=8x^8$ at $x=\frac{1}{2}$

$f'\left(\frac{1}{2}\right)=64\left(\frac{1}{2}\right)^7=\frac{1}{2}$

7. D With $f(x)=x+\frac{k}{x}$, we need $0=f'(-2)=1-\frac{k}{4}$ and so $k=4$. Since $f''(-2)<0$ for $k=4$, f does have a relative maximum at $x=-2$.

8. B $p(x)=q(x)(x-1)+12$ for some polynomial $q(x)$ and so $12=p(1)=(1+2)(1+k)\Rightarrow k=3$

9. C $A=\pi r^2$, $\frac{dA}{dt}=2\pi r\cdot\frac{dr}{dt}$ and from the given information in the problem $\frac{dA}{dt}=2\frac{dr}{dt}$.

So, $2\frac{dr}{dt}=2\pi r\cdot\frac{dr}{dt}\Rightarrow r=\frac{1}{\pi}$

10. E $x=e^y\Rightarrow y=\ln x$

11. B Let L be the distance from $\left(x,-\frac{x^2}{2}\right)$ and $\left(0,-\frac{1}{2}\right)$.

$L^2=(x-0)^2+\left(\frac{x^2}{2}-\frac{1}{2}\right)^2$

$2L\cdot\frac{dL}{dx}=2x+2\left(\frac{x^2}{2}-\frac{1}{2}\right)(x)$

$\frac{dL}{dx}=\frac{2x+2\left(\frac{x^2}{2}-\frac{1}{2}\right)(x)}{2L}=\frac{2x+x^3-x}{2L}=\frac{x^3+x}{2L}=\frac{x\left(x^2+1\right)}{2L}$

$\frac{dL}{dx}<0$ for all $x<0$ and $\frac{dL}{dx}>0$ for all $x>0$, so the minimum distance occurs at $x=0$.

The nearest point is the origin.

12. If $f(x) = \dfrac{4}{x-1}$ and $g(x) = 2x$, then the solution set of $f(g(x)) = g(f(x))$ is

(A) $\left\{\dfrac{1}{3}\right\}$ (B) $\{2\}$ (C) $\{3\}$ (D) $\{-1, 2\}$ (E) $\left\{\dfrac{1}{3}, 2\right\}$

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13. The region bounded by the x-axis and the part of the graph of $y = \cos x$ between $x = -\dfrac{\pi}{2}$ and $x = \dfrac{\pi}{2}$ is separated into two regions by the line $x = k$. If the area of the region for $-\dfrac{\pi}{2} \le x \le k$ is three times the area of the region for $k \le x \le \dfrac{\pi}{2}$, then $k =$

(A) $\arcsin\left(\dfrac{1}{4}\right)$ (B) $\arcsin\left(\dfrac{1}{3}\right)$ (C) $\dfrac{\pi}{6}$

(D) $\dfrac{\pi}{4}$ (E) $\dfrac{\pi}{3}$

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14. If the function f is defined by $f(x) = x^5 - 1$, then f^{-1}, the inverse function of f, is defined by $f^{-1}(x) =$

(A) $\dfrac{1}{\sqrt[5]{x}+1}$ (B) $\dfrac{1}{\sqrt[5]{x+1}}$ (C) $\sqrt[5]{x-1}$

(D) $\sqrt[5]{x}-1$ (E) $\sqrt[5]{x+1}$

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15. If $f'(x)$ and $g'(x)$ exist and $f'(x) > g'(x)$ for all real x, then the graph of $y = f(x)$ and the graph of $y = g(x)$

(A) intersect exactly once.
(B) intersect no more than once.
(C) do not intersect.
(D) could intersect more than once.
(E) have a common tangent at each point of intersection.

Answers

12. A $\dfrac{4}{2x-1}=2\left(\dfrac{4}{x-1}\right)\Rightarrow x-1=4x-2;\ x=\dfrac{1}{3}$

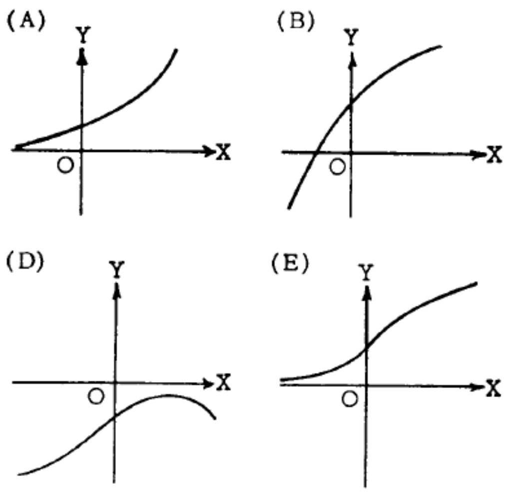
13. C $\int_{-\pi/2}^{k}\cos x\,dx=3\int_{k}^{\pi/2}\cos x\,dx;\ \sin k-\sin\left(-\dfrac{\pi}{2}\right)=3\left(\sin\dfrac{\pi}{2}-\sin k\right)$

$\sin k+1=3-3\sin k;\ 4\sin k=2\Rightarrow k=\dfrac{\pi}{6}$

14. E $y=x^5-1$ has an inverse $x=y^5-1\Rightarrow y=\sqrt[5]{x+1}$

15. B The graphs do not need to intersect (eg. $f(x)=-e^{-x}$ and $g(x)=e^{-x}$). The graphs could intersect (e.g. $f(x)=2x$ and $g(x)=x$). However, if they do intersect, they will intersect no more than once because $f(x)$ grows faster than $g(x)$.

16. If y is a function of x such that $y' > 0$ for all x and $y'' < 0$ for all x, which of the following could be part of the graph of $y = f(x)$?



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17. The graph of $y = 5x^4 - x^5$ has a point of inflection at

(A) $(0,0)$ only
(B) $(3,162)$ only
(C) $(4,256)$ only
(D) $(0,0)$ and $(3,162)$
(E) $(0,0)$ and $(4,256)$

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18. If $f(x) = 2 + |x-3|$ for all x, then the value of the derivative $f'(x)$ at $x = 3$ is

(A) -1
(B) 0
(C) 1
(D) 2
(E) nonexistent

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19. A point moves on the x-axis in such a way that its velocity at time t $(t > 0)$ is given by $v = \frac{\ln t}{t}$. At what value of t does v attain its maximum?

(A) 1
(B) $e^{\frac{1}{2}}$
(C) e
(D) $e^{\frac{3}{2}}$
(E) There is no maximum value for v.

Answers

16. B $y' > 0 \Rightarrow y$ is increasing; $y'' < 0 \Rightarrow$ the graph is concave down. Only B meets these conditions.

17. B $y' = 20x^3 - 5x^4$, $y'' = 60x^2 - 20x^3 = 20x^2(3-x)$. The only sign change in y'' is at $x = 3$. The only point of inflection is $(3,162)$.

18. E There is no derivative at the vertex which is located at $x = 3$.

19. C $\frac{dv}{dt} = \frac{1-\ln t}{t^2} > 0$ for $0 < t < e$ and $\frac{dv}{dt} < 0$ for $t > e$, thus v has its maximum at $t = e$.

20. An equation for a tangent to the graph of $y = \arcsin\frac{x}{2}$ at the origin is

(A) $x - 2y = 0$ (B) $x - y = 0$ (C) $x = 0$ (D) $y = 0$ (E) $\pi x - 2y = 0$

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21. At $x = 0$, which of the following is true of the function f defined by $f(x) = x^2 + e^{-2x}$?

(A) f is increasing.
(B) f is decreasing.
(C) f is discontinuous.
(D) f has a relative minimum.
(E) f has a relative maximum.

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22. $\frac{d}{dx}\left(\ln e^{2x}\right) =$

(A) $\frac{1}{e^{2x}}$ (B) $\frac{2}{e^{2x}}$ (C) $2x$ (D) 1 (E) 2

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23. The area of the region bounded by the curve $y = e^{2x}$, the x-axis, the y-axis, and the line $x = 2$ is equal to

(A) $\frac{e^4}{2} - e$ (B) $\frac{e^4}{2} - 1$ (C) $\frac{e^4}{2} - \frac{1}{2}$

(D) $2e^4 - e$ (E) $2e^4 - 2$

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24. If $\sin x = e^y$, $0 < x < \pi$, what is $\frac{dy}{dx}$ in terms of x ?

(A) $-\tan x$ (B) $-\cot x$ (C) $\cot x$ (D) $\tan x$ (E) $\csc x$

Answers

20. A $y(0)=0$ and $y'(0)=\left.\dfrac{1/2}{\sqrt{1-\dfrac{x^2}{4}}}\right|_{x=0}=\left.\dfrac{1}{\sqrt{4-x^2}}\right|_{x=0}=\dfrac{1}{2}$. The tangent line is $y=\dfrac{1}{2}x\Rightarrow x-2y=0$.

21. B $f'(x)=2x-2e^{-2x}$, $f'(0)=-2$, so f is decreasing

22. E $\ln e^{2x}=2x\Rightarrow\dfrac{d}{dx}\left(\ln e^{2x}\right)=\dfrac{d}{dx}(2x)=2$

23. C $\displaystyle\int_0^2 e^{2x}\,dx=\left.\frac{1}{2}e^{2x}\right|_0^2=\frac{1}{2}\left(e^4-1\right)$

24. C $y=\ln\sin x$, $y'=\dfrac{\cos x}{\sin x}=\cot x$

25. A region in the plane is bounded by the graph of $y = \frac{1}{x}$, the x-axis, the line $x = m$, and the line $x = 2m$, $m > 0$. The area of this region

(A) is independent of m.

(B) increases as m increases.

(C) decreases as m increases.

(D) decreases as m increases when $m < \frac{1}{2}$; increases as m increases when $m > \frac{1}{2}$.

(E) increases as m increases when $m < \frac{1}{2}$; decreases as m increases when $m > \frac{1}{2}$.

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26. $\int_0^1 \sqrt{x^2 - 2x + 1}\, dx$ is

(A) -1

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) 1

(E) none of the above

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27. If $\frac{dy}{dx} = \tan x$, then $y =$

(A) $\frac{1}{2}\tan^2 x + C$ (B) $\sec^2 x + C$ (C) $\ln|\sec x| + C$

(D) $\ln|\cos x| + C$ (E) $\sec x \tan x + C$

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28. The function defined by $f(x) = \sqrt{3}\cos x + 3\sin x$ has an amplitude of

(A) $3 - \sqrt{3}$ (B) $\sqrt{3}$ (C) $2\sqrt{3}$ (D) $3 + \sqrt{3}$ (E) $3\sqrt{3}$

Answers

25. A $\int_m^{2m} \frac{1}{x} dx = \ln x \Big|_m^{2m} = \ln(2m) - \ln(m) = \ln 2$ so the area is independent of m.

26. C $\int_0^1 \sqrt{x^2 - 2x + 1}\, dx = \int_0^1 |x-1|\, dx = \int_0^1 -(x-1)\, dx = -\frac{1}{2}(x-1)^2 \Big|_0^1 = \frac{1}{2}$

Alternatively, the graph of the region is a right triangle with vertices at (0,0), (0,1), and (1,0). The area is $\frac{1}{2}$.

27. C $\int \tan x\, dx = \int \frac{\sin x}{\cos x} dx = -\ln|\cos x| + C = \ln|\sec x| + C$

28. C $\sqrt{3}\cos x + 3\sin x$ can be thought of as the expansion of $\sin(x+y)$. Since $\sqrt{3}$ and 3 are too large for values of $\sin y$ and $\cos y$, multiply and divide by the result of the Pythagorean Theorem used on those values, i.e. $2\sqrt{3}$. Then

$$\sqrt{3}\cos x + 3\sin x = 2\sqrt{3}\left(\frac{\sqrt{3}}{2\sqrt{3}}\cos x + \frac{3}{2\sqrt{3}}\sin x\right) = 2\sqrt{3}\left(\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x\right)$$
$$= 2\sqrt{3}(\sin y \cos x + \cos y \sin x) = 2\sqrt{3}\sin(y+x)$$

where $y = \sin^{-1}\left(\frac{1}{2}\right)$. The amplitude is $2\sqrt{3}$.

Alternatively, the function $f(x)$ is periodic with period 2π. $f'(x) = -\sqrt{3}\sin x + 3\cos x = 0$ when $\tan x = \sqrt{3}$. The solutions over one period are $x = \frac{\pi}{3}, \frac{4\pi}{3}$. Then $f\left(\frac{\pi}{3}\right) = 2\sqrt{3}$ and $f\left(\frac{4\pi}{3}\right) = -2\sqrt{3}$. So the amplitude is $2\sqrt{3}$.

29. $\int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin x} dx =$

(A) $\ln\sqrt{2}$ (B) $\ln\frac{\pi}{4}$ (C) $\ln\sqrt{3}$ (D) $\ln\frac{\sqrt{3}}{2}$ (E) $\ln e$

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30. If a function f is continuous for all x and if f has a relative maximum at $(-1,4)$ and a relative minimum at $(3,-2)$, which of the following statements must be true?

(A) The graph of f has a point of inflection somewhere between $x=-1$ and $x=3$.
(B) $f'(-1)=0$
(C) The graph of f has a horizontal asymptote.
(D) The graph of f has a horizontal tangent line at $x=3$.
(E) The graph of f intersects both axes.

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31. If $f'(x)=-f(x)$ and $f(1)=1$, then $f(x)=$

(A) $\frac{1}{2}e^{-2x+2}$ (B) e^{-x-1} (C) e^{1-x} (D) e^{-x} (E) $-e^{x}$

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32. If a, b, c, d, and e are real numbers and $a \neq 0$, then the polynomial equation $ax^7+bx^5+cx^3+dx+e=0$ has

(A) only one real root.
(B) at least one real root.
(C) an odd number of nonreal roots.
(D) no real roots.
(E) no positive real roots.

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33. What is the average (mean) value of $3t^3-t^2$ over the interval $-1 \le t \le 2$?

(A) $\frac{11}{4}$ (B) $\frac{7}{2}$ (C) 8 (D) $\frac{33}{4}$ (E) 16

Answers

29. A $\int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin x}\,dx = \ln(\sin x)\Big|_{\pi/4}^{\pi/2} = \ln 1 - \ln\frac{1}{\sqrt{2}} = \ln\sqrt{2}$

30. E Because f is continuous for all x, the Intermediate Value Theorem implies that the graph of f must intersect the x-axis. The graph must also intersect the y-axis since f is defined for all x, in particular, at $x = 0$.

31. C $\frac{dy}{dx} = -y \Rightarrow y = ce^{-x}$ and $1 = ce^{-1} \Rightarrow c = e;\ y = e \cdot e^{-x} = e^{1-x}$

32. B If $a < 0$ then $\lim_{x\to-\infty} y = \infty$ and $\lim_{x\to\infty} y = -\infty$ which would mean that there is at least one root.
If $a > 0$ then $\lim_{x\to-\infty} y = -\infty$ and $\lim_{x\to\infty} y = \infty$ which would mean that there is at least one root.
In both cases the equation has at least one root.

33. A $\frac{1}{3}\int_{-1}^{2} 3t^3 - t^2\,dt = \frac{1}{3}\left(\frac{3}{4}t^4 - \frac{1}{3}t^3\right)\Big|_{-1}^{2} = \frac{1}{3}\left(\left(12 - \frac{8}{3}\right) - \left(\frac{3}{4} + \frac{1}{3}\right)\right) = \frac{11}{4}$

34. Which of the following is an equation of a curve that intersects at right angles every curve of the family $y=\frac{1}{x}+k$ (where k takes all real values)?

(A) $y=-x$ (B) $y=-x^2$ (C) $y=-\frac{1}{3}x^3$ (D) $y=\frac{1}{3}x^3$ (E) $y=\ln x$

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35. At $t=0$ a particle starts at rest and moves along a line in such a way that at time t its acceleration is $24t^2$ feet per second per second. Through how many feet does the particle move during the first 2 seconds?

(A) 32 (B) 48 (C) 64 (D) 96 (E) 192

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36. The approximate value of $y=\sqrt{4+\sin x}$ at $x=0.12$, obtained from the tangent to the graph at $x=0$, is

(A) 2.00 (B) 2.03 (C) 2.06 (D) 2.12 (E) 2.24

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37. Which is the best of the following polynomial approximations to $\cos 2x$ near $x=0$?

(A) $1+\frac{x}{2}$ (B) $1+x$ (C) $1-\frac{x^2}{2}$ (D) $1-2x^2$ (E) $1-2x+x^2$

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38. $\int \frac{x^2}{e^{x^3}}dx=$

(A) $-\frac{1}{3}\ln e^{x^3}+C$ (B) $-\frac{e^{x^3}}{3}+C$ (C) $-\frac{1}{3e^{x^3}}+C$

(D) $\frac{1}{3}\ln e^{x^3}+C$ (E) $\frac{x^3}{3e^{x^3}}+C$

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39. If $y=\tan u$, $u=v-\frac{1}{v}$, and $v=\ln x$, what is the value of $\frac{dy}{dx}$ at $x=e$?

(A) 0 (B) $\frac{1}{e}$ (C) 1 (D) $\frac{2}{e}$ (E) $\sec^2 e$

Answers

34. D $y' = -\frac{1}{x^2}$, so the desired curve satisfies $y' = x^2 \Rightarrow y = \frac{1}{3}x^3 + C$

35. A $a(t) = 24t^2$, $v(t) = 8t^3 + C$ and $v(0) = 0 \Rightarrow C = 0$. The particle is always moving to the right, so distance = $\int_0^2 8t^3 dt = 2t^4 \Big|_0^2 = 32$.

36. B $y = \sqrt{4 + \sin x}$, $y(0) = 2$, $y'(0) = \frac{\cos 0}{2\sqrt{4 + \sin 0}} = \frac{1}{4}$. The linear approximation to y is $L(x) = 2 + \frac{1}{4}x$. $L(1.2) = 2 + \frac{1}{4}(1.2) = 2.03$

37. D All options have the same value at $x = 0$. We want the one that has the same first and second derivatives at $x = 0$ as $y = \cos 2x$: $y'(0) = -2\sin 2x \Big|_{x=0} = 0$ and $y''(0) = -4\cos 2x \Big|_{x=0} = -4$.

For $y = 1 - 2x^2$, $y'(0) = -4x \Big|_{x=0} = 0$ and $y''(0) = -4$ and no other option works.

38. C $\int \frac{x^2}{e^{x^3}} dx = -\frac{1}{3}\int e^{-x^3}(-3x^2 dx) = -\frac{1}{3}e^{-x^3} + C = -\frac{1}{3e^{-x^3}} + C$

39. D $x = e \Rightarrow v = 1$, $u = 0$, $y = 0$; $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dx} = \left(\sec^2 u\right)\left(1 + \frac{1}{v^2}\right)\left(\frac{1}{x}\right) = (1)(2)\left(e^{-1}\right) = \frac{2}{e}$

40. If n is a non-negative integer, then $\int_0^1 x^n\,dx = \int_0^1 (1-x)^n\,dx$ for

(A) no n (B) n even, only (C) n odd, only
(D) nonzero n, only (E) all n

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41. If $\begin{cases} f(x) = 8 - x^2 & \text{for } -2 \le x \le 2, \\ f(x) = x^2 & \text{elsewhere}, \end{cases}$ then $\int_{-1}^{3} f(x)\,dx$ is a number between

(A) 0 and 8 (B) 8 and 16 (C) 16 and 24 (D) 24 and 32 (E) 32 and 40

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42. What are all values of k for which the graph of $y = x^3 - 3x^2 + k$ will have three distinct x-intercepts?

(A) All $k > 0$ (B) All $k < 4$ (C) $k = 0, 4$ (D) $0 < k < 4$ (E) All k

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43. $\int \sin(2x+3)\,dx =$

(A) $\frac{1}{2}\cos(2x+3) + C$ (B) $\cos(2x+3) + C$ (C) $-\cos(2x+3) + C$
(D) $-\frac{1}{2}\cos(2x+3) + C$ (E) $-\frac{1}{5}\cos(2x+3) + C$

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44. The fundamental period of the function defined by $f(x) = 3 - 2\cos^2 \frac{\pi x}{3}$ is

(A) 1 (B) 2 (C) 3 (D) 5 (E) 6

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45. If $\frac{d}{dx}(f(x)) = g(x)$ and $\frac{d}{dx}(g(x)) = f(x^2)$, then $\frac{d^2}{dx^2}\left(f(x^3)\right) =$

(A) $f\left(x^6\right)$ (B) $g\left(x^3\right)$ (C) $3x^2 g\left(x^3\right)$
(D) $9x^4 f\left(x^6\right) + 6x\,g\left(x^3\right)$ (E) $f\left(x^6\right) + g\left(x^3\right)$

Answers

40. E One solution technique is to evaluate each integral and note that the value is $\frac{1}{n+1}$ for each.

Another technique is to use the substitution $u = 1 - x$; $\int_0^1 (1-x)^n\,dx = \int_1^0 u^n\,(-du) = \int_0^1 u^n\,du$

Integrals do not depend on the variable that is used and so $\int_0^1 u^n\,du$ is the same as $\int_0^1 x^n\,dx$

41. D $\int_{-1}^{3} f(x)\,dx = \int_{-1}^{2}\left(8 - x^2\right)dx + \int_2^3 x^2 dx = \left(8x - \frac{1}{3}x^3\right)\Big|_{-1}^{2} + \frac{1}{3}x^3\Big|_2^3 = 27\,{}^1\!/_3$

42. D $y = x^3 - 3x^2 + k$, $y' = 3x^2 - 6x = 3x(x-2)$. So f has a relative maximum at $(0, k)$ and a relative minimum at $(2, k-4)$. There will be 3 distinct x-intercepts if the maximum and minimum are on the opposite sides of the x-axis. We want $k - 4 < 0 < k \Rightarrow 0 < k < 4$.

43. D $\int \sin(2x+3)\,dx = -\frac{1}{2}\cos(2x+3) + C$

44. C Since $\cos 2A = 2\cos^2 A - 1$, we have $3 - 2\cos^2\frac{\pi x}{3} = 3 - (1 + \cos\frac{2\pi x}{3})$ and the latter expression has period $\frac{2\pi}{\left(\frac{2\pi}{3}\right)} = 3$

45. D Let $y = f(x^3)$. We want y'' where $f'(x) = g(x)$ and $f''(x) = g'(x) = f(x^2)$

$$
\begin{aligned}
y &= f(x^3) \\
y' &= f'(x^3)\cdot 3x^2 \\
y'' &= 3x^2\left(f''(x^3)\cdot 3x^2\right) + f'(x^3)\cdot 6x \\
&= 9x^4 f''(x^3) + 6x\,f'(x^3) = 9x^4 f((x^3)^2) + 6x\,g(x^3) = 9x^4 f(x^6) + 6x\,g(x^3)
\end{aligned}
$$