Marginal Cost and Revenue … Set 3

Review Exercise Set 8

Exercise 1: The total cost (in hundreds of dollars) to produce x units of a product is defined by the given function.

$$C(x) = \frac{3x+2}{x+4}$$

a) Find the marginal cost function
b) What is the marginal cost when x = 20

Exercise 2: The profit (in hundreds of dollars) from the sale of x units of a product is defined by the given function. Find and interpret the marginal profit when 30 units are sold.

$$P(x) = \frac{x^2}{x-1} \text{ ; where } x > 1$$

Exercise 3: Suppose the cost in dollars of producing x units is given by the function $C(x) = .2x^2 + 6x + 50$. Find the average cost function and the average marginal cost function.

Review Exercise Set 8 Answer Key

Exercise 1: The total cost (in hundreds of dollars) to produce x units of a product is defined by the given function.

$$C(x)=\frac{3x+2}{x+4}$$

a) Find the marginal cost function

$$C(x)=\frac{3x+2}{x+4}$$

$$C'(x)=\frac{(x+4)D_x(3x+2)-(3x+2)D_x(x+4)}{(x+4)^2}$$

$$=\frac{(x+4)(3)-(3x+2)(1)}{(x+4)^2}$$

$$=\frac{3x+12-3x-2}{(x+4)^2}$$

$$=\frac{10}{(x+4)^2}$$

b) What is the marginal cost when x = 20

$$C'(x)=\frac{10}{(x+4)^2}$$

$$C'(20)=\frac{10}{(20+4)^2}$$

$$=\frac{10}{576}$$

$$=\frac{5}{288}$$

Marginal Cost and Revenue … Set 3

Exercise 4: Suppose the cost in dollars of producing x units is given by the function $C(x) = (x^2 + 3)^3$. Find the average marginal cost function and evaluate it when x = 10.

Exercise 5: A company has the given cost and revenue functions where x is between 0 and 1000. Find the marginal profit function and evaluate it when x = 500

$C(x) = 1000x - .2x^2$; $R(x) = .0008x^3 - 2.4x^2 + 2400x$

Exercise 2: The profit (in hundreds of dollars) from the sale of x units of a product is defined by the given function. Find and interpret the marginal profit when 30 units are sold.

$$P(x) = \frac{x^2}{x-1}\text{; where } x > 1$$

Find the derivative

$$\begin{aligned} P'(x) &= \frac{(x-1)D_x(x^2) - (x^2)D_x(x-1)}{(x-1)^2} \\ &= \frac{(x-1)(2x) - (x^2)(1)}{(x-1)^2} \\ &= \frac{2x^2 - 2x - x^2}{(x-1)^2} \\ &= \frac{x^2 - 2x}{(x-1)^2} \end{aligned}$$

Evaluate the marginal profit function at x = 30

$$\begin{aligned} P'(x) &= \frac{x^2 - 2x}{(x-1)^2} \\ P'(30) &= \frac{30^2 - 2(30)}{(30-1)^2} \\ &= \frac{900 - 60}{841} \\ &= \frac{840}{841} \\ &\approx 0.99881 \end{aligned}$$

Interpret results

The profit from selling the 31st unit is approximately $99.88.

Exercise 3: Suppose the cost in dollars of producing x units is given by the function $C(x) = .2x^2 + 6x + 50$. Find the average cost function and the marginal average cost function.

Find the average cost function

$$\overline{C(x)} = \frac{C(x)}{x}$$
$$= \frac{.2x^2 + 6x + 50}{x}$$
$$= .2x + 6 + \frac{50}{x}$$

Find the marginal average cost function

$$\overline{C(x)} = .2x + 6 + 50x^{-1}$$
$$\overline{C'(x)} = .2 + 0 - 50x^{-2}$$
$$= .2 - \frac{50}{x^2}$$

Exercise 4: Suppose the cost in dollars of producing x units is given by the function $C(x) = (x^2 + 3)^3$. Find the marginal average cost function and evaluate it when x = 10.

Find the average cost function

$$\overline{C(x)} = \frac{C(x)}{x}$$
$$= \frac{(x^2+3)^3}{x}$$

Find the marginal average cost function

$$\overline{C'(x)} = \frac{(x)D_x(x^2+3)^3 - (x^2+3)^3 D_x(x)}{(x)^2}$$
$$= \frac{(x)(3)(x^2+3)^2(2x) - (x^2+3)^3(1)}{x^2}$$
$$= \frac{6x^2(x^2+3)^2 - (x^2+3)^3}{x^2}$$
$$= \frac{(x^2+3)^2\left[6x^2 - (x^2+3)\right]}{x^2}$$
$$= \frac{(x^2+3)^2(5x^2-3)}{x^2}$$

Exercise 4 (Continued):

Evaluate the marginal average cost function at x = 10

$$\overline{C'(x)} = \frac{(x^2+3)^2(5x^2-3)}{x^2}$$

$$\overline{C'(10)} = \frac{(10^2+3)^2(5(10)^2-3)}{10^2}$$

$$= \frac{(103)^2(497)}{100}$$

$$= 52,726.73$$

Exercise 5: A company has the given cost and revenue functions where x is between 0 and 1000. Find the marginal profit function and evaluate it when x = 500

$C(x) = 1000x - .2x^2$; $R(x) = .0008x^3 - 2.4x^2 + 2400x$

Find the profit function

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= (.0008x^3 - 2.4x^2 + 2400x) - (1000x - .2x^2) \\ &= .0008x^3 - 2.4x^2 + 2400x - 1000x + .2x^2 \\ &= .0008x^3 - 2.2x^2 + 1400x \end{aligned}$$

Find the marginal profit function

$$\begin{aligned} P'(x) &= .0008x^3 - 2.2x^2 + 1400x \\ &= .0024x^2 - 4.4x + 1400 \end{aligned}$$

Evaluate the marginal profit function at x = 500

$$\begin{aligned} P'(500) &= .0024(500)^2 - 4.4(500) + 1400 \\ &= 600 - 2200 + 1400 \\ &= -200 \end{aligned}$$