Marginal Cost and Revenue … Set 6

Notes on Marginal Cost, Revenue and Profit

Manufacturing cost ("cost" for short) is represented by a function $C(x)$, where x is quantity of units manufactured. Cost responds to several factors, both fixed and variable. **Sales revenue** ("revenue" for short) and **profit** are represented by functions $R(x)$ and $P(x)$, where x is quantity of units sold (that is, demand).

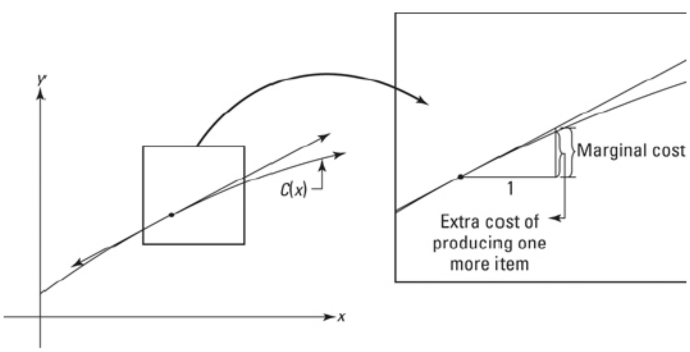
(Demand, in turn, is a function of price, p. Inversely, price can be a function of demand. For the first part of the discussion in the book and here, price is given as a constant; we analyzed profit accordingly.)

Marginal cost, marginal revenue, and marginal profit measure the change in these functions for one additional unit of production or sales.

We're interested in marginal functions because they are important indicators of the behavior of the particular processes. Important information is obtained by knowing the value of x where marginal costs go to zero, as will be seen.

Production or sales quantity x is a *discrete* variable (that is, a rational number—and usually a non-negative integer) and as such its graph versus cost or profit or revenue would be a series of dots, we "smooth out" the data as if it were points on the entire real number line. Then the graph of, say, cost, would look like the continuous curve below.

This smoothing of the data lets methods of calculus be used to *derive* the marginal functions from those given. The change in a function for each additional unit at any point on its curve is seen, for all practical purposes, as the *instantaneous rate of change* of the function. This new function, the *derivative*, is derived geometrically as in the figure below.



On the tangent to the curve, the run is marked off as 1, for "one additional unit produced." Thus, rise/run = rise/1 = instantaneous rate of change: the derivative.

In fact, "Extra cost" reaches up to the curve, but "Marginal cost" goes up a tiny amount more, to the tangent line. Thus, marginal cost is a bit *more* than the extra cost (if a cost function were concave up instead of concave down like it is here, the marginal cost would be a tiny bit *less* than the extra cost). The approximation is acceptable.

The *derivative* (the limit of the difference quotient) thus represents *marginal cost.*

The figure shows that a tangent line lies very close to the curve for very small intervals of x. Thus, it is an *approximation* of a cost function *marginal cost* — is the *approximate* increase in cost of producing one more item. Marginal revenue and marginal profit work the same way.

Thus, marginal cost $C'(x)$ describes the *behavior* of slope of tangent lines at any point along the curve. If a function is linear to begin with, then marginal cost is merely the slope of the function itself. But cost, revenue, and profit functions are generally not linear, so calculus methods are essential in describing the corresponding marginal functions.

EXAMPLE:

The cost of producing x widgets is given by the following cost function:

$$C(x) = 10x + 100\sqrt{x} + 10{,}000$$

Determine the marginal cost, marginal revenue, and marginal profit at $x = 100$ widgets.

SOLUTION:

Marginal cost $C'(x)$ is evaluated at $x = 100$.

$$\begin{aligned} C(x) &= 10x + 100\sqrt{x} + 10{,}000 \\ &= 10x + 100x^{1/2} + 10{,}000 \\ C'(x) &= 10 + \frac{1}{2}\cdot 100\cdot x^{-1/2} = 10 + \frac{50}{x^{1/2}} \quad \text{(power rule)} \\ C'(100) &= 10 + \frac{50}{100^{1/2}} = 10 + \frac{50}{10} = \$15 \end{aligned}$$

Thus, at $x = 100$ units the marginal cost is \$15; this is the approximate cost of producing the 101st widget.

Marginal revenue

Revenue $R(x) = px$. Suppose instead of a fixed price, p is given as a function of demand:

$$p(x) = 1000/\sqrt{x}\ .$$

The revenue function is thus:

$$R(x) = p\cdot x = \frac{1000}{\sqrt{x}}\cdot x$$

To find marginal revenue $R'(x)$, we could use product and quotient rules on the above function. But, it would be *much* easier to write the x factor with a rational exponent, then simply apply the power rule:

$$\begin{aligned} R(x) &= \frac{1000}{\sqrt{x}}\cdot x = 1000\cdot x\cdot x^{-1/2} = 1000x^{1/2} \\ R'(x) &= \frac{1}{2}\cdot 1000x^{-1/2} = 500x^{-1/2} = \frac{500}{x^{1/2}} = \frac{500}{\sqrt{x}} \end{aligned}$$

Finally, evaluating at $x = 100$ widgets:

$$R'(100) = \frac{500}{\sqrt{100}} = \frac{500}{10} = 50$$

Thus, the approximate revenue from selling the 101st widget is \$50.

Marginal profit

Profit $P(x) = R(x) - C(x)$. Marginal profit = marginal revenue – marginal cost:

$$P'(x) = R'(x) - C'(x)$$

Evaluated at $x = 100$ is simply a matter of subtracting the values we already found for marginal revenue and cost at that level of demand:

$$P'(x) = 50 - 15 = \$35.$$

The sale of the 101st widget thus results in an approximate profit of \$35.

$$P'(x) = R'(x) - C'(x)$$

Notes on Demand Function

Demand for a commodity refers to the quantity of a commodity people are willing to purchase at a specific *price*. Instead of being represented by the variable x, demand is often given by q.

In cost, revenue, and profit functions, x or q is the independent variable. But in a demand function, q is the *dependent* variable, as we saw in problem #4 in Sec 10.

$q(x)$ or $q(p)$ is a typical way to represent a demand function.

But as we did in the above example, it can be written as $p(x)$ or $p(q)$.