Exponents and Logarithms … (set 2)

Example

Differentiate $\log_e(x^2 + 3x + 1)$.

Answers

Solution

We solve this by using the chain rule and our knowledge of the derivative of $\log_e x$.

$$\begin{aligned}
\frac{d}{dx}\log_e(x^2+3x+1) &= \frac{d}{dx}(\log_e u) \qquad (\text{where } u = x^2+3x+1) \\
&= \frac{d}{du}(\log_e u) \times \frac{du}{dx} \qquad (\text{by the chain rule}) \\
&= \frac{1}{u} \times \frac{du}{dx} \\
&= \frac{1}{x^2+3x+1} \times \frac{d}{dx}(x^2+3x+1) \\
&= \frac{1}{x^2+3x+1} \times (2x+3) \\
&= \frac{2x+3}{x^2+3x+1}.
\end{aligned}$$

Exponents and Logarithms … (set 2)

Example

Find $\frac{d}{dx}(e^{3x^2})$.

Exponents and Logarithms … (set 2)

Answers

Solution

This is an application of the chain rule together with our knowledge of the derivative of e^x.

$$\begin{aligned} \frac{d}{dx}(e^{3x^2}) &= \frac{de^u}{dx} \qquad \text{where } u = 3x^2 \\ &= \frac{de^u}{du} \times \frac{du}{dx} \qquad \text{by the chain rule} \\ &= e^u \times \frac{du}{dx} \\ &= e^{3x^2} \times \frac{d}{dx}(3x^2) \\ &= 6xe^{3x^2}. \end{aligned}$$

Exponents and Logarithms … (set 2)

Example

Find $\frac{d}{dx}(e^{x^3+2x})$.

Answers

Solution

Again, we use our knowledge of the derivative of e^x together with the chain rule.

$$
\begin{aligned}
\frac{d}{dx}(e^{x^3+2x}) &= \frac{de^u}{dx} \qquad \text{(where } u = x^3 + 2x\text{)} \\
&= e^u \times \frac{du}{dx} \qquad \text{(by the chain rule)} \\
&= e^{x^3+2x} \times \frac{d}{dx}(x^3 + 2x) \\
&= (3x^2 + 2) \times e^{x^3+2x}.
\end{aligned}
$$

Example

Differentiate $\ln(2x^3 + 5x^2 - 3)$.

Answers

Solution

We solve this by using the chain rule and our knowledge of the derivative of

$$\begin{aligned}
\frac{d}{dx}\ln\left(2x^3+5x^2-3\right) &= \frac{d\ln u}{dx} \qquad \text{(where } u=(2x^3+5x^2-3)\text{)}\\
&= \frac{d\ln u}{du}\times\frac{du}{dx} \qquad \text{(by the chain rule)}\\
&= \frac{1}{u}\times\frac{du}{dx}\\
&= \frac{1}{2x^3+5x^2-3}\times\frac{d}{dx}(2x^3+5x^2-3)\\
&= \frac{1}{2x^3+5x^2-3}\times(6x^2+10x)\\
&= \frac{6x^2+10x}{2x^3+5x^2-3}.
\end{aligned}$$

Exponents and Logarithms … (set 2)

Exercise 1

Differentiate the following functions.

a. $f(x) = \ln(2x^3)$ **b.** $f(x) = e^{x^7}$ **c.** $f(x) = \ln(11x^7)$

d. $f(x) = e^{x^2+x^3}$ **e.** $f(x) = \log_e(7x^{-2})$ **f.** $f(x) = e^{-x}$

g. $f(x) = \ln(e^x + x^3)$ **h.** $f(x) = \ln(e^x x^3)$ **i.** $f(x) = \ln\left(\dfrac{x^2+1}{x^3-x}\right)$

Exponents and Logarithms … (set 2)

Answers

Solutions to Exercise 1

a. $f'(x) = \frac{6x^2}{2x^3} = \frac{3}{x}$

Alternatively write $f(x) = \ln 2 + 3\ln x$ so that $f'(x) = 3\frac{1}{x}$.

b. $f'(x) = 7x^6 e^{x^7}$

c. $f'(x) = \frac{7}{x}$

d. $f'(x) = (2x + 3x^2)e^{x^2+x^3}$

e. Write $f(x) = \log_e 7 - 2\log_e x$ so that $f'(x) = -\frac{2}{x}$.

f. $f'(x) = -e^{-x}$

g. $f'(x) = \frac{e^x + 3x^2}{e^x + x^3}$

h. Write $f(x) = \ln e^x + \frac{3}{\ln x}$ so that $f'(x) = 1 + \frac{3}{x}$.

i. Write $f(x) = \ln(x^2+1) - \ln(x^3 - x)$ so that $f'(x) = \frac{2x}{x^2+1} - \frac{3x^2-1}{x^3-x}$.