Piecewise Functions

Let's analyze the piecewise function defined by

$$f(x) = \begin{cases} -x+1, & x \le -1 \\ 2, & -1 < x < 3 \\ x^2-4, & x \ge 3. \end{cases}$$

To help with input, think of $f(x)$ as follows:

$$f(x) = \begin{cases} 1^{\text{st}} \text{ piece}, & x \le -1 \\ 2^{\text{nd}} \text{ piece}, & -1 < x < 3 \\ 3^{\text{rd}} \text{ piece}, & x \ge 3. \end{cases}$$

In general, first look at the input intervals to select the appropriate piece to use for output.

1) Evaluate the following: $f(-3)$, $f(-1)$, $f(e)$, $f(3)$.

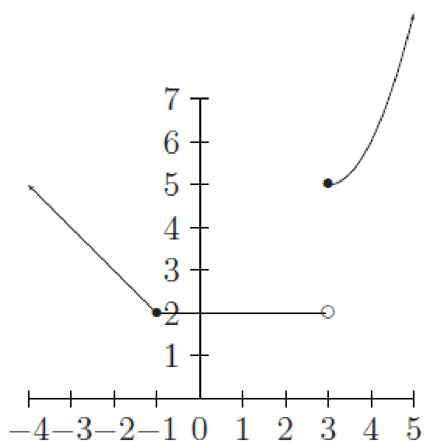
   Ans.

   $\left.\begin{matrix} f(-3) = 4 \\ f(-1) = 2 \end{matrix}\right\}$ from 1^{st} piece

   $f(e) = 2$ from 2^{nd} piece, $e \approx 2.7$

   $f(3) = 5$ from 3^{rd} piece

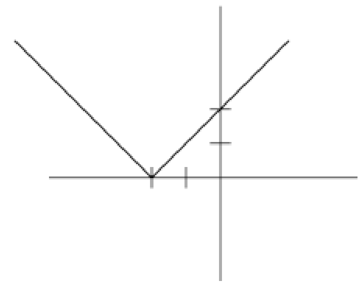
2) Here is the graph of f. Pay particular attention to the endpoints of the input intervals. Notice how this graph still passes the Vertical Line Test.



PRACTICE PROBLEMS for Topic 2 – Piecewise-Defined Functions

2.1. Define each absolute value function in piecewise form. Sketch a graph.

Ex. $f(x) = |x+2|$ Ans. $f(x) = \begin{cases} x+2, & x \geq -2 \\ -(x+2), & x < -2 \end{cases}$



a) $f(x) = |x-1|$ b) $f(x) = |2x+3|$ Answers

2.2. Let a function be 'defined' as follows

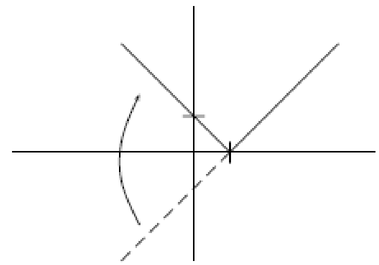
$$f(x) = \begin{cases} -x^2-1, & x \leq 0 \\ 2, & 0 < x < 4 \\ \sqrt{x}, & x \geq 4. \end{cases}$$

a) Find $f(-2)$, $f(0)$, $f(\pi)$, $f(x^2+5)$.

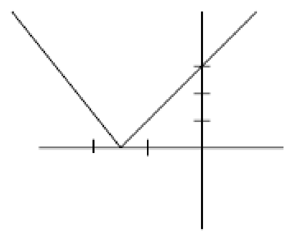
b) Sketch a graph of f. Answers

Answers

2.1. a) $f(x) = \begin{cases} x-1, & x \geq 1 \\ -(x-1), & x < 1 \end{cases}$



2.1. b) $f(x) = \begin{cases} 2x+3, & x \geq -3/2 \\ -(2x+3), & x < -3/2 \end{cases}$

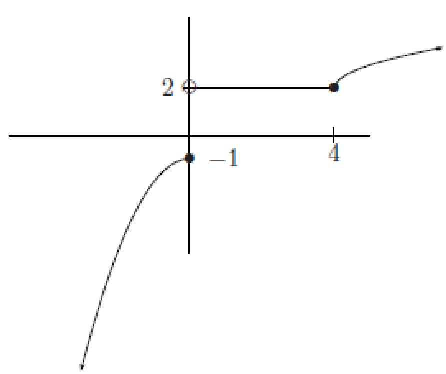


Return to Problem

2.2. a)

$$\begin{aligned} f(-2) &= -5 \\ f(0) &= -1 \\ f(\pi) &= \;\; 2 && \text{because } \pi \approx 3.14 \\ f(x^2+5) &= \sqrt{x^2+5} && \text{because } x^2+5 > 4 \text{ for all } x. \end{aligned}$$

b)



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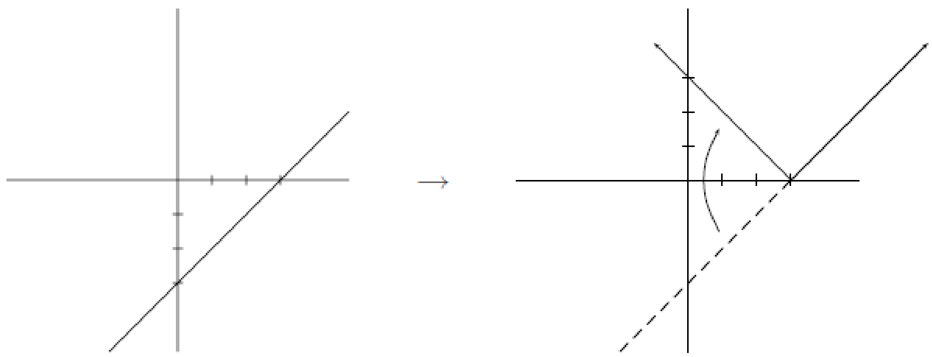
Write a piecewise definition for $f(x) = |x-3|$. Sketch the graph of f.

Answers:

$$f(x) = |x-3| = \begin{cases} x-3 & \text{if } x \geq 3 \\ -(x-3) & \text{if } x < 3 \end{cases}$$

$f(x) = x - 3$ $\qquad\qquad$ $f(x) = |x-3|$



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