Factor $x^2+8x+15$ completely.

Product	Sum
$\_\_\_\bullet\_\_\_=15$	$\_\_\_+\_\_\_=8$
$1\bullet15=15$	$1+15=16$
$3\bullet5=15$	$\boxed{3+5=8}$

Choral Response Questions
Expression, equation, or inequality?
Expression.
What type of polynomial? *Trinomial*.
Is there a GCF other than 1? *No*.

Factoring by Grouping

Area Model

$x^2+8x+15$

Based on our product/sum chart, we will use $3x$ and $5x$.

$=x^2+3x+5x+15$

Fill in the area model.

x^2	$3x$
$5x$	15

Factor the GCF from each row and column.

	x	$+3$
x	x^2	$3x$
$+5$	$5x$	15

$=(x+5)(x+3)$

Algebraically

$x^2+8x+15$

$=x^2+3x+5x+15$

Group the first two terms and the last two terms.

$=(x^2+3x)+(5x+15)$

Factor the GCF from each quantity.

$=x(x+3)+5(x+3)$
$=(x+5)(x+3)$

Factoring by Guess & Check

Area Model

$x^2+8x+15$

Fill in the first and last terms.

x^2	
	15

The first factors have to be x and x. Strategically guess which factors of 15 to try.

	x	$+3$
x	x^2	
$+5$		15

Fill in the remaining areas and check the diagonal like terms.

	x	$+3$
x	x^2	$3x$
$+5$	$5x$	15

$=(x+5)(x+3)$

Algebraically

$x^2+8x+15$
$=(x\quad)(x\quad)$
$=(x+\quad)(x+\quad)$
$=(x+5)(x+3)$

The First and Last terms are the same when we multiply the trial factors. What changes is the middle term. Thus, it is only necessary to check the Outer and Inner terms.

$(x+5)(x+3)$

$3x+5x=8x$ ☑

*Note that when teaching area models, the area represents the trinomial and the dimensions represent the factors.

Factor $x^2 + 19x + 60$ completely. Use guess and check with an area model and algebraically.

If needed students can still use a product/sum chart. The goal is for students to develop their number sense and intuition to factor quadratics without using a product/sum chart.

Product	**Sum**
___ • ___ =	___ + ___ =

Choral Response Questions

Expression, equation, or inequality?

What type of polynomial?

Is there a GCF other than 1?

Area Model using a Generic Rectangle

$x^2 + 19x + 60$

Algebraically

$x^2 + 19x + 60$

SIGNS OF FACTORS

Example 2: Factor the following expressions completely.

x^2+4x+3	x^2-4x+3	x^2+2x-3	x^2-2x-3
$=(x\quad 3)(x\quad 1)$	$=(x\quad 3)(x\quad 1)$	$=(x\quad 3)(x\quad 1)$	$=(x\quad 3)(x\quad 1)$

By choosing a trinomial whose first terms have to be x and x, and whose last terms have to be 3 & 1, it will help students to see how the signs are determined.

$=(x+3)(x+1)$	$=(x-3)(x-1)$	$=(x+3)(x-1)$	$=(x-3)(x+1)$

You Try 2:

x^2+8x+7	x^2-8x+7	x^2+6x-7	x^2-6x-7

Discuss if the third term is positive, the signs are either both positive or both negative. At this point, if the middle term is positive then they are both positive. If the middle term is negative then they are both negative.

Discuss if the third term is negative, one sign is negative and one sign is positive. The middle term determines the sign of the larger factor (note this is only always true when *a* = 1).

USING THE MIDDLE TERM TO GUESS MORE EFFECTIVELY

<u>You Try 3:</u> Factor the following expressions completely. Pay attention to the affect the middle term has on the factors.

$x^2+61x+60$ $=(x\quad)(x\quad)$	$x^2-61x+60$ $=(x\quad)(x\quad)$	$x^2+59x-60$ $=(x\quad)(x\quad)$	$x^2-59x-60$ $=(x\quad)(x\quad)$
$x^2+32x+60$ $=(x\quad)(x\quad)$	$x^2-32x+60$ $=(x\quad)(x\quad)$	$x^2+28x-60$ $=(x\quad)(x\quad)$	$x^2-28x-60$ $=(x\quad)(x\quad)$
$x^2+23x+60$ $=(x\quad)(x\quad)$	$x^2-23x+60$ $=(x\quad)(x\quad)$	$x^2+17x-60$ $=(x\quad)(x\quad)$	$x^2-17x-60$ $=(x\quad)(x\quad)$
$x^2+19x+60$ $=(x\quad)(x\quad)$	$x^2-19x+60$ $=(x\quad)(x\quad)$	$x^2+11x-60$ $=(x\quad)(x\quad)$	$x^2-11x-60$ $=(x\quad)(x\quad)$
$x^2+17x+60$ $=(x\quad)(x\quad)$	$x^2-17x+60$ $=(x\quad)(x\quad)$	$x^2+7x-60$ $=(x\quad)(x\quad)$	$x^2-7x-60$ $=(x\quad)(x\quad)$
$x^2+16x+60$ $=(x\quad)(x\quad)$	$x^2-16x+60$ $=(x\quad)(x\quad)$	$x^2+4x-60$ $=(x\quad)(x\quad)$	$x^2-4x-60$ $=(x\quad)(x\quad)$

What do you notice about how the middle term of the trinomial affects what factors you try first?

- Discuss the closer the middle term is to zero in the trinomial, the closer the factors will be in the binomials.
- Also, discuss the relationship between the sign of larger factor in the binomial and the sign of the middle term from the trinomial for the last two columns. When the last term of the trinomial is negative, the sign of the larger factor in the binomials should match the sign of the sum (note again this is only always true when *a* = 1).
- By choosing a trinomial whose first term is x^2, and whose last term is ± 60, it will help students understand how to make better guesses about which factors to use. Also, by choosing the number 60 which has 12 whole number factors, and varying its sign, students will have the ability to factor 24 similar trinomials.

Factor the following expression using guess and check with an area model and algebraically.

$$x^2 + 5x - 24$$

Guess & Check

Area Model	**Algebraically**

Factor the following 24 expressions. Be ready to discuss how we know the signs of the binomials and how the middle term of the trinomial helps you decide which factors of 60 to use.

$x^2+61x+60$ $=(x\quad)(x\quad)$	$x^2-61x+60$ $=(x\quad)(x\quad)$	$x^2+59x-60$ $=(x\quad)(x\quad)$	$x^2-59x-60$ $=(x\quad)(x\quad)$
$x^2+32x+60$ $=(x\quad)(x\quad)$	$x^2-32x+60$ $=(x\quad)(x\quad)$	$x^2+28x-60$ $=(x\quad)(x\quad)$	$x^2-28x-60$ $=(x\quad)(x\quad)$
$x^2+23x+60$ $=(x\quad)(x\quad)$	$x^2-23x+60$ $=(x\quad)(x\quad)$	$x^2+17x-60$ $=(x\quad)(x\quad)$	$x^2-17x-60$ $=(x\quad)(x\quad)$
$x^2+19x+60$ $=(x\quad)(x\quad)$	$x^2-19x+60$ $=(x\quad)(x\quad)$	$x^2+11x-60$ $=(x\quad)(x\quad)$	$x^2-11x-60$ $=(x\quad)(x\quad)$
$x^2+17x+60$ $=(x\quad)(x\quad)$	$x^2-17x+60$ $=(x\quad)(x\quad)$	$x^2+7x-60$ $=(x\quad)(x\quad)$	$x^2-7x-60$ $=(x\quad)(x\quad)$
$x^2+16x+60$ $=(x\quad)(x\quad)$	$x^2-16x+60$ $=(x\quad)(x\quad)$	$x^2+4x-60$ $=(x\quad)(x\quad)$	$x^2-4x-60$ $=(x\quad)(x\quad)$