Solving Systems of Equations (Graphing & Substitution)

Two intersecting linear equations will have one solution.

Two parallel linear equations, equations with the same slope, will have no solutions.

Two linear equations that create the same line, equations with the same slope and the same y-intercept, will have infinitely many solutions.

Solve each system by graphing (and show your work). To use the method of graphing to solve a system of two equations in x and y, perform the following steps.

- 1. Solve both equations for y in terms of x.
- 2. Use a graphing utility to graph both equations in the same viewing window.
- 3. Use the intersect feature to obtain the point(s) of intersection.
- 4. Check that each solution satisfies both of the original equations.

1)
$$y = \frac{5}{4}x - 2$$

 $y = \frac{1}{4}x + 2$

2)
$$y = \frac{5}{4}x + 2$$

 $y = \frac{5}{4}x - 2$

3)
$$x + 3y = 12$$

 $5x - 3y = 6$

4)
$$x - 4y = 16$$

 $x + 2y = -2$

5)
$$x + y = -1$$

 $x - y = -3$

6)
$$2x + y = -3$$

 $2x + 3y = 3$

7)
$$x + 2y = 2$$

 $3x - 4y = 16$

8)
$$6x - y = -2$$

 $x - y = 3$

9)
$$3x - 2y = 8$$

 $x - 2y = 4$

10)
$$2x - y = -4$$

 $5x + y = -3$

11)
$$x - y = -2$$

 $x - y = -3$

12)
$$x - 4y = 16$$

 $x + y = 1$

13)
$$x + 4y = -16$$

 $7x - 4y = -16$

14)
$$2x + y = -1$$

 $3x - y = -4$

15)
$$0 = -3 + 3y - 12x$$

 $-8 - 2y = 2x$

16)
$$-y + 1 = -2x$$

 $-3y = -6x - 3$

Solve each system by substitution (and show your work). To use the method of substitution to solve a system of equations in x and y, perform the following steps.

- 1. Solve one of the equations for one variable in terms of the other.
- 2. Substitute the expression found in Step 1 into the other equation to obtain an equation in one variable.
- 3. Solve the equation obtained in Step 2.
- 4. Back-substitute the value(s) obtained in Step 3 into the expression obtained in Step 1 to find the value(s) of the other variable.
- 5. Check that the solution satisfies both of the original equations.

17)
$$x + 4y = -19$$

 $3x - 4y = 7$

18)
$$4x + y = 4$$

 $-12x - 3y = -1$

19)
$$x - 4y = -13$$

 $-x + 4y = 13$

20)
$$x - 8y = -6$$

 $2x - y = -12$

21)
$$4x - 7y = -7$$

 $x + 5y = -22$

22)
$$-x + y = 12$$

 $7x + 8y = -9$

23)
$$x - 8y = 7$$

 $-3x + 24y = 6$

24)
$$7x - 3y = 19$$

 $2x + y = -2$

25)
$$-4x + 5y = 5$$

 $x + 2y = -11$

26)
$$x + 4y = 16$$

 $4x - 2y = 10$

27)
$$x + 8y = 5$$

 $-x - 3y = 0$

28)
$$x + y = -3$$

 $3x + 3y = -9$

29)
$$-5x - 3y = -13$$

 $-3x - 6y = -12$

30)
$$7x - y = -21$$

 $-7x + 6y = 21$

Solve each system of equations using substitution. Show your work.

31)
$$-4x^2 + y^2 + 3x + 105y + 111 = 0$$

 $x + 3y + 4 = 0$

32)
$$2x^2 + 2y^2 - 13x + 2y + 21 = 0$$

 $x + 2y = 1$

33)
$$3x^2 + 19x - 4y + 80 = 0$$

 $2x + y = -3$

34)
$$3x^2 + 3y^2 - 40x + 120 = 0$$

 $x - 3y = 0$

35)
$$2x^2 - 5y^2 + 10x + y + 6 = 0$$

 $3x - y = 1$

36)
$$-x^2 - 12x - 2y - 23 = 0$$

 $2x + y + 4 = 0$

37)
$$x^2 + y^2 - 24x - 2y + 113 = 0$$

 $x - y - 3 = 0$

38)
$$2x^2 + 5y^2 - x + 84y + 190 = 0$$

 $x + y = 2$

39)
$$-5x^2 + 2y^2 - 24x - 48 = 0$$

 $x - y = 0$

40)
$$x^2 + 3y^2 - 54y + 15 = 0$$

 $x - 3y = 3$

41)
$$-6x^2 + y^2 + 3x - 59y - 102 = 0$$

 $x + y - 1 = 0$

42)
$$2x^2 - 4y^2 + 5x - 5y - 98 = 0$$

 $2x - y = 2$

43)
$$2x^2 + 5y^2 + 93x + y - 95 = 0$$

 $3x + y - 3 = 0$

44)
$$5x^2 + 2y^2 + 21x - y + 1 = 0$$

 $x + y - 2 = 0$

Solve by graphing.

45)
$$y = e^x$$

 $x - y + 1 = 0$

46)
$$y = -4e^{-x}$$

 $y + 3x + 8 = 0$

$$47) x + 2y = 8$$
$$y = 2 + \ln x$$

48)
$$y = -2 + \ln(x - 1)$$

 $3x + 2y = 9$

$$49) \quad y = \sqrt{x} + 4$$
$$y = 2x + 1$$

$$50) x - y = 3$$

$$\sqrt{x} - y = 1$$

51)
$$x^2 + y^2 = 169$$

 $x^2 - 8y = 104$

52)
$$x^2 + y^2 = 4$$

 $2x^2 - y = 2$

Think about it.

- 53) A total of \$12,000 is invested in two funds paying 9% and 11% simple interest. The yearly interest is \$1180. How much is invested at each rate?
- 54) A total of \$25,000 is invested in two funds paying 7% and 4% simple interest. The yearly interest is \$1405. How much is invested at each rate?
- 55) A small business invests \$10,000 in equiptment to produce a new soft drink. Each bottle of the soft drink costs \$0.65 to produce and is sold for \$1.20. How many bottles must be sold for the business to break even?
- 56) A small business has an initial investment of \$6,000. The unit cost of the product is \$23.20, and the selling price is \$35.20. How many units must be sold to break even?
- 57) You are offered two jobs selling college textbooks. One company offers an annual salary of \$30,000 plus a year-end bonus of 1% of your total sales. The other company offers an annual salary of \$25,000 plus a year-end bonus of 2% of your total sales. How much would you have to sell in a year to make the second offer the better offer?
- 58) If a system consists of a parabola and a circle, how many solutions can the system have?
- 59) When solving a system by substitution, how can you recognize that the system has no solution?