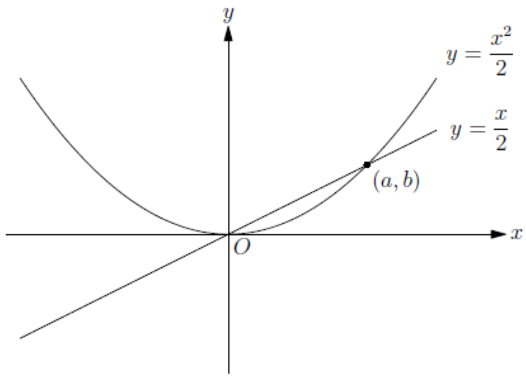
Geometry



1. The curve $y = x^2/2$ and the line $y = x/2$ intersect at the origin and at the point (a, b), as shown in the figure above. What is the value of b?

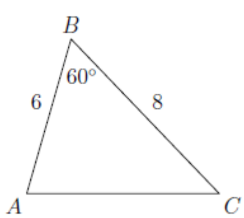
(A) $\frac{1}{8}$

(B) $\frac{1}{4}$

(C) $\frac{1}{2}$

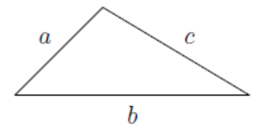
(D) 1

(E) 2



2. In the figure above, $AB = 6$ and $BC = 8$. What is the area of triangle ABC?

(A) $12\sqrt{2}$
(B) $12\sqrt{3}$
(C) $24\sqrt{2}$
(D) $24\sqrt{3}$
(E) $36\sqrt{3}$



Note: Figure not drawn to scale.

3. In the figure above, $3 < a < 5$ and $6 < b < 8$. Which of the following represents all possible values of c?

(A) $0 < c < 3$
(B) $1 < c < 3$
(C) $0 < c < 13$
(D) $1 < c < 13$
(E) $3 < c < 13$

4. Line l goes through points P and Q, whose coordinates are $(0, 1)$ and $(b, 0)$, respectively. For which of the following values of b is the slope of line l greater than $-\frac{1}{2}$?

(A) $\frac{1}{2}$

(B) 1

(C) $\frac{3}{2}$

(D) $\frac{5}{3}$

(E) $\frac{5}{2}$

Answers

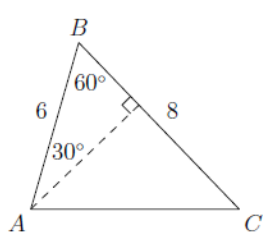
Geometry

1. C (Estimated Difficulty Level: 4)

To determine where two curves intersect, set the equations equal to one another and solve for x. (Hint: know this for the SAT!) In this question, we need to figure out which values of x satisfy: $x^2/2 = x/2$. If $x = 0$, this equation works, but we need the solution when $x \neq 0$. Dividing both sides of the equation by x gives: $x/2 = 1/2$ so that $x = 1$. By plugging in $x = 1$ to either of the two curves, you should find that $y = 1/2$. So, the point of intersection is $(1, 1/2)$, making answer C the correct one.

2. B (Estimated Difficulty Level: 5)

This is the kind of problem that would be too hard and/or require things you aren't expected to know for the SAT (such as trigonometry), *unless* you draw a construction line in the figure. (This is a very hard question anyway.) In this case, you want to draw a line from A perpendicular to the opposite side of the triangle:



This forms a 30-60-90 triangle whose hypotenuse has length 6. Now, use the 30-60-90 triangle diagram given to you at the beginning of each SAT math section: The length of the side opposite the 30° angle is 3, and the length of the side opposite the 60° angle (the dashed line) is $3\sqrt{3}$.

So finally, if the base of the triangle is segment $\overline{BC}$, then the dashed line is the height of the triangle, and the area of the triangle is $(1/2) \cdot 8 \cdot 3\sqrt{3} = 12\sqrt{3}$.

3. D (Estimated Difficulty Level: 5)

You need to know the "third-side rule" for triangles to solve this question: *The length of the third side of a triangle is less than the sum of the lengths of the other two sides and greater than the positive difference of the lengths of the other two sides.* Applied to this question, the first part of the rule says that the value of c must be less than $a + b$. Since we are interested in *all* possible values of c, we need to know the greatest possible value of $a + b$. With $a < 5$ and $b < 8$, $a + b < 13$ so that c must be less than 13.

For the second part of the rule, c must be greater than $b - a$. (Note that b is always bigger than a, so that $b - a$ is positive.) We are interested in *all* possible values of c, so we need to know the least possible value of $b - a$. The least value occurs when b is as small as possible and a is as large as possible: $b - a > 6 - 5 = 1$. Then, c must be greater than 1. Putting this together, $1 < c < 13$, making answer D the correct one.

4. E (Estimated Difficulty Level: 4)

First, calculate the slope of line l using the given points:

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{0-1}{b-0} = -\frac{1}{b}.$$

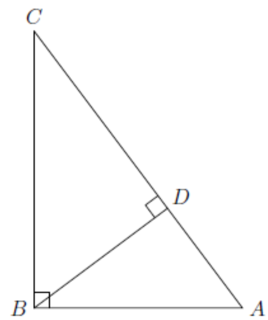
At this point, a good approach is to work with the answers by plugging them into the expression for slope above until you get a value greater than $-1/2$. For example, using answer A gives a slope of $-1/(1/2) = -2$, which is not greater than $-1/2$, so answer A is incorrect. You should find that answer E is the correct one, since $-1/(5/2) = -2/5$ is greater than $-1/2$. (Knowing the decimal equivalents of basic fractions will really help speed this process up.)

Here is the algebraic solution:

$$-\frac{1}{b} > -\frac{1}{2} \quad \Rightarrow \quad \frac{1}{b} < \frac{1}{2} \quad \Rightarrow \quad b > 2.$$

(Remember to flip the inequality when multiplying by negative numbers or when taking the reciprocal of both sides.) Only answer E makes $b > 2$.

Geometry



5. In the figure above, $AB = 6$ and $BC = 8$. What is the length of segment $\overline{BD}$?

(A) 2

(B) $\frac{12}{5}$

(C) 4

(D) $\frac{24}{5}$

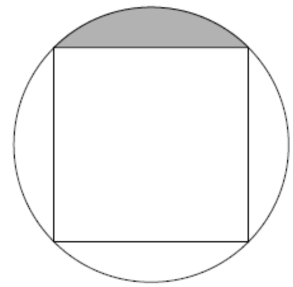
(E) 6

6. If four distinct lines lie in a plane, and exactly two of them are parallel, what is the least possible number of points of intersection of the lines?

(A) Two
(B) Three
(C) Four
(D) Five
(E) More than five

7. The perimeter of a particular equilateral triangle is numerically equal to the area of the triangle. What is the perimeter of the triangle?

(A) 3
(B) 4
(C) $4\sqrt{3}$
(D) $12\sqrt{3}$
(E) $18\sqrt{3}$



8. In the figure above, a square is inscribed in a circle. If the area of the square is 36, what is the perimeter of the shaded region?

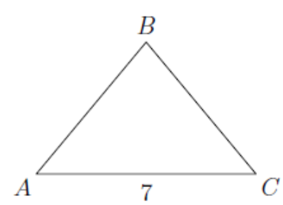
(A) $6 + \frac{3\sqrt{2}}{2}\pi$

(B) $6 + 3\pi$

(C) $6 + 3\sqrt{2}\pi$

(D) $36 + 6\sqrt{2}\pi$

(E) $\frac{9}{2}\pi - 9$



Note: Figure not drawn to scale.

9. In the figure above, $AC = 7$ and $AB = BC$. What is the smallest possible integer value of AB?

Answers

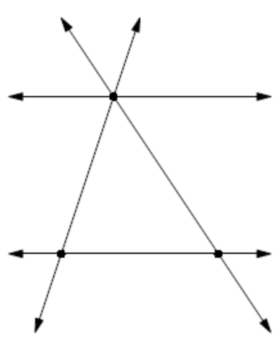
5. D (Estimated Difficulty Level: 5)

Since ABC is a right triangle, the length of segment $\overline{AC}$ is $\sqrt{6^2+8^2}=10$. (Hint: you will see 3-4-5 and 6-8-10 triangles a lot on the SAT.) The area of triangle ABC is $(1/2)\cdot b\cdot h$, where b is the base and h is the height of triangle ABC.

The key thing to remember for this problem is that the base can be *any* of the three sides of a triangle, not just the side at the bottom of the diagram. If the base is AB, then the height is BC and the area of the triangle is $(1/2)(6)(8)=24$. If the base is AC, then the height is BD and the area of the triangle is still 24. This means that $(1/2)(AC)(BD)=(1/2)(10)(BD)=24$ so that $BD=24/5$.

6. B (Estimated Difficulty Level: 4)

Draw a diagram for this problem! With exactly two parallel lines, the other two lines cannot be parallel to themselves or to the first two lines. Your diagram may seem to suggest five points of intersection; however, the point of intersection of the two non-parallel lines can overlap with a point of intersection on one of the parallel lines:



From the figure, the least possible number of intersection points is then three.

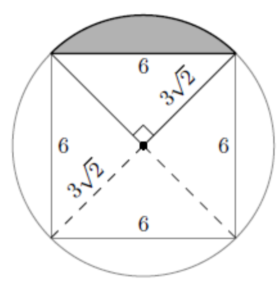
7. D (Estimated Difficulty Level: 5)

One formula that a good math student such as yourself may want to memorize for the SAT is the area of an equilateral triangle. If the length of each side of the triangle is s, then the area is $\sqrt{3}s^2/4$. The perimeter of this triangle is $3s$.

Now, since the perimeter equals the area for this triangle, we have: $3s=\sqrt{3}s^2/4$ so that $3=\sqrt{3}s/4$ and $s=12/\sqrt{3}=4\sqrt{3}$. The perimeter is then $12\sqrt{3}$, making answer D the correct one. (Did you get $s=4\sqrt{3}$ and then choose answer C? Sorry about that.)

8. A (Estimated Difficulty Level: 5)

For many difficult SAT questions, it can be very helpful to know some "extra" math along with the "required" math. First, when a square is inscribed in a circle, the diagonals are diameters of the circle. Second, the diagonals of a square meet at right angles. Third, a diagonal of a square is $\sqrt{2}$ times as long as the length of one of the sides. (A diagonal of a square makes a 45-45-90 triangle with two sides.) For this question, the length of each side of the square is 6 (since the area is $6^2=36$), and the length of a diagonal is $6\sqrt{2}$, so the radius of the circle is $3\sqrt{2}$, as shown below:



A final piece of needed math: the arc length of a portion of a circle is the circumference times the central angle of the arc divided by 360°. Here, the central angle is 90°, so the needed arc length (shown darkened in the figure above) is just 1/4 times the circle's circumference. The arc length is then $2\pi r/4=2\pi\cdot3\sqrt{2}/4=3\pi\sqrt{2}/2$ and the perimeter of the shaded region is $6+3\pi\sqrt{2}/2$.

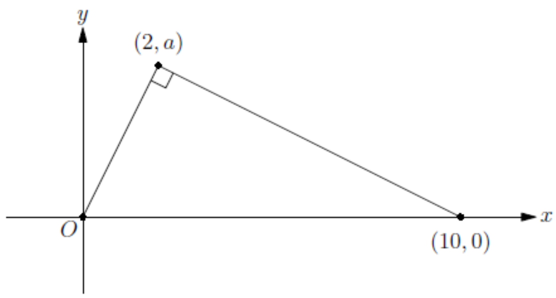
Answers

9. 4 (Estimated Difficulty Level: 5)

You need to know half of the "third-side rule" for triangles to solve this question: *The length of the third side of a triangle is less than the sum of the lengths of the other two sides.* For this question, we will make AC the third side.

Now, suppose that the length of each of the other two sides of the triangle is x, so that $AB = BC = x$. Then, the third-side rule says that AC is less than the sum of AB and BC: $7 < x + x$. Simplifying gives: $2x > 7$ so that $x > 3.5$. The smallest possible integer value for x is 4.

Geometry



10. In the figure above, two line segments in the x-y plane form a right triangle with the x-axis. What is the value of a?

(A) $2\sqrt{2}$
(B) 4
(C) 5
(D) $4\sqrt{2}$
(E) $5\sqrt{2}$

11. The perimeter of square $ABCD$ is x, and the perimeter of isosceles triangle EFG is y. If $AB = EF = FG$, which of the following must be true?

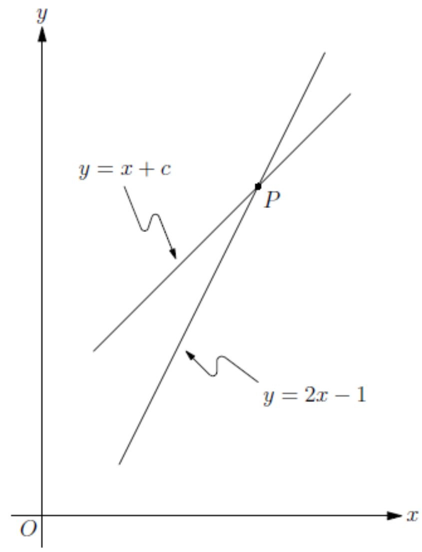
(A) $0 < y < \frac{x}{4}$

(B) $\frac{x}{4} < y < \frac{x}{2}$

(C) $\frac{x}{2} < y < x$

(D) $x < y < 2x$

(E) $2x < y < 4x$



Note: Figure not drawn to scale.

12. In the x-y plane, the lines $y = 2x - 1$ and $y = x + c$ intersect at point P, where c is a positive number. Portions of these lines are shown in the figure above. If the value of c is between 1 and 2, what is one possible value of the x-coordinate of P?

Answers

10. B (Estimated Difficulty Level: 5)

One way to do this question is to use the fact that the product of the slopes of two perpendicular lines (or line segments) is -1. The slope of the line segment on the left is $(a-0)/(2-0) = a/2$. The slope of the line segment on the right is $(0-a)/(10-2) = -a/8$. The two slopes multiply to give -1:

$$\frac{a}{2} \cdot \frac{-a}{8} = -\frac{a^2}{16} = -1.$$

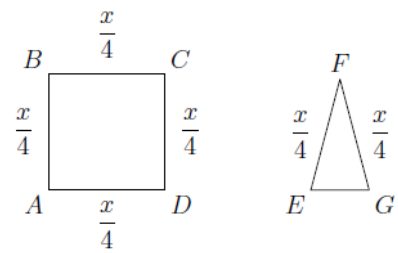
Solving for a gives $a^2 = 16$ so that $a = 4$. A messier way to do this problem is to use the distance formula and the Pythagorean theorem. The length of the line segment on the left is $\sqrt{2^2 + a^2}$, and the length of line segment on the right is $\sqrt{(10-2)^2 + (0-a)^2}$. Then, the Pythagorean theorem says that:

$$\left(\sqrt{2^2 + a^2}\right)^2 + \left(\sqrt{(10-2)^2 + (0-a)^2}\right)^2 = 10^2.$$

Simplifying the left-hand side gives: $2a^2 + 68 = 100$ so that $2a^2 = 32$. Then, $a^2 = 16$, making $a = 4$.

11. C (Estimated Difficulty Level: 5)

Make a diagram, and fill it in with the information that is given. (You should do this for any difficult geometry question without a figure.) Since the perimeter of square $ABCD$ is x, each side of the square has length $x/4$, so your figure should look something like this:



Now, use the third-side rule for triangles: *The length of the third side of a triangle is less than the sum of the lengths of the other two sides and greater than the positive difference of the lengths of the other two sides.* When the rule is applied to EG as the third side, we get: $0 < EG < x/2$. If y is the perimeter of the triangle, then $y = x/4 + x/4 + EG = x/2 + EG$. Solving for EG gives $EG = y - x/2$. Substituting into the inequality gives $0 < y - x/2 < x/2$ so that $x/2 < y < x$, making answer C the correct one. To make this problem less abstract, it may help to make up a number for the perimeter of the square. (A good choice might be 4 so that $x = 1$. You'll find $1/2 < y < 1$, the same as answer C when $x = 1$.)

12. $2 < x < 3$ (Estimated Difficulty Level: 5)

In order to determine at what point two lines intersect, set the equations of the lines equal to one another. In this case, we have: $2x - 1 = x + c$ so that $x = c + 1$. In other words, $x = c + 1$ is the x-coordinate of P, the point where the lines intersect. Now, if c is between 1 and 2, then $c + 1$ is between 2 and 3. Any value for the x-coordinate of P between 2 and 3 is correct.